Sudden Stops with Heterogeneous Agents^{*}

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Abstract

This paper develops a heterogeneous agent model of a small open economy and studies how households differ in their responses to aggregate productivity and interest rate shocks. Poor households display stronger consumption responses to an aggregate productivity shock because they are more likely to be constrained in liquid assets. In contrast, rich households display stronger consumption responses to an interest rate shock because they are more likely to be unconstrained in liquid assets. When the economy experiences a sudden stop, defined as transitory contractionary shocks to productivity and the interest rate, the interest rate effect neutralizes the productivity effect. As a consequence, the sudden stop generates consumption-income elasticities that display little variation along the income distribution, similar to a permanent shock. My finding captures the observed behavior of households in the Mexican Peso Crisis of 1994. (*JEL* D31, E21, E32, F32, F41)

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1 Introduction

Emerging markets frequently experience sudden stops: sharp recessions that coincide with a reversal of the trade balance. An open question is what drives sudden stops. Two candidates are unexpected, transitory shocks that throw the economy off balance and permanent shocks to the fundamentals of the economy. Until recently, these shocks have only been considered in the context of representative agent (RA) models. This paper investigates the ability of transitory shocks and permanent shocks to explain sudden stops through the lens of a heterogeneous agent (HA) model. Using the HA model, I study the ability of each shock to match the data along two dimensions. At the household level, I study the ability of each shock to match the consumption responses of households during the Mexican Peso Crisis of 1994, documented in Guntin et al. (2023). At the aggregate level, I study the ability of each shock to match the signature features of the Mexican Peso Crisis: a sharp drop in investment and a reversal of the trade balance.

To this end, I build a heterogeneous agent small open economy (HASOE) model in which households face idiosyncratic income risk, limited access to financial markets, and store the vast majority of their wealth in illiquid assets. I consider two approaches to generate a sudden stop. Under the first approach, the economy experiences a transitory decline in productivity that coincides with a transitory increase in the interest rate. This captures the procyclical nature of interest rates in emerging markets, documented in Kaminsky et al. (2004): in bad times, emerging markets typically face an increase in the interest rate which increases the cost of smoothing aggregate fluctuations.¹ Under the second approach, the economy experiences a permanent decline in productivity, as in Aguiar and Gopinath (2007) and Guntin et al. (2023).² I show that each approach is able to recreate the consumption responses observed in the household data. In contrast, the transitory shocks generate a stronger decline in investment and reversal of the trade balance that characterize the Mexican Peso Crisis. This occurs because the increase in the interest rate motivates households to substitute from capital into the external bond of the economy.

For the permanent shock, the intuition of the representative agent benchmark of Aguiar and Gopinath (2007) holds at the household level: because households cannot sustain their consumption level given a permanent decline in income, their optimal choice is to immediately decrease their consumption with income.³ This holds for both low income households that display a large response to transitory income fluctuations,

¹For further discussion of procyclical interest rates, see Calvo (1998), Calvo and Reinhart (2002), and Calvo et al. (2006). Within my case study of the Mexican Peso Crisis, I explicitly motivate the interest rate increase using the data.

²Guntin et al. (2023) differs in that it features a permanent decline in an income endowment. Within my model, a permanent decline in productivity generates a permanent decline in the general equilibrium labor income endowment that households receive. This paper abstracts from more sophisticated models of highly persistent declines in productivity, like that of Queralto (2020).

 $^{^{3}}$ How strongly consumption decreases depends on an individual household's financial position. In the model, unconstrained households that behave more like permanent income consumers respond to the long run decline in income. In contrast, constrained households that would like to consume more only respond to the immediate decline in income.

and high income households that display a smaller response to transitory income fluctuations.

The ability of the transitory approach to generate the observed consumption responses is more nuanced. Because they are likely to be financially constrained, low income households display a large consumption response to the transitory decline in labor income and a small consumption response to the increase in the interest rate generated by the transitory approach. Because they are more likely to hold liquid assets, high income households display a smaller consumption decline to a temporary decline in income. However, precisely because they hold liquid assets, the increase in the interest rate incentivizes high income households to increase their liquid asset position, financed through a decrease in consumption. When the two effects are added together, the stronger response to the interest rate overcomes the weaker response to income, so that high income households display a weakly larger consumption response than low income households, as is observed in the data. This occurs despite substantial heterogeneity in income, wealth, and access to financial markets.

The two shocks differ in their implications for aggregate variables. The transitory shocks generate a sudden stop which features a sharp decline in investment and reversal of the trade balance, as is observed in the data.⁴ The permanent shock generates a similar decline in consumption, but weaker responses of investment and the trade balance. The stronger investment decline experienced during the transitory shocks occurs due to increase in the interest rate, which motivates households to substitute from domestic capital to the external bond of the economy.⁵ In the context of the Mexican Peso Crisis, this supports the episode as being driven by the procyclical increase in the interest rate rather than a permanent decline in productivity.

The household problem of the model features a two asset environment similar to that of Kaplan et al. (2014), Kaplan et al. (2018), and Hong (2023a). Households have access to a liquid bond and an illiquid asset that is subject to convex adjustment costs. Both assets are subject to a non-negativity constraint. While the non-negativity constraint prohibits borrowing, it generates a large portion of low income households that hold neither debts nor savings, aligning with the data.⁶ I compare the two asset model to a traditional single asset model. The two asset model improves on the single asset model along two dimensions. First, the two asset model can capture both the portion of constrained households and the average MPC of households. The former governs how many households display a substitution effect to an interest rate change whereas the latter governs how responsive households are to income fluctuations. Secondly, the two asset model

⁴Within the Mexican Peso Crisis there exists a disconnect between the aggregate and household data. Because I target nondurable consumption, the household data features a consumption to income response of no more than one to one. This contrasts with the aggregate data that features a more than one to one consumption to GDP response. Following the national expenditure decomposition of GDP, the overreaction of aggregate consumption partially supports a reversal of the trade balance. Because I target the household consumption response, which does not overreact relative to income, both the transitory and permanent approaches generate weaker trade balance reversals relative to the reversal observed in the data.

 $^{^{5}}$ Within the resource constraint of the economy, this is expressed as an explicit tradeoff between investment and the trade balance.

⁶I support this using household data from the Mexican Family Life Survey.

features a weaker relationship between income and the likelihood of being financially constrained, which is overstated in the single asset model. Because the single asset model overstates this relationship, the lowest income decile displays a one to one consumption response for even a temporary decrease in income. This leads the single asset model to overstate heterogeneity in households responses for both a change in income and a change in the interest rate.

I motivate this paper using four stylized facts from the data. First, I characterize the Mexican Peso Crisis at the aggregate level. Second, I discuss the procyclical nature of the interest rate in emerging markets, which features most prominently in Uribe and Yue (2006) and Neumeyer and Perri (2005). Third, closely following the empirical work of Guntin et al. (2023), I replicate that high income households display a weakly larger consumption response than low income households during the Mexican Peso Crisis using the Mexican National Survey of Household Income and Expenditure (ENIGH).⁷ Lastly, I document that access to financial markets increases with income using the Mexican Family Life (MFL) survey.

Literature This paper contributes to a variety of literatures. First, I contribute to the literature that studies the drivers of emerging markets, which includes Neumeyer and Perri (2005), Uribe and Yue (2006), Aguiar and Gopinath (2007), García-Cicco et al. (2010), and Chang and Fernandez (2013).⁸ I contribute to this literature by studying transitory shocks and permanent shocks through the lens of an HA model and comparing their ability to match both the aggregate and household data. My analysis supports interest rates as a driver of emerging markets, although I focus on a crisis episode rather than the business cycle.

I contribute to the literature that studies sudden stops in emerging markets, which includes contributions from Calvo (1998), E. G. Mendoza (2002), E. Mendoza and Smith (2004), Calvo et al. (2006), E. G. Mendoza (2010), and Bianchi and Mendoza (2020). Contributions to the nascent HA literature include Cugat (2019), Guntin et al. (2023), and Villalvazo (2023). Similar to Cugat (2019), my model is able to replicate the consumption response of the sudden stop because a significant portion of households lack access to financial markets.⁹ My model differs from Cugat (2019) in that the inability to consumption smooth using a bond is generated endogenously through an occasionally binding constraint and idiosyncratic income risk, whereas Cugat (2019) features hand to mouth households like those of Campbell and Mankiw (1989). In addition, I abstract from heterogeneity in household employment in the tradeable and nontradeable sectors. This paper differs from Villalvazo (2023) in that it focuses on heterogeneity along the income distribution rather than the wealth distribution and focuses on the Mexican Peso Crisis rather than the Global Financial Crisis. In

⁷The primary measure of consumption responses, also used in this paper, is the consumption-income elasticity, the percentage change in consumption divided by the percentage change in income.

⁸García-Cicco et al. (2010) include both permanent shocks and interest rate shocks in a small open economy model. Their analysis finds a small contribution of permanent shocks.

 $^{^{9}}$ This differs from the typical collateral constraint and eventual Fisherian deflation that generates a strong consumption decline in RA models.

addition, my model lacks the boom bust episodes that Villalvazo (2023) generates through aggregate risk.¹⁰

This paper contributes to the literature that studies the distributional effects of aggregate shocks. Contributions, which primarily focus on interest rate fluctuations, include Auclert (2019), Di Maggio et al. (2017), and Amberg et al. (2022). Contributions specific to small open economies include de Ferra et al. (2020), Guo et al. (2021), Zhou (2022), Guntin et al. (2023), and Oskolkov (2023). I contribute to this literature by studying the heterogeneous effects of aggregate productivity and interest rate shocks.

While similar in objectives, this paper contrasts most strongly with Guntin et al. (2023), who interpret the weak relationship between consumption-income elasticities and income as supporting a permanent decline in income.¹¹ I show that a two asset model with a realistic combination of transitory productivity and interest rate shocks can match the observed consumption-income elasticities of the data. In addition, my model can comment on the larger dynamics of the sudden stop because it features a production sector.

Unlike Hong (2023a), the interest rate plays a critical role in my model. This occurs because I do not introduce the debt-elastic interest rate studied in Schmitt-Grohé and Uribe (2003).¹² Hong (2023a)'s Bayesian estimation produces a strong countercyclical relationship between the aggregate debt position of the economy and the interest rate which nullifies the contribution of exogenous interest rate shocks. Instead, I allow for complete pass through of interest rates shocks to households and simply match the aggregate consumption responses. This allows for a significant response of unconstrained households to the interest rate increase which drives the sudden stop.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 presents the calibration. Section 4 studies the ability of the transitory and permanent approaches to match the Mexican Peso Crisis of 1994. Section 5 presents stylized facts from the aggregate and household data. Section 6 concludes.

2 Model

This section describes the model. Time is discrete and infinite. The model is a small open economy that saves on the international market. The economy is populated by a representative firm and a unit continuum of households that are heterogeneous in income and asset holdings. The consumption good is produced by the representative firm using a combination of capital, managed by the firm, and labor, hired from a labor

 $^{^{10}}$ Because Villalvazo (2023) uses a global model, the economy features a buildup preceding the sudden stop similar to the RA model of E. G. Mendoza (2010). The use of a global solution method is out of the scope of this paper because this paper needs a rich distribution of income, whereas global HA models typically use binary low and high income states.

¹¹Within the model, I generate a permanent decline in income by introducing a permanent decline in the productivity of the representative firm.

 $^{^{12}}$ An original motivation of debt-elastic interest rates is to induce stationarity in perturbed RA models. This is not necessary in my model as stationarity is induced by the non-negativity constraint of the bond.

union that represents the households. Household heterogeneity is generated by idiosyncratic income risk. Households have access to two assets: a liquid asset that provides a certain return and an illiquid asset that is subject to convex adjustment costs.. Both the liquid and illiquid assets are subject to non-negativity constraints. Aggregate savings from the liquid asset are saved on the international market. The illiquid asset is shares of the representative firm.

The rest of this section is organized as follows. Section 2.1 describes the household problem. Section 2.2 describes the production side of the economy. Section 2.3 defines market clearing, and Section 2.4 defines the perfect foresight equilibrium.

2.1 Households

This section describes the household problem. The economy is populated by a unit continuum of households, indexed by *i*. Household *i* has preferences over infinite streams of the consumption good $\{c_t^i\}_{t=0}^{\infty}$ given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^i) \tag{1}$$

where $0 < \beta < 1$ is the discount factor. In each period, household *i* must satisfy the budget constraint

$$c_t^i + b_t^i + a_t^i + \chi(a_t^i, a_{t-1}^i) = (1 + r_t^b)b_{t-1}^i + (1 + r_t^a)a_{t-1}^i + e_t^i w_t L_t$$
(2)

where b_t^i denotes the liquid asset and a_t^i denotes the illiquid asset, both in terms of the consumption good, and $\chi(\cdot, \cdot) \geq 0$ is a convex adjustment cost function.¹³ At time t, the returns of the liquid and illiquid assets are given by r_t^b , which is known in period t - 1, and r_t^a , which is determined in period t. Aggregate labor income is given by $w_t L_t$, and household *i*'s individual labor income is given by $e_t^i w_t L_t$, where e_t^i is mean one, exogenous and follows a known stochastic process. In each period, household *i*'s asset holdings are subject to the non-negativity constraints

$$b_t^i \ge 0,\tag{3}$$

and
$$a_t^i \ge 0,$$
 (4)

respectively.¹⁴

¹³I use the convention that b_t^i denotes liquid asset holdings between period t and t + 1, and similarly for a_t^i .

¹⁴The use of a non-negativity constraint rather than a more generous constraint that allows for borrowing does not have a material effect on the ability of the model to generate constrained households that display a significant marginal propensity to consume. It does have a significant effect on the heterogeneity in responses to interest rate shocks because there are no constrained debtors that display negative wealth effects to an interest rate increase.

When solving their problem, households take as given the path of aggregate labor income $\{w_t L_t\}_{t=0}^{\infty}$, the path of the interest rate $\{r_t^b\}_{t=0}^{\infty}$, and the path of the illiquid asset return $\{r_t^a\}_{t=0}^{\infty}$. I collect these as $\Gamma = \{w_t, L_t, r_t^b, r_t^a\}_{t=0}^{\infty}$, which I refer to as the 'household inputs'.¹⁵ Taking Γ as given, household *i* chooses the paths of their consumption $\{c_t^i\}_{t=0}^{\infty}$, liquid asset holdings $\{b_t^i\}_{t=0}^{\infty}$, and illiquid asset holdings $\{a_t^i\}_{t=0}^{\infty}$ to maximize (1) subject to the budget constraint (2) and the non-negativity constraints (3) and (4).

When solving their problem, households face idiosyncratic income risk. For a generic household, the idiosyncratic income component e satisfies

$$\log e_t = \rho_e \log e_{t-1} + \sigma_e \epsilon^e_t, \epsilon^e_t \sim \mathcal{N}(0, 1), \tag{5}$$

where $0 \le \rho_e < 1$ is the autocorrelation of idiosyncratic income risk, σ_e is its standard deviation, and $\mathcal{N}(0, 1)$ is the standard normal distribution.

Dropping i, we can compute the first order conditions

$$u(c_t) = \mu_t^b + \beta E_t (1 + r_{t+1}^b) u(c_{t+1}), \tag{6}$$

$$u(c_t)(1+\chi_1(a_t, a_{t-1})) = \mu_t^a + \beta E_t(1+r_{t+1}^a - \chi_2(a_{t+1}, a_t))u(c_{t+1}),$$
(7)

where μ_t^b is the Lagrange multiplier of the liquid asset non-negativity constraint (3), and μ_t^a is the Lagrange multiplier of the illiquid asset non-negativity constraint (4). Equation (6) denotes the Euler equation of the liquid asset and equation (7) denotes the Euler equation of the illiquid asset. In addition, households satisfy the conditions

$$\mu_t^b \ge 0, \ \mu_t^a \ge 0, \ \mu_t^b b_t = 0, \text{ and } \mu_t^a a_t = 0.$$
 (8)

Solving the household problem produces a series of policies $\{c_t(e, b, a; \Gamma)\}_{t=0}^{\infty}, \{b_t(e, b, a; \Gamma)\}_{t=0}^{\infty}, \{a_t(e, b, a; \Gamma)\}_{t=0}^{\infty}\}_{t=0}^{\infty}$ that depend on the entire path of households inputs Γ .

The distribution of households at time t is described by the cumulative density function (CDF) Ψ_t , where

$$\Psi_t(e, a, b; \Gamma) = Pr(e_t \le e, a_{t-1} \le a, b_{t-1} \le b; \Gamma).$$

$$\tag{9}$$

The distribution function Ψ_t satisfies the law of motion

$$\Psi_{t+1}(e',b',a';\Gamma) = \int_{e,b,a} Pr(e_{t+1} \le e'|e_t = e) \mathcal{I}\left[a_t(e,b,a;\Gamma) \le a', b_t(e,b,a;\Gamma) \le b'\right] d\Psi_t(e,b,a;\Gamma), \quad (10)$$

¹⁵In practice, the household problem only depends on $w_t L_t$ and not w_t and L_t separately.

where \mathcal{I} is the indicator function and $b_t(e, b, a; \Gamma)$ and $a_t(e, b, a; \Gamma)$ denote a household's policies in period t as a function of their idiosyncratic income e and asset positions b, and a. At time t, aggregate consumption, liquid assets, illiquid assets, and adjustment costs are defined as

$$C_t = \int_{e,b,a} c_t(e,b,a;\Gamma) d\Psi_t(e,b,a;\Gamma),$$
(11)

$$B_t = \int_{e,b,a} b_t(e,b,a;\Gamma) d\Psi_t(e,b,a;\Gamma), \qquad (12)$$

$$A_t = \int_{e,b,a} a_t(e,b,a;\Gamma) d\Psi_t(e,b,a;\Gamma),$$
(13)

and
$$\chi_t = \int_{e,b,a} \chi(a_t(e,b,a;\Gamma),a) d\Psi_t(e,b,a;\Gamma),$$
 (14)

respectively.¹⁶

At time t, I define the marginal propensity to consume (MPC) out of liquid assets at household position (e, b, a) as

$$MPC_t(e, b, a; \Gamma) = \frac{c_t(e, b + \epsilon, a; \Gamma) - c_t(e, b, a; \Gamma)}{\epsilon}$$
(15)

for a small $\epsilon > 0.17$

2.2Production

The consumption good is produced by a representative firm using capital and labor. The firm maximizes

$$E_0 \sum_{t=0}^{\infty} Q_{0,t} \pi_t \tag{16}$$

where $\{Q_{0,t}\}_{t=0}^{\infty}$ is a discount factor and $\{\pi_t\}_{t=0}^{\infty}$ is the path of dividends distributed to equity owners. The discount factor $Q_{0,t}$ is given by

$$Q_{0,t} = \begin{cases} 1 & t = 0\\ \Pi_{s=0}^{t} \frac{1}{1+r_s^a} & t > 0, \end{cases}$$
(17)

where $\{r_t^a\}_{t=0}^{\infty}$ is the path of the illiquid asset return. The firm generates the consumption good using the Cobb-Douglass production function

$$Y_t = z_t K_{t-1}^{\alpha} L_t^{1-\alpha} \tag{18}$$

¹⁶Because the mass of households is size one, aggregates of household variables coincide with the mean of household variables. ¹⁷When solved in the discretized state space, the linear approximation of the MPC at point (e, b_j, a) is given by MPC $(e, b_j, a) = 1$ $\frac{c(e,b_{j+1},a)-c(e,b_j,a)}{b_{j+1}-b_j}$ where the liquid asset grid takes the form $\{\ldots,b_{j-1},b_j,b_{j+1},\ldots\}$.

where z_t is the firm's productivity at time t, K_{t-1} is the capital stock chosen in period t-1, and L_t is labor. The budget constraint of the firm is given by

$$\pi_t + I_t + \Phi(K_t, K_{t-1}) = z_t K_{t-1}^{\alpha} L_t^{1-\alpha} - w_t L_t$$
(19)

$$I_t = K_t - (1 - \delta) K_{t-1} \tag{20}$$

where w_t is the market wage for labor, $\Phi(\cdot, \cdot) \ge 0$ is a convex adjustment cost function, and δ is the depreciation rate of capital.

Given the path of productivity and the discount factor $\{z_t, Q_{0,t}\}_{t=0}^{\infty}$, the firm chooses dividends, capital, and labor $\{\pi_t, K_t, L_t\}_{t=0}^{\infty}$, to maximize (16) subject to the discount factor (17), given the constraints (19) and (20). The first order condition with respect to capital and labor are given by

$$(1 + r_{t+1}^{a})(1 + \Phi_{1}(K_{t}, K_{t-1})) = E_{t}\left(z_{t+1}\alpha K_{t}^{\alpha}L_{t+1}^{1-\alpha} + 1 - \delta - \Phi_{2}(K_{t+1}, K_{t})\right),$$
(21)

and
$$w_t = z_t (1 - \alpha) K_{t-1}^{\alpha} L_t^{-\alpha}$$
, (22)

respectively.

The quantity of equity shares is normalized to one. Given the path of the price of equity shares $\{q_t\}_{t=0}^{\infty}$, the gross return on equity is given by

$$1 + r_t^a = \frac{q_t + \pi_t + \chi_t}{q_{t-1}}.^{18}$$
(23)

Here the illiquid asset adjustment costs are reimbursed with firm profits.¹⁹

Following Hong (2023a), labor is supplied at the aggregate level by a labor union. Taking the market wage w_t as given, the labor union solves

$$\max_{L_t} w_t L_t - \kappa \frac{1}{1+\omega} (L_t)^{1+\omega} \tag{24}$$

where $\kappa > 0$. This provides the labor supply curve²⁰

$$w_t = \kappa(L_t)^{\omega}.\tag{25}$$

¹⁸Starting with $q_0 = \frac{q_1 + \pi_1 + \chi_1}{1 + r_1^a} = \frac{\pi_1 + \chi_1}{1 + r_1^a} + \frac{q_2 + \pi_2 + \chi_2}{1 + r_2^a}$ and iterating forward, we find that $q_0 = \sum_{t=0}^{\infty} \frac{\pi_t + \chi_t}{1 + r_t^a}$. This reveals that the firm maximizes $\pi_0 + q_0 + \chi_0$, the present value of the firm.

 $^{^{19}}$ This removes illiquid asset adjustment costs from the resource constraint so that output follows the standard decomposition into consumption, investment, and the trade balance.

2.3 Prices and Clearing

The interest rate on the bond is given by

$$r_t = r^* + \mu_t - 1, \tag{26}$$

where $r^* > 0$ is the steady state interest rate and μ_t is an exogenous shock that equals one at the steady state. The realized return on liquid asset holdings at time t is known one period in advance:

$$r_t^b = r_{t-1}.$$
 (27)

Given a firm share price of q_t , the clearing condition of the illiquid asset market is given by

$$A_t = q_t, \tag{28}$$

which equalizes the value of equity shares held by households and the market value of the firm. The trade balance is given by

$$TB_t = B_t - (1 + r_t)B_{t-1} \tag{29}$$

and the trade balance to output ratio is given by

$$TBY_t = TB_t/Y_t.$$
(30)

Integrating over the household budget constraint (2) with respect to Ψ_t , applying the illiquid asset clearing condition (28), and the definitions of the trade balance (29) and investment (20) provides the aggregate resource constraint

$$Y_t = C_t + I_t + \Phi_t + TB_t, \tag{31}$$

which decomposes output into consumption, investment, and the trade balance.

2.4 Equilibrium

Decentralized Equilibrium. Given the path of productivity and the deviation of the interest rate $\{z_t, \mu_t\}_{t=0}^{\infty}$, a decentralized equilibrium is a path of prices $\{w_t, r_t, r_t^b, r_t^a, q_t\}_{t=0}^{\infty}$,

²⁰This is equivalent to individual households having Greenwood-Hercowitz-Huffman preferences $u(c, l) = \frac{c^{1-\gamma}}{1-\gamma} - \kappa \frac{1}{1+\omega} l_t^{1+\omega}$.

a path of household policies $\{c_t(e, b, a; \Gamma), b_t(e, b, a; \Gamma), a_t(e, b, a; \Gamma)\}_{t=0}^{\infty}$, a path of the distribution of households $\{\Psi_t\}_{t=0}^{\infty}$, and a path of quantities $\{C_t, A_t, B_t, \chi_t, Y_t, \pi_t, K_t, L_t, I_t, TB_t, TBY_t\}_{t=0}^{\infty}$ such that, given $\Gamma = \{w_t, L_t, r_t^b, r_t^a\}_{t=0}^{\infty}$:

- 1. $\{c_t(e, b, a; \Gamma), b_t(e, b, a; \Gamma), a_t(e, b, a; \Gamma)\}_{t=0}^{\infty}$ satisfy conditions (2), (6) (8).
- 2. The distribution $\{\Psi_t(e, b, a; \Gamma)\}_{t=0}^{\infty}$ follows the law of motion given by equation (10).
- 3. $\{Y_t, \pi_t, K_t, w_t, L_t, I_t, TB_t, TBY_t\}_{t=0}^{\infty}$ satisfy (18), (19), (21), (22), (25), (20), (29), and (30) respectively.
- 4. $\{C_t, A_t, B_t, \chi_t\}_{t=0}^{\infty}$ are given by the aggregation equations (11) (14).
- 5. The interest rates $\{r_t, r_t^b\}_{t=0}^{\infty}$ are given by equations (26) and (27), respectively, and the illiquid asset return $\{r_t^a\}_{t=0}^{\infty}$ is given by (23).
- 6. The illiquid asset clearing condition (28) is satisfied.

2.5 Solution Method

I solve for the solutions to perfect foresight shocks using the methods and toolkit developed in Auclert et al. (2021). In every exercise, I assume the economy is initially at the stationary steady state that arises with the presence of idiosyncratic income risk. I consider both 'transitory' shocks and 'permanent' shocks. Under a transitory shock, both z_t and μ_t returns to their initial steady state values. Under a permanent shock, z_t transitions to a new long run steady state. For the transitory shock, I assume the economy returns to the stationary steady state within T = 400 periods. For the permanent shock, I compute the new long run steady state and compute the transition path to the new steady state. Given $\{z_t, \mu_t\}_{t=0}^T$, I solve for the equilibrium by iterating on the path of the illiquid asset return $\{r_t^a\}_{t=0}^T$ to satisfy the illiquid asset market clearing condition (28).²¹ To inspect the household problem, I compute the path of the distribution of households $\{\Psi_t(e, b, a; \Gamma)\}_{t=0}^T$ and household policies $\{c_t(e, b, a; \Gamma)\}_{t=0}^T$, $\{a_t(e, b, a; \Gamma)\}_{t=0}^T$, and household policies $\{c_t(e, b, a; \Gamma)\}_{t=0}^T$, $\{a_t(e, b, a; \Gamma)\}_{t=0}^T$.

3 Functional Forms, Calibration, and Steady State

This section describes the model's functional forms, calibration, and steady state. I use standard household preferences and functional forms. I calibrate the model at the steady state using a combination of parameters

²¹Because the interest rate does not vary with debt, like in Schmitt-Grohé and Uribe (2003) and García-Cicco et al. (2010), I can avoid iterating on r_t because it is effectively exogenous. Secondly, I directly clear the labor market by substituting the labor supply condition (25) into the representative firm's labor demand condition (22).

from the literature and targets from the household data. I then characterize heterogeneity at the steady state.

3.1 Functional Forms

This section describes the functional forms of household preferences and the adjustment costs for the illiquid asset and capital. Households have constant relative risk aversion (CRRA) preferences over consumption given by

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \tag{32}$$

where $\gamma > 0$ is the inverse intertemporal elasticity of substitution.

Adjustment costs of the illiquid asset are given by

$$\chi(a_t, a_{t-1}) = \frac{\chi_1}{2} \left(\frac{a_t - (1 + r_t^a) a_{t-1}}{(1 + r_t^a) a_{t-1} + \chi_0} \right)^2 \left((1 + r_t^a) a_{t-1} + \chi_0 \right), \tag{33}$$

where $\chi_0 > 0$, and $\chi_1 > 0$. Parameter χ_0 ensures adjustment costs are well-defined for $a_t = a_{t-1} = 0$. Equation (33) represents a growing standard in two asset HA models, and originates from Auclert et al. (2021)'s discrete time implementation of the adjustment costs presented in Kaplan et al. (2018).

Adjustment costs of the capital stock are given by

$$\Phi(K_t, K_{t-1}) = \frac{\phi}{2} K_{t-1} \left(\frac{K_t}{K_{t-1}} - 1\right)^2, \tag{34}$$

where $\phi \geq 0$.

3.2 Calibration

This section presents the calibration of the model. The time unit is one year. Parameters can be placed in two groups: parameters that are set externally and internally calibrated parameters.

Table 1 displays the set of parameters that are set externally. I set the inverse intertemporal elasticity of substitution γ to a standard value of 2. I set r^* to 5% per annum, a standard annual value for emerging markets such as Mexico. I draw the idiosyncratic income process from Villalvazo (2023) estimate for Mexico, which provides $\rho_e = 0.91$, $\sigma_e = 0.18$. The depreciation rate δ is set to 10% per year. Following the widely used parameters of García-Cicco et al. (2010), I set the capital share α to 0.32 and the elasticity of labor supply ω to 0.60.

Table 2 describes the set of parameters that are internally calibrated. I normalize κ to 1.86 so that the

aggregate labor supply L_{ss} is equal to one at the steady state. Within the household problem, I jointly calibrate β and χ_1 so that 60% of households are constrained and households display an average MPC of 0.55.²² This provides $\beta = 0.89$ and $\chi_1 = 2.73$, respectively.²³ Finally, I solve for the steady state illiquid asset return r_{ss}^a that clears the illiquid market, which provides $r^a = 0.078$. In the baseline model, I set capital adjustment costs to zero, $\phi = 0$. In RA models such as García-Cicco et al. (2010) or Uribe and Yue (2006) this would lead to a dramatic over-response of investment. Such an over-response does not occur in this model because the firm finances investment through equity owned by the households, which faces significant convex adjustment costs. As a consequence, the firm investment response inherits the equity adjustment costs that households face.

Parameter	Value	Source / Target	Description	
γ	2	Standard	Inverse IES	
r^*	0.05	Standard	Steady State Interest Rate	
$ ho_e$	0.91	Villalvazo (2023)	Persistence income risk	
σ_{e}	0.18	Villalvazo (2023)	Standard deviation income risk	
δ	0.10	Standard	Depreciation rate	
α	0.32	García-Cicco et al. (2010)	Capital share	
ω	0.60	García-Cicco et al. (2010)	Labor supply elasticity	
ϕ	0.0	Varies	Capital adjustment costs	

Table 1. Externally Calibrated Parameters

Notes: The time unit is one year.

Parameter	Value	Source / Target	Description
κ	1.86	$L_{ss} = 1$	Labor disutility
β	0.89	60% Households Constrained	Subjective discount factor
χ_1	2.73	Average MPC 55%	Illiquid asset adjustment costs
r^a_{ss}	0.078	Illiquid Market Clearing	Illiquid asset return

Table 2. Internally Calibrated Parameters

 $^{^{22}}$ These two targets govern the aggregate dynamics of the model. The percent of constrained households controls how many households display a direct response to interest rate fluctuations. The MPC governs the average response to labor income fluctuations.

 $^{^{23}}$ As is standard, I calibrate a smaller value of β relative to a representative agent benchmark to rationalize households' observed liquid asset holdings with the large precautionary savings effect generated by idiosyncratic income risk.



Figure 1. Heterogeneity at the Steady State

Notes: This figure characterizes the stationary steady state of the model. The left panel plots the average MPC within each income decile. The right panel plots the percent of liquidity constrained households within each income decile. The MPC at position (e, b, a) is computed as $MPC(e, b, a) = \frac{c_{ss}(e, b+\epsilon, a)-c_{ss}(e, b, a)}{\epsilon}$ for small $\epsilon > 0$. A household is constrained if $b_{ss}(e, b, a) = 0$. For each panel, the average within each income decile is computed by interpolating along the idiosyncratic income distribution.

I now characterize the steady state of the model that develops in the presence of idiosyncratic income risk and no aggregate risk. In this case, the exogenous variables z_t and μ_t are set to their steady state values $z_{ss} = 1$ and $\mu_{ss} = 1$, respectively, which produces static values for labor income $w_{ss}L_{ss}$, the interest rate r_{ss} , and the illiquid asset return r_{ss}^a . Given $\Gamma_{ss} = \{w_{ss}, L_{ss}, r_{ss}, r_{ss}^a\}$, solving the household problem with only idiosyncratic risk produces a stationary distribution $\Psi_{ss}(e, b, a; \Gamma_{ss})$ and household policies $c_{ss}(e, b, a; \Gamma_{ss}), b_{ss}(e, b, a; \Gamma_{ss}), a_{ss}(e, b, a; \Gamma_{ss})$.

I now consider how households differ at the steady state along the income dimension. The left panel of Figure 1 plots the percentage of constrained households within each income decile. As is explicitly targeted, 60% of households are constrained. The likelihood of being constrained varies significantly with income. Within the poorest two income deciles, 88% of households are constrained to 12% in the highest two income deciles. Relative to the data in Section 5.4, the model overstates the relationship between income and the likelihood of being liquidity constrained. The right panel of Figure 1 plots the average MPC within each income decile. As is targeted, households display an average MPC of 0.55. The MPC displays significant heterogeneity with respect to income. Households in the bottom two deciles of income display an average MPC of 0.25.

Only 7% percent of households are constrained in their illiquid asset holdings. From the perspective of

	Income		Wealth	
Moment	Model	Data (ENIGH)	Model	Data (WID)
Share Top 5	0.11	0.11	0.22	0.60
Share Top 10	0.21	0.23	0.36	0.72
Share Bottom 50	0.33	0.45	0.12	0.02
Share Bottom 20	0.13	0.17	0.02	-0.01

Table 3. Moments of Income and Wealth Distributions, Model and Data

Notes: This table compares moments of the steady state distribution of the model and the empirical data. ENIGH denotes Mexican National Survey of Household Income and Expenditure. WID denotes the World Inequality Database. In the model, income denotes labor income wL. Wealth is defined as the sum of liquid and illiquid asset holdings, b + a.

the household data in Section 5.4, the measure of households that are constrained in illiquid asset holdings can be viewed as generous or conservative. For the most generous definition of illiquid assets that includes durable goods, the portion of constrained households is accurate. For more stringent measures that require housing or financial assets, the distribution understates the quantity of households that possess no illiquid assets.

I now compare inequality within the steady state distribution relative to the data. Table 3 compares inequality in income and wealth. I consider the top five, top ten, bottom fifty, and bottom twenty shares, and the gini index. For income, I draw my empirical counterpart from the residualized distribution of after tax income computed using MFL. For wealth, I draw my empirical counterpart from the World Inequality Database, which uses the methodology described in Bajard et al. (2022).²⁴ In general, the model overstates inequality at the bottom end of the income distribution. The model predicts bottom 50% and bottom 20% income shares of 0.33 and 0.13, respectively, whereas the data provides shares of 0.45 and 0.17, respectively.

Similar to Hong (2023a) and Villalvazo (2023), who feature the same household problem, the model faces difficulty in capturing wealth inequality.²⁵ I draw wealth shares from the World Inequality Database (WID). Within the model, I define wealth as b+a, the sum of liquid and illiquid asset holdings.²⁶ The model predicts a top five and top 10 wealth shares of 0.24 and 0.39, respectively, whereas the data provides shares of 0.60 and 0.72, respectively. The model predicts bottom 50% and bottom 20% income shares of 0.11 and 0.02, respectively, whereas the data provides shares of 0.02 and -0.01, respectively.

²⁴Bajard et al. (2022) impute measures of wealth inequality in Mexico using a cluster of similar countries.

 $^{^{25}}$ Hong (2023a) matches wealth inequality by introducing entrepreneurs that lack income risk or borrowing constraints. I do not include entrepreneurs because there is no way to place them within the income distribution.

 $^{^{26}}$ Because agents hold the vast majority of their wealth in illiquid assets, inequality in net wealth is largely determined by inequality in illiquid assets.

4 Results

In this section I evaluate the ability of the transitory and permanent approaches to replicate the Mexican Peso Crisis of 1994. The focus of each exercise is twofold: to capture the heterogeneous consumption responses of households discussed in Section 5.3 and the aggregate responses of the economy discussed in Section 5.1.

4.1 View I: Transitory Shocks

This section studies the ability of the transitory approach to generate a sudden stop. The transitory approach features simultaneous contractionary shocks to the productivity of the representative firm and the external interest rate of the economy. The path of productivity z_t and the interest rate shock μ_t are given by

$$\log z_t = \rho_z^t dz_0,\tag{35}$$

and
$$\log \mu_t = \rho^t_\mu d\mu_0,$$
 (36)

where $0 \le \rho_z < 1, \ 0 \le \rho_\mu < 1$.

Building the transitory shocks requires choosing paths for productivity and the interest rate, each determined by their initial fluctuations $dz_0, d\mu_0$ and persistences, ρ_z, ρ_μ . I calibrate the shocks in two steps. First, I introduce aggregate risk to calibrate the persistence of the productivity process. I assume productivity z_t and the interest rate μ_t follows the processes $\log z_t = \rho_z \log z_{t-1} + \sigma_z \epsilon_t^z, \epsilon_t^z \sim \mathcal{N}(0, 1)$ and $\log \mu_t = \rho_\mu \log \mu_{t-1} + \sigma_\mu \epsilon_t^\mu, \epsilon_t^\mu \sim \mathcal{N}(0, 1)$, respectively. I fix $\rho_\mu = 0.62$ and $\sigma_\mu = 0.019$, calibrated from the Moody's Baa corporate bond series as a measure of the world interest rate. Next, I linearize the model and jointly calibrate ρ_z and σ_z to match the autocorrelation and standard deviation of Mexican GDP.²⁷ This provides $\rho_z = 0.53$ and $\sigma_z = 0.0166$.

In the second stage, I calibrate the magnitude of the initial productivity and interest rate shocks to match the sudden stop. Table 4 displays the calibrated parameters. Regardless of the path of the interest rate, the initial decline in output is completely determined by the initial decline in productivity. Therefore, I calibrate $dz_0 = -0.054$ to match the decline in the cyclical component of GDP of 8.9%. Calibrating the consumption response requires more nuance. The household data, which feature nondurable consumption, displays an average two-year consumption-income elasticity of 0.86. In contrast, the aggregate data, which features durable consumption goods, delivers a much larger two-year consumption-GDP elasticity of 1.31 and a one-year elasticity of 1.33.²⁸ To remain grounded from the perspective of the household data, I target a

 $^{^{27}}$ I use the annual HP filtered series of GDP with a smoothing parameter of 6.25 from 1965 to 2010.

 $^{^{28}}$ I compute the 'aggregate' elasticities over the HP filtered cyclical component of the annual series.

Parameter	Value	Source/Target	Description			
View I: Transitory Shocks						
dz_0	-0.054	$dY_0/Y_{ss} = -8.9\%$	Initial decline in productivity			
ρ_z	0.53	Mexico GDP autocorr. 1965-2010	Persistence of decline in productivity			
$d\mu_0$	0.12	$dC_0/C_{ss} = dY_0/Y_{ss}$	Initial increase in interest rate			
$ ho_{\mu}$	0.62	Moody's Baa Yield	Persistence of increase in interest rate			
View II : Permanent Shock						
$d\zeta_0$	-0.054	$dY_0/Y_{ss} = -8.9\%$	Initial decline in productivity			
$ ho_P$	0.00	$dC_0/C_{ss} = dY_0/Y_{ss}$	Persistence of permanent decline			

Table 4. Sudden Stop Drivers: View I (Transitory) and View II (Permanent)

Notes: This table describes the aggregate shocks under the transitory and permanent views.

one-to-one initial response of aggregate consumption relative to output (GDP).²⁹ Critically, I only target the aggregate consumption response and leave any heterogeneity in household responses completely untargeted. Given the calibration, this provides $d\mu_0 = 0.12$. Relative to the observed increase in the EMBI+ rate for Mexico discussed in section 5.2, I view this increase as reasonable.

Aggregates I now characterize the response of the model to the transitory shocks at the aggregate level. Figure 2 displays the impulse responses of output, aggregate consumption, investment, and the trade balance to output ratio as percentage deviations from their steady state values. Output and consumption display the calibrated decrease of 8.9%. Investment displays an initial decrease of 26.9%, slightly larger than the decrease in the annual cyclical component of 24.7% observed in the data.³⁰ The trade balance to output ratio displays an initial increase of 3.5%, whereas the annual data displays a trade balance reversal of 5.0%.³¹ The exercise successfully generates the signature features of a sudden stop: a recession that features a sharp decline in consumption and investment that coincides with a reversal of the trade balance.

Relative to the Mexican Peso Crisis, the trade balance displays a weak reversal. This also holds in E. G. Mendoza (2010)'s simulated sudden stop, Cugat (2019)'s similar sudden stop exercise, and Villalvazo (2023)'s simulated sudden stop.³² The exercise predicts a weak trade balance response because it targets

 $^{^{29}}$ Because labor income is proportional to output, this implies a one to one consumption to labor income elasticity.

 $^{^{30}}$ I compute the cyclical component using the Hodrick-Prescott Filter with a smoothing parameter of 6.25.

 $^{^{31}\}mathrm{Here}\ \mathrm{I}$ refer to the cyclical component, as with investment.

 $^{^{32}}$ Cugat (2019) is similar in that the sudden stop is triggered by contractionary shocks to productivity and the interest rate. We can view Villalvazo (2023)'s simulated sudden stop as conservative because it only features aggregate fluctuations to the interest rate and excludes productivity or endowment fluctuations.

Broadly speaking, each paper and this paper use the resource constraint $Y_t = C_t + I_t + TB_t$, with the exception that Cugat (2019), and Villalvazo (2023) lack investment. This paper is more similar to E. G. Mendoza (2010) in that the trade balance reversal is financed through an overreaction of investment relative to GDP where as Cugat (2019) and Villalvazo (2023) generate the reversal through an overreaction of consumption relative to GDP. Each paper falls short of the observed trade balance reversal in that it either lacks i) the overreaction of investment or ii) the overreaction of consumption.



Figure 2. View I (Transitory): Impulse Responses

Notes: This figure displays the impulse responses of aggregate variables to the transitory shocks described in Table 4. Consumption, output, and investment are expressed as percentage deviations from their respective steadystates. The trade balance to output ratio is expressed as percentage point deviations from its steadystate.

the weaker consumption response exhibited in the household data rather than the stronger consumption response observed in the aggregate data. In an exercise that targets the aggregate consumption response, the trade balance would display a sharper reversal.



Figure 3. View I (Transitory Shocks): Consumption-Income Elasticities

Notes: This figure plots the two-year consumption-income elasticities in response to the transitory shocks described in Table 4.

Heterogeneity I now study how households differ in their consumption responses to the transitory shocks. Figure 3 plots the average consumption-income elasticity (the elasticity) within each income decile.³³ Households display an average elasticity of 0.78, close to the average elasticity of 0.86 observed in the household data. The lowest income decile displays an elasticity of 0.68. In contrast, the highest income decile displays a larger elasticity of 0.94. This replicates Guntin et al. (2023)'s empirical finding that income is a poor predictor of consumption-income elasticities during sudden stops.

Figure 3 shows that the transitory approach can match the consumption responses observed in the data but does not provide a transparent explanation for why low and high income households display similar responses. To study this, I follow the decomposition exercise of Kaplan et al. (2018) and separately input the general equilibrium fluctuations of labor income $\{w_t L_t\}_{t=0}^T$, the interest rate $\{r_t\}_{t=0}^T$, and the illiquid asset

 $^{^{33}}$ To follow the data, I compute the elasticity using a two-year difference in observed consumption and income. Within the model, this coincides with period t = -1, where the economy is at the steady state, and period t = 1, one period after the initial impact of the shocks.

return $\{r_t^a\}_{t=0}^T$. The methodology is described in more detail in Section D. Because the household problem is nonlinear, I include a separate nonlinearity term that captures the difference between perfect foresight responses to a collection of simultaneous shocks and the sum of consumption responses to individual shocks.

Figure 4 plots the decomposition. The contribution of labor income is flat at nearly 0.65 for the first six deciles, after which it declines to 0.52 for the highest income decile. The interest rate generates a small negative consumption response of about -0.1 for the first four income deciles, after which it increases to 0.41 for the highest income decile.³⁴ The illiquid asset makes a small average contribution of 0.06 across all income deciles, and displays a weak decline in income. The increasing relationship between income and the response to the interest rate overwhelms the decreasing relationship between income and the consumption response to the decline in income. As a consequence, high income households display weakly larger consumption responses than low income households.

Why are high income households less responsive to the decline in labor income? The driver is the relationship between income and the likelihood of having access to financial markets. Low income households are more likely to be constrained in their asset holdings. As a consequence, low income households consume a larger portion of their immediate increase in labor and illiquid asset income. In contrast, high income households are more likely to hold liquid assets. This allows them to better consumption smooth over the partial equilibrium transitory decline in income. Relative to the MPCs presented in Section 3.2, the response to labor income displays less heterogeneity. This occurs because the sudden stop generates a persistent more fluctuation of income whereas the MPC computes the response to a one period cash infusion. While constrained households only respond to the immediate change in income, unconstrained households behave more like a permanent income consumer and respond to future changes in income. Because high income households are more likely to be unconstrained, their consumption response to the labor income shock increases relative to their MPC.

The contribution of the interest rate to the consumption response increases in income. Perhaps surprisingly, the driver of this relationship is the same as that of the labor income shock. Low income households are more likely to be constrained. This introduces a wedge in their Euler equation so that they lack a direct response to the interest rate increase. In contrast, high income households are more likely to be unconstrained. These households feature a standard Euler equation in which consumption varies directly with the interest rate.³⁵ This relationship is supported in two empirical studies. Vissing-Jørgensen (2002) and Havranek et al.

³⁴The negative contribution of the interest rate is a feature of the two-year consumption-income elasticities. The decomposed one-year consumption-income elasticity, displayed in figure A.5, features a strictly contractionary contribution of the interest rate to consumption across all income deciles.

 $^{^{35}}$ Because we've decomposed the consumption response at the household level, any 'indirect effects' of the interest rate, like those discussed in Kaplan et al. (2018), are included in the labor income and illiquid asset return terms.



(2015) both show that liquid asset holders display larger responses to interest rate fluctuations.³⁶

Figure 4. View I (Transitory): Decomposed Consumption-Income Elasticities

Notes: This figure decomposes the two-year consumption-income elasticities within each income decile with respect to labor income, the interest rate, the illiquid asset return, and a nonlinearity term, conditional on the sudden stop. The decomposed responses are computed by separately inputting in the general equilibrium paths of labor income, the interest rate and the illiquid return. The nonlinearity is computed as the difference between the consumption-income elasticity computed using all inputs and the sum of the consumption responses, weighted by the percentage change in income, conditional on each input.

The Role of Illiquid Assets Heterogeneity in consumption responses to the illiquid asset return fluctuation are driven by two effects: First, low income households respond more to changes in income generated by the illiquid asset. Secondly, high income households feature stronger absolute monetary income changes generated by the illiquid asset. Because the model features wealthy hand to mouth households, high income households respond significantly to the higher changes in capital income they experience. Conditional on a change in the illiquid asset return, the first effect generates a declining response of consumption with respect to income, whereas the second effect supports an increasing response. In this calibration, the first effect dominates the second effect, which produces a declining consumption response conditional on an illiquid asset return shock. The dominance of the first effect is small, so that the illiquid asset only makes a weak

³⁶The two studies differ in that Vissing-Jørgensen (2002) studies asset holders within the United States whereas Havranek et al. (2015) performs a meta analysis of countries with varying levels of financial development.

contribution to heterogeneity in responses. See Section 4.5 for an analysis that features a more prominent contribution of the illiquid asset.

Relationship with Guntin et al. (2023)'s Credit Tightening View Unlike Guntin et al. (2023) credit tightening view, I have shown that transitory shocks can replicate the sudden stop at both the household and aggregate level. This paper differs in that it uses two shocks, contractionary productivity and interest rate shocks, whereas Guntin et al. (2023) uses a single contractionary endowment shock. Within their household problem, Guntin et al. (2023) implement a borrowing constraint of the form

$$b_t \ge -\theta Y_t^{\nu} \tag{37}$$

where b_t is the bond position of a generic household, $\theta > 0$ scales the borrowing constraint, Y_t is an exogenous aggregate income endowment and ν determines how strongly the constraint contracts with the income endowment.³⁷ When aggregate income contracts, the borrowing constraint contracts with it, forcing constrained households to deleverage. At the aggregate level, this is helpful as it generates a stronger contraction of consumption and the deleveraging explicitly forces an increase in household savings. At the household level, the approach is problematic because the contraction disproportionately falls on low income households, since they are more likely to be constrained. This amplifies the consumption response of poor households so that the relationship between income and consumption is even more strongly decreasing than with a static constraint.³⁸ Lastly, because the constraint depends on the aggregate endowment Y_t and not the individual endowment $Y_t e_t^i$, households with the lowest levels of income have the same access to borrowing as households with the highest levels of income. This leads to extreme levels of leverage for poor households.

At the stationary steady state that arises when households borrow subject to equation (37), the vast majority of low income households are indebted. From a purely empirical perspective, this is problematic because the data in Section 5.4 shows that the vast majority of low income households are financially constrained in that they hold neither savings nor debts. The presence of indebted households also has implications for responses to an interest rate shock. If poor households borrow up to a nontrivial collateral constraint, they display negative wealth effects in response to an interest rate increase. This significantly changes the consumption-income elasticity curve conditional on an interest rate shock, which provides another motivation to use the non-negativity constraint rather than a borrowing constraint that allows for debt.

Lastly, households make their asset decisions at the stationary steady state without knowledge of an

³⁷Similar to this paper, household *i* receives individual income $e_t^i Y_t$ where e_t^i is an exogenous idiosyncratic income shock.

³⁸Here, a 'static' constraint denotes constant borrowing constraints and a non-negativity constraint. See Villalvazo (2023) for an HA model that features a non-static collateral constraint. Villalvazo (2023) differs in that collateral constraint depends on individual levels of asset holdings, so that poor households exhibit reasonable leverage levels.

impending contraction. If households could anticipate a contraction, they would be less likely to borrow up to the constraint to begin with.³⁹ This would, however, require coding a global model like that of Villalvazo (2023). I do not use a global model because I need to build a full distribution of income, rather than using binary low and high income states that the majority of global HA models are bound to.



4.2 View II: Permanent Shock

Figure 5. View I (Transitory) and View II (Permanent): Consumption-Income Elasticities

Notes: This figure displays the average two-period consumption-income elasticity within each income decile under three approaches: a transitory decline in productivity that coincides with a transitory increase in the interest rate, a permanent decline in productivity, and a permanent decline in productivity that coincides the transitory increase in the interest rate from the first approach. Consumption-income elasticities are computed over two periods.

This section studies the ability of a permanent decline in productivity to generate a sudden stop. As with the transitory shocks, I evaluate the ability of the permanent shock to capture the features of the sudden stop at the household and aggregate level. I consider long run changes to productivity of the form

$$\log z_t = \log z_{t-1} + \zeta_t \tag{38}$$

$$\zeta_t = \rho_P^t d\zeta_0,\tag{39}$$

where $d\zeta_0$ is the initial change in productivity and $|\rho_P| < 1$ is its persistence. Given $d\zeta_0$ and ρ_P , productivity z_t moves to a new long run steady state value that produces a new and unique stationary steady state of the model. To solve for the new long run steady state, I leave all parameters unchanged and solve for the new

³⁹E. G. Mendoza (2010) uses this line of reasoning to explain the infrequent nature of sudden stops.

illiquid asset return that clears the illiquid asset market, r_{ss}^a . I then solve the perfect foresight path from the initial steady state to the long run steady state to produce a path of all aggregate variables, the distribution of households $\{\Psi_t\}_{t=0}^T$, and the series of household policies $\{c_t(e, b, a; \Gamma), b_t(e, b, a; \Gamma), a_t(e, b, a; \Gamma)\}_{t=0}^T$.

Building the permanent shock requires choosing the initial decline in productivity $d\zeta_0$ and the persistence term ρ_P . Table 4 describes the calibrated parameters. As with the transitory shocks, I set the initial decline in productivity to $d\zeta_0 = -0.054$ to match the observed cyclical decline in output of 8.9%. I then calibrate ρ_P to target a one-to-one initial consumption to output response. This provides $\rho_P = 0.00$, similar to Aguiar and Gopinath (2007)'s quarterly persistence of 0.00.⁴⁰

Heterogeneous Responses Figure 5 displays the average consumption-income elasticity within each income decile for the transitory and permanent approaches. As with the transitory approach, the permanent shock succeeds at recreating the consumption responses observed in the data. This expands the lack of heterogeneity in consumption responses conditional on a permanent shock observed in Guntin et al. (2023)'s single asset model to the two asset model. Households display large consumption responses for a different reason than under the transitory approach. Broadly, the intuition of Aguiar and Gopinath (2007) holds at the household level for all levels of income. Because the decline in labor income is permanent, households know they cannot maintain their current consumption level in the long run. As a consequence, the optimal choice is to immediately decrease consumption with income. This holds for low income households, who are more likely to be constrained, and high income households, who are less likely to be constrained.

Figure A.6 decomposes the consumption responses conditional on the permanent shock with respect to labor income and the illiquid asset return. In this case, the contribution of labor income strictly increases with income. This occurs because the long run decline in labor income is larger than the initial decline in labor income, as seen in Figure A.7. Constrained households, who would like to consume more, only decrease their consumption by the initial amount that is forced by the initial income decline. Unconstrained households, that behave more like a permanent income consumer and are more likely to be high income, display a larger response because they respond to the long run decline in labor income.

In addition, I plot the consumption responses that arise from introducing both the permanent decline in productivity and the temporary increase in the interest rate. In this case, high income households display too large of a consumption response relative to the data. This occurs because high income households respond

⁴⁰Here I refer to Aguiar and Gopinath (2007)'s estimate in Column IV, Table 4. The estimate of ρ^P is smaller than that of Guntin et al. (2023)'s because Guntin et al. (2023) generates a direct permanent decline in income. Within the model of this paper, a permanent decline in productivity with $\rho^P = 0.00$ still generates a persistent decline in the equilibrium path of labor income. This occurs because the marginal product of labor depends on the capital stock. Following a permanent shock, the firm does not unproductively 'burn' any capital on the transition to the new steady state. As the capital stock gradually deteriorates to its long run value, the marginal product of labor, and hence labor income, further declines from its initial decrease to its lower long run value.



simultaneously to their long run decline in labor income and the temporary increase in the interest rate.

Figure 6. View I (Transitory) and View II (Permanent): Impulse Responses

Notes: This figure compares the aggregate impulse responses of the model to the transitory and permanent approaches described in Table 4. The transitory approach features a 5.4% decline in productivity with a persistence of 0.62 and a 12% increase in the interest rate with a persistence of 0.62. The permanent approach features a permanent decline in productivity of 5.4%. All variables are in percentage deviations from the initial steady state.

Aggregates Up to this point, I've shown that both the transitory and permanent approaches can match the consumption responses observed in the household data. From the household perspective, this leaves it ambiguous which type of shock drove the Mexican Peso Crisis. To further differentiate between the views, I study how they differ in their aggregate responses. Figure 6 plots the impulse responses of the exogenous drivers, productivity and the interest rate, along with the aggregate responses of consumption, output, investment, and the trade balance to output ratio. All variables are presented as percentage deviations from the initial steady state except the interest rate and trade balance to output ratio which are expressed in percentage points. In the long run, aggregates in the transitory approach revert to their original steady



Figure 7. View I (Transitory) and View II (Permanent): Comparison with Mexican Peso Crisis

Notes: This figure compares the responses of the transitory and permanent approaches to generate a sudden stop with the observed cyclical components of the Mexican Peso crisis. Each view of the data is normalized by the period preceding the sudden stop. For the data, this coincides with 1994; for the model, this coincides with the steady state. The annual cyclical component is detrended using an HP Filter with a smoothing parameter 6.25. Source: WDI.

states while aggregates in the permanent approach revert to their lower long run steady state.

As is explicitly calibrated, each approach delivers identical initial responses of consumption and output. The two models differ in their implications for investment and the trade balance. The transitory approach displays a stronger initial decline in investment of 27%, whereas the permanent approach displays an initial decline of 15%. Similarly, the transitory approach displays a stronger trade balance reversal of 3.5%, compared to 0.9% for the permanent approach. Inspecting the resource constraint in equation (31), because the initial responses of output and consumption are matched, the initial stronger trade balance reversal under the transitory approach is financed exclusively through its stronger initial investment decrease.

The transitory and permanent views differ in what drives the decline in investment. Under the transitory approach, the increase in the interest rate motivates households to temporarily substitute away from the illiquid asset to the liquid asset. The selloff of the illiquid asset generates a decline in household financing of equity which forces the firm to scale back investment. Because the increase in the interest rate is sharp and brief, investment displays a large but brief initial decline. While the permanent shock generates a permanent decline in investment, it fails to account for the large initial response.

Figure 7 compares each approach to the cyclical components observed in the Mexican Peso Crisis.⁴¹ Specific to the Peso Crisis, we can see that output, consumption, and investment quickly revert to their trend. This is better matched by the transitory approach than the permanent approach. As stated before, the transitory approach delivers a weak trade balance response relative to the data because it undershoots the aggregate consumption response.

4.3 **Productivity and Interest Rate Shocks**

Section 4.1 features simultaneous productivity and interest rate shocks. To build intuition, this section studies the separate contributions of the productivity and interest rate shocks to the aggregate and household responses.

The Productivity Shock This section studies the response of the model to the transitory productivity shock. I consider the transitory productivity shock used to build the sudden stop: a 5.4% decrease in productivity that reverts to its steady state with a persistence of 0.53. The left panel of Figure 8 displays the impulse responses. Output displays an initial decrease of 8.9%, driven by a combination of lower productivity and labor usage. Consumption displays an initial decrease of 4.9% and investment displays an initial decrease of 18.9%. Because households do not absorb the entirety of the output decrease, the trade balance to output

 $^{^{41}}$ Because I introduce the cyclical component of the crisis, this approach is slightly biased towards the transitory approach in terms of aggregates reverting to their long run trends.



Figure 8. Productivity Shock: IRFs and Heterogeneous Responses

Notes: This figure studies the responses of the model to the decline in productivity that contributes to the sudden stop. Productivity features an initial decline of 0.054that reverts to its steady state value with a persistence of 0.53. The left figure plots the aggregate impulse responses of the model. All variables are in percentage steady state deviations from their initial value. The right figure plots the average two-year consumption-income elasticity within each income decile, along with its decomposition.



Figure 9. Interest Rate Shock: IRFs and Heterogeneous Responses

Notes: This figure studies the responses of the model to the increase in the interest rate that contributes to the sudden stop. The interest rate features an initial increase of 0.12 that reverts to the steady state with a persistence of 0.62. The left figure plots the aggregate impulse responses of the model. All variables are in percentage steady state deviations from their initial value. The right figure plots the average two-year percentage change in consumption within each income decile, along with its decomposition.

ratio depreciates by 1.7%. Because the interest rate is exogenous, it is unchanged throughout the productivity shock.⁴²

I now study how households differ in their consumption responses, conditional on the productivity shock. The right panel of Figure 9 plots the average consumption-income elasticity for each income decile and its decomposition. Households display an average elasticity of 0.71.⁴³ The poorest income decile displays an elasticity of 0.74. The elasticity declines to 0.56 for the highest income decile.

The transitory productivity shock alone fails to replicate the sudden stop. This is not surprising given that a significant portion of households have some ability to consumption smooth. At the household level, I reject the productivity shock as the driver of the sudden stop because the consumption-income elasticity is declining in income and does not match the high consumption-income elasticity observed for high income households. At the aggregate level, consumption does not display a one to one response with income, and the trade balance to output ratio fails to display a reversal. The household level and aggregate failures are tied to each other through the behavior of high income households. Because high income households take up a larger share of aggregate consumption, it will be challenging to generate a large aggregate consumption response so long as high income households use their assets to consumption smooth.

The Interest Rate Shock This section studies the responses of the model to the transitory interest rate shock. I consider the interest rate increase introduced in the transitory approach: a 12% percent increase in the interest rate that reverts to the steady state with a persistence of 0.62. The left panel of Figure 9 displays the impulse responses. Output does not display an initial response because productivity is unchanged, capital is determined in the previous period, and the labor supply does not feature wealth effects. The increase in the interest rate increases the relative price of current consumption, which motivates households to save. This leads to an initial consumption decrease of 5.1%, which supports a 6% appreciation of the trade balance to output ratio. The interest rate shock leads to a decline in investment of 10.0%.

I now consider how households differ in their responses to the interest rate shock. The right panel of Figure 9 displays the two-year percentage change in consumption for each income decile.⁴⁴ Households display an average percentage change in consumption is 0.86%. The lowest income decile displays a consumption increase of 0.38%. In contrast, the highest income decile displays a consumption decrease of 2.81%.

The interest rate shock makes two important contributions. First, aggregate consumption displays variation that is generated independently of the productivity of the firm and hence the labor income that households receive. This is the driver of papers such as Neumeyer and Perri (2005) and Uribe and Yue

⁴²Models that follow Schmitt-Grohé and Uribe (2003) can feature feedback between productivity shocks and the interest rate. ⁴³The average elasticity of consumption among households does not coincide with the aggregate consumption elasticity because richer households take up a disproportionate share of consumption.

 $^{^{44}}$ Because income is unchanged in the initial period, the consumption-income elasticity is undefined.

(2006) that generates an increase in the volatility of consumption relative to output and a countercyclical trade balance.⁴⁵ Secondly, the increase in consumption variation is generated disproportionately by high income households. Both of these contributions play an important role in successfully generating a realistic sudden stop at the aggregate and household level.



4.4 The Role of Persistence

Figure 10. Heterogeneous Consumption Responses: The Role of Persistence

Notes: This figure studies how consumption responses differ with the persistence of transitory aggregate shocks. The left panel plots the average one-period consumption-income elasticity within each income decile conditional on a 1% increase in productivity for four values of the persistence of productivity: $\rho_z = 0.50, 0.70, 0.80, 0.90$. The right panel plots the average one-period percentage change in consumption within each income decile conditional on a 1% decrease in the interest rate for four values of the persistence of the persistence of the interest rate shock: $\rho_{\mu} = 0.50, 0.70, 0.80, 0.90$.

This section studies how the heterogeneous responses of consumption vary with the persistence of a transitory aggregate shock. I first consider how the consumption responses change with the persistence of productivity, ρ_z . The left panel of Figure 10 plots the average consumption-income elasticity to a 1% increase in productivity for four different values of the persistence, $\rho_z = 0.50, 0.70, 0.80, 0.90$. The consumption-income elasticity is computed using the general equilibrium paths of labor income and the illiquid asset return developed under each value of the persistence. As ρ_z increases, the consumption response increases for all households because the present value of the labor income and illiquid asset value fluctuations is higher. Low income households display a weaker increase in consumption relative to high income households. This

⁴⁵The contribution of the interest rate to consumption variation is much more involved in this model because it features a significant contribution of direct effects. In a RA model the contribution of interest rates fluctuations to consumption variation is independent of labor income at the first order.

occurs because low income households are more likely to be constrained in which case they can only respond to the immediate change in income or the value of illiquid assets. High income households, however, display a stronger increase in consumption with respect to the increase in persistence. This occurs because high income households are less likely to be constrained so that they behave more like a permanent income consumer and display a significant response to future changes in income.

I now consider how consumption responses change with the persistence of the interest rate, ρ_{μ} . The right panel of Figure 10 plots the average percentage change in consumption for each income decile to a 1% decrease in the interest rate for four values of the persistence, $\rho_{\mu} = 0.50, 0.70, 0.80, 0.90$. For a low persistence of $\rho_{\mu} = 0.50$, low income households display a small consumption increase relative to high income households. This occurs because low income households are constrained and do not display a direct response to the interest rate.⁴⁶ As ρ_{μ} increases, low income households display a small increase in their consumption response and high income households display a significant increase in their response. This occurs because low income households display a significant increase in their response. This occurs because low income households behave more like a permanent consumer and care about future fluctuations in the interest rate.

4.5 Sensitivity Analysis

Stronger Indirect Effects In the calibrated sudden stop, the market value of the illiquid asset displays a muted contribution to household consumption responses. I consider a stronger contribution of the illiquid asset by increasing the capital adjustment costs of the firm. I consider three values of capital adjustment costs, $\phi = 2.0, 5.0, 10.0$. As discussed in Alves et al. (2020), stronger capital adjustment costs increase the decline in the value of the firm conditional on an interest rate increase. As in the baseline model, I calibrate the initial increase in the interest rate to match a one to one initial consumption to output response. Conveniently, capital adjustment costs have no bearing on the steady state, so all initial household consumption and asset positions are identical. I keep the same path of productivity as in Table 4. Figure A.8 plots the general equilibrium pathway of the illiquid asset return, the value of the firm, and the calibrated increase in the interest rate for each value of ϕ . Notably, higher capital adjustment costs induce a stronger decline in the value of the firm but also require a lower interest rate increase to match a one to one consumption to output response. Why does this happen? For a given increase in the interest rate, households display their original direct consumption response to the interest rate and also respond to the stronger decline in the value of their equity shares. The stronger decline in the value of equity amplifies the consumption

 $^{^{46}\}mbox{Because}$ constrained households do not hold any assets, they also do not display any wealth effects to the interest rate fluctuation.

responses so that a smaller increase in the interest rate is necessary to match the consumption response target. Each value of capital adjustment costs $\phi = 2.0, 5.0, 10.0$ lowers the calibrated interest rate increase to $d\mu_0 = 0.11, 0.10$, and 0.09, respectively.

Figure A.10 decomposes the consumption responses for the middle level of capital adjustment costs, $\phi = 5.0$. Unlike in Figure 4, Similar to the contribution of labor income, the contribution of the illiquid asset declines in income. I emphasize that this model understates illiquid asset inequality along two dimensions. First, only 7% of households are constrained in illiquid asset holdings, so the majority of households display a substitution effect to the illiquid asset return. From the perspective of the household data in Section 5.4, this is a reasonable approximation for durable goods but a bad approximation for purely financial assets. Second, the model understates total wealth inequality. A model that better captures the wealth shares of high income households would feature stronger wealth effects for high income households and weaker wealth effects for low income households. Within the decomposed responses, this could present as a flat or even increasing contribution of the illiquid asset to consumption responses.

Figure A.9 plots the one and two-period consumption-income elasticities for each value of ϕ . We can see the two-period elasticities are still able to match the consumption responses, despite a smaller calibrated increase in the interest rate. The stronger contribution of the illiquid asset return leads to a flattening effect of consumption responses. This occurs because the illiquid asset return displays a stronger effect for low income households. In addition, because the calibrated increase in the interest rate decreases, the stronger direct response of high income households decreases. Combined, the stronger illiquid asset contribution and weaker direct contribution further weaken household heterogeneity in responses.

Consumption Measurement Timing An important difference between the model and the data is that ENIGH surveys households in 1994:Q3 and 1996:Q3, a time difference of two years, whereas the time period of the model is one year. Figure A.11 plots the one, two, and three-year consumption-income elasticities.

Smaller and Larger Interest Rate Increases This section studies the sensitivity of the main result to the magnitude of the interest rate increase. Figure A.12 plots the two-year consumption-income elasticity conditional on the transitory productivity shock described in Section 4. I consider three increases in the interest rate, $dr_0 = 0.06, 0.12, 0.18$. As dr_0 increases, low income households display smaller consumption responses and high income households display larger responses.⁴⁷

 $^{^{47}}$ The smaller response of low income households conditional on a larger interest rate increase is specific to the two-period elasticity and not the one-period elasticity.

4.6 Comparison with a Single Asset Model

This section compares the two asset household problem with a single asset household problem. I consider a Bewley (1977) model where households have the same consumption preferences and satisfy the constraints

$$c_t^i + b_t^i = (1 + r_t^b)b_{t-1}^i + e_t^i w_t L_t$$
(40)

$$b_t^i \ge 0 \tag{41}$$

where r_t^b is the interest rate, $w_t L_t$ is the average labor income, and e_t^i is the idiosyncratic component of household *i*'s income. I assume households face the same interest rate, average income, and idiosyncratic income risk as in the stationary steady state of the two asset model. I consider three calibrations of β : leaving β fixed to its value in the two asset model, and calibrating β to match the percent of constrained households or the average MPC of the two asset model, respectively.

Figure A.13 compares the steady states of the two and single asset models. The left panel displays the percent of constrained households within each income decile, and the right panel displays the average MPC within each income decile. The fixed β model and MPC matching model feature a sharp change in the percent of constrained households around the sixth decile of income. This implies that all households in the top four deciles of income have access to financial markets. The model that matches the percent of constrained households more closely tracks the two asset model, but features an average MPC of nearly one for the first four income deciles.

Figure A.14 plots the one and two-period consumption-income elasticities for the single asset model that targets the average MPC. I input the path of the general equilibrium decline in labor income generated during the calibrated sudden stop and allow the initial increase in the interest rate to vary from 0.01 points to 0.15 points, all of which revert to the steady state with a persistence of 0.62. As seen in the one-period elasticities, the single asset model can recover the consumption responses of high income households, but has a unique consumption-income elasticity of one for the low income households. The lowest income decile displays a nearly fixed consumption-income elasticity because nearly all households are constrained and the average MPC is nearly one. We might presume that moving from a two asset model to a single asset model eliminates wealthy hand to mouth households, so that high income households display a lower consumptionincome elasticity. However, because they behave less like a hand to mouth household and more like a permanent income consumer, the high income households of the single asset model display a larger response to the interest rate increase. In this case, the latter effect overwhelms the former effect so that high income households actually display a larger consumption-income elasticity than in the baseline calibration of the two asset model. From this perspective we can view the two asset model as being conservative in how it models high income households.

Figure A.15 studies how responses of the single asset model vary with the persistence of aggregate shocks. The upper panels display the responses to labor income fluctuations that feature increasing persistence. I use the general equilibrium labor income pathways produced for aggregate productivity declines that feature increasing persistences of 0.50, 0.70, 0.80, 0.90. This ensures the responses are driven by the different household problems and are not features of the production side of the two asset economy. I ignore the movement of the illiquid asset return which is not featured in the single asset model. We can see the consumption responses of the single asset model to a labor income shock are always decreasing in income because the lowest income decile always displays an elasticity of nearly one, regardless of the increase in the interest rate. This differs from the two asset model which displays an increasing consumption response to a productivity shock for a persistence of 0.90.

5 Data

This section presents the data. I characterize four stylized facts. First, I present the frequently studied aggregate data of the Mexican Peso Crisis. Second, I discuss the procyclical nature of interest rates in emerging markets. Third, using the Mexican National Survey of Household Income and Expenditure (ENIGH) dataset, I compute how household differ in their consumption responses during the Mexican Peso Crisis.⁴⁸ Finally, I use the Mexican Family Life Survey (MFL) to show that the likelihood of having access to financial markets increases with income.⁴⁹

5.1 Aggregates: The Mexican Peso Crisis

This section describes the Mexican Peso Crisis from the perspective of the aggregate data. Leading up until the fourth quarter of 1994, the Mexican economy experienced a boom in borrowing and investment. In late December, the Mexican Peso devalued, triggering an outflow of capital and a recession that reached a trough in the second quarter of 1995.

I collect quarterly consumption, GDP, investment, and the trade balance to GDP ratio from the International Monetary Fund International Financial Statistics dataset (hereafter IMF-IFS). All series are in real, per capita terms and seasonally adjusted.⁵⁰ Figure 11 displays the series in log levels, excluding the trade balance to GDP ratio, with each term normalized by the first quarter of 1993. The crisis displays the

 $^{^{48}}$ I closely follow Guntin et al. (2023)'s computations for Mexico.

⁴⁹I use MFL because it features balance sheet data rather whereas ENIGH only features expenditure data.

 $^{^{50}}$ I compute quarterly population levels by linearly interpolating the log level of annual population levels.



Figure 11. Mexican Peso Crisis: Quarterly Aggregates

Notes: This figure plots the evolution of quarterly aggregate GDP, consumption, investment, and the trade balance to GDP ratio in Mexico during the mid 1990s. All variables excluding the trade balance to GDP ratio are in real, log-level, per capita terms. Quarterly population is computed by linear interpolating the log level of the annual population. All variables are normalized by their 1993:Q1 values. Vertical line on 1995:Q2. Source: IMF-IFS

characteristic features of a sudden stop: an abrupt decline in consumption and GDP that coincides with a reversal of the trade balance. From peak (1994:Q4) to trough (1995:Q2), GDP and consumption display contractions of 9.2% and 11.2%, respectively. Investment displays a much sharper decline of 40%, and the trade balance displays a reversal of 6.4% from -2.3% to 4.1%. Leading up to the crisis, the economy displays a characteristic buildup in GDP, consumption, and borrowing before the sudden stop.⁵¹ Figure B.2 and B.1 of the appendix depict the crisis in terms of quarterly cyclical components and growth rates.



Figure 12. Mexican Peso Crisis: Annual Aggregates

Notes: This figure plots the evolution of annual aggregate GDP, consumption, investment, and the trade balance to GDP ratio in Mexico during the mid 1990s. All variables excluding the trade balance to GDP ratio are in real, log-level, per capita terms. All variables are normalized by their 1993 values. Vertical line on 1995. Source: WDI.

Figure 12 displays the annual series. All variables are in annual, per capita log levels, excluding the trade balance to GDP ratio which is in levels. For the annual data, GDP and consumption displays decrease by 8.4% and 12.7% from 1994 to 1995, respectively. Inspecting the quarterly data, annual consumption displays a larger decline than GDP because it remains depressed throughout 1995. Relative to the quarterly data,

 $^{^{51}}$ Because I consider perfect foresight shocks from the stationary steady state, my model excludes the 'boom-bust' that develops in global models such as E. G. Mendoza (2010) and Villalvazo (2023).
investment displays a smaller drop of 25.6% and the trade balance to GDP ratio displays a slightly smaller reversal of 5.9%.



Figure 13. Neumeyer and Perri (2005): Procyclical Interest Rates in Emerging Economies

Notes: This figure characterizes the cyclical component of output and interest rates in emerging markets. Output is seasonally adjusted and detrended using a log-linear trend. For each country, the interest rate measure is the EMBI index of dollar denominated bonds specific to each country. Source: Neumeyer and Perri (2005) replication files.

5.2 Aggregates: Procyclical Interest Rates in Emerging Markets

In this section, I characterize the cyclicality of the interest rate in emerging and developed markets. Using the replication data available in Neumeyer and Perri (2005), I measure the interest rate in emerging markets using the 90 day treasury bill plus the J.P. Morgan EMBI+ spread.⁵² Figure 13 plots the interest rate and cyclical component for several emerging markets, including Mexico, Argentina, Brazil, and the Philippines. We can observe that when output experiences a decline, the interest rate simultaneously increases. This

 $^{^{52}}$ As discussed in Neumeyer and Perri (2005) and Uribe and Yue (2006), because the EMBI spread is in denominated in US dollars, the real rate is computed by subtracting a measure of US inflation. Using measures of the lending and deposit rate from IMF-IFS deliver negative interest rates with a variety of inflation expectation measures due to the inflation experienced during the Mexican Peso Crisis.



Figure 14. Neumeyer and Perri (2005): Countercyclical Interest Rates in Developed Economies

Notes: This figure characterizes the cyclical component of output and interest rates in emerging markets. Output is seasonally adjusted and detrended using a log-linear trend. Source: Neumeyer and Perri (2005) replication files.

captures the 'when it rains it pours' phenomenon described in Kaminsky et al. (2004): during bad times the cost of borrowing to maintain consumption has increased.

While most dramatic for the Argentinian recessions, the increase in the interest rate during the Mexican Peso Crisis is clear. In the first quarter of 1994, the rate was below 5% per annum. By the second quarter of 1995, the rate had increased to a peak of 19%. Afterwards, the rate displayed a trough of 6.5% in the third quarter of 1997.

Figure 14 plots the interest rate and cyclical component of output for several developed markets, including Australia, Canada, New Zealand, and the Netherlands. Here we can see that when output decreases, the interest rate closely follows with a decrease, and the reverse occurs during an output increase.⁵³ As noted in Kaminsky et al. (2004), the interest rate acts as a buffer for output fluctuations: in bad times it is affordable to borrow to maintain consumption and in good times the interest rate increase puts downward pressure on consumption.

5.3 Household: Consumption Responses During the Crisis

This section studies how households differ in their consumption responses during the Mexican Peso Crisis. I replicate Guntin et al. (2023)'s finding that households display little variation in consumption responses across the income distribution during the Mexican Peso Crisis. My primary measure is the consumptionincome elasticity: the percentage change in consumption with respect to the percentage change in income. I compute consumption-income elasticities using ENIGH, which surveys Mexico biennially from 1992 to 2014 and documents household income and expenditures. For the income measure I include after tax salaries, business income, and transfers. For the consumption measure I include expenditures on food, personal items, and clothing.

Using the 1994 and 1996 datasets, I characterize the responses of consumption and income using the model

$$X_{it} = \alpha + \beta Y_{it} + \gamma \text{POST}_t \times Z_{it} + \zeta d_{it} + \delta \text{POST}_t \times d_{it} + \epsilon_{it}$$
(42)

where X_{it} denotes the log consumption or log income of household *i* at time *t*. Following Blundell et al. (2008), the term Y_{it} includes controls for household size, locality size, and the sex, education, and a quadratic function of the age of the household head. POST denotes whether an observation occurs during 1996. d_{it} denotes the household *i*'s income decile at time *t*, and POST_t × d_{it} denotes the household *i*'s income decile, interacted with POST_t. POST_t × Z_{it} includes the sex and education of the household head, interacted with POST_t. Using this method, δ measures the percentage change of consumption or income for households

 $^{^{53}}$ We can interpret the lag in the interest rate change as the endogenous response of the monetary policy setter.

within each income decile from 1994 to 1996. I compute the consumption-income elasticities across the income distribution by dividing the estimate of the percentage change in consumption for each income decile with the estimate of the percentage deviation in income for each decile. Finally, I compute bootstrapped errors by taking 2000 samples from the household data with replacement, using the sample weights provided in the data.

Figure 15 plots average the consumption-income elasticity within each income decile. Notably, the average consumption-income elasticity is nearly one for one with income and displays little variation across the income distribution. For our purposes, the average response of nearly one to one is not remarkable. In fact, in the aggregate time series, the cyclical component of consumption moved more than one for one with the cyclical component of GDP.⁵⁴ To the extent that the household data can replicate the aggregate data, we would expect to see a similar average response.



Figure 15. Mexican Peso Crisis: Consumption-Income Elasticities

Notes: This figure plots the average consumption-income elasticity within each income decile from 1994 to 1996. Data is drawn from the 1994 and 1996 samples of ENIGH. The heterogeneous changes of consumption and income for each income decile are identified using equation (42) and residualized by household size, locality size, and the sex, education, and a quadratic function of the age of the household head. Standard errors are computed using 2000 bootstrap replications. Figure B.4 displays the separate heterogeneous responses of consumption and income.

Figure 15 motivates this paper from the perspective of the household data. Why do households display little heterogeneity in consumption responses? From a certain perspective, Figure 15 supports the representative agent framework: households follow similar consumption policies and can be abstracted into a single representative household. Empirically, this is rebutted by Hong (2023b), who documents heterogeneity in

 $^{^{54}}$ This holds for both peak (1994:Q4) to trough (1995:Q2) and from the 1994 sample of ENIGH (1994:Q3) to the 1996 sample of ENIGH (1996:Q3).

	Liquid: Formal	Savings: Formal	Debt: Formal	Liquid: Informal	Savings: Informal	Debt: Informal
2nd Income Quartile	0.079^{***} (0.001)	-0.036^{***} (0.002)	0.264^{***} (0.002)	0.090^{***} (0.001)	0.123^{***} (0.002)	0.025^{***} (0.002)
3rd Income Quartile	(0.001) 0.286^{***} (0.001)	(0.002) (0.275^{***}) (0.002)	(0.002) 0.106^{***} (0.002)	(0.001) (0.112^{***}) (0.001)	(0.063^{***}) (0.002)	(0.002) 0.114^{***} (0.002)
4th Income Quartile	(0.001) 0.837^{***} (0.001)	(0.002) 0.831^{***} (0.002)	(0.002) 0.596^{***} (0.002)	0.362^{***} (0.001)	(0.002) 0.173^{***} (0.002)	(0.002) (0.375^{***}) (0.002)
Controls	Y	(0.002) Y	(0.002) Y	Y	(0.002) Y	(0.002) Y
Observations	2946	2946	2946	2946	2946	2946
					4	dubit o o t

Note:

*p<0.1; **p<0.05; ***p<0.01

Standard Errors in Parentheses

Table 5. MFL: Income and Access to Financial Markets

Notes: This table characterizes the relationship between income and access to financial markets. Data is drawn from the 2005 wave of the Mexican Family Life (MFL) Survey. Access to financial markets is indicated by whether a household possesses a given type of asset. Assets include formal savings, formal debts, informal savings, and informal debts, categorized in Tables (10) and (11). A household holds formal liquid assets if it holds either formal savings or formal debts. Informal liquid assets is similarly defined.

The relationship between income and access to financial markets is measured using the probit model of equation (43), which includes a set of income quartiles and a set of controls. Income includes salaried and business income. The set of controls includes the sex, education, a quadratic function of the age of the household head, and controls for the state of residence.

the marginal propensity to consume (MPC) across the income distribution using the methodology presented in Blundell et al. (2008). Hong (2023b)'s computation and Figure 15 differ in that Hong (2023b) studies the response to an identified small increase in liquid assets, whereas the aggregate data suggests that Figure 15 is almost certainly driven by multiple large shocks.

5.4 Household: Heterogeneity in Access to Financial Markets

This section characterizes the financial environment of households in Mexico using the Mexican Family Life Survey (MFL). Similar to Allen et al. (2016), I show that the likelihood of having access to financial markets is increasing in income.

MFL underwent three waves in 2002, 2005-2006, and 2009-2012 and surveys households on their income, liquid assets, and illiquid assets. I use the 2005-2006 wave which includes 5785 households. I include households that have a household head between 25 and 60 years of age and earn some income over the year. The first restriction reduces the sample to 4363 households; the second restriction further reduces the sample to 2946 households. Income includes after tax salaries, wages, piecework, tips, bonuses, and net business income.⁵⁵ Household income is computed by summing over the income of individuals within the household

⁵⁵MFL either reports a participants salary or provides a decomposition of their salary into wages, piecework, tips, etc.

	Durable	Property	Animal	Financial
2nd Income Quartile	0.405^{***}	-0.046***	-0.156***	-0.138^{***}
3rd Income Quartile	(0.003) 0.296^{***}	(0.001) 0.017^{***}	(0.002) -0.321^{***}	(0.002) 0.266^{***}
4th Income Quartile	(0.003) 0.425^{***}	(0.001) -0.037^{***}	(0.002) - 0.303^{***}	(0.002) 0.713***
Controls	(0.003) Y	(0.001) Y	(0.002) Y	(0.002) Y
Observations	2946	2946	2946	2946
Note:		*p<().1: **p<0.05	: ***p<0.01

*p<0.1; **p<0.05; ***p<0.01 Standard Errors in Parentheses

Table 6. MFL: Income and Illiquid Assets

Notes: This table characterizes how households differ in their possession of illiquid assets. Data is drawn from the 2005 wave of the Mexican Family Life (MFL) Survey. Illiquid assets are classified into durable goods, property, animals, and financial assets. A household only owns property if its owns its home outright. The relationship between income and illiquid asset possession is measured using the probit model of equation (43), which includes a set of income quartiles and a set of controls. Income includes salaried and business income. The set of controls includes the sex, education, a quadratic function of the age of the household head, and controls for the state of residence.

between 25 and 60 years of age.

MFL surveys households about their savings and debts at both the household and individual level. I use data at the individual level because it includes more detailed information on the type of savings and debts. Households hold savings in banks, cooperatives, the *caja solidaria* program, within the household, *afores* programs, with a friend, and other. Debts originate from banks, savings funds, moneylenders, friends, relatives, work, pawnshops, verbal agreements, and government programs. I define a savings or debt as 'formal' if it is likely to charge interest.⁵⁶ Formal savings are held at banks, cooperatives, *afores*, and *caja solidaria*. I define formal debts as those from banks, savings funds, moneylenders, and pawnshops.⁵⁷ I define informal savings as those held at the house, with a friend that is not the household head, and at work, and I define informal debts as those from relatives, friends, pawnshops, and verbal agreements. A household has a formal saving if any of its members between the ages of 25 and 60 possess a formal saving, and similarly for formal debts, informal savings or formal debts. I define a household as possessing formal liquid assets if it possesses formal savings or formal debts, and I similarly define informal liquid assets.⁵⁸ Lastly, I also consider how households differ in their possession of illiquid assets, which I categorize into durable goods, property, animals, and financial assets.⁵⁹

⁵⁶Within MFL, interest rates are explicitly documented for debts. Rates of return are not documented for savings.

 $^{^{57}}$ All debts are originated within the previous 12 months.

 $^{^{58}}$ Under this definition, a household that has both debts and savings and a zero liquid wealth position is defined as possessing liquid assets.

⁵⁹These assets make up the vast majority of the illiquid asset stock. I only document a household as owning housing if it

I measure the relationship between income and access to financial markets using the probit model

$$Y_{i} = \alpha + \beta Z_{i} + \kappa D_{i} + \epsilon_{i}$$

$$X_{i} = \begin{cases} 1 \quad Y_{i} \ge 0 \\ 0 \quad Y_{i} < 0 \end{cases}$$

$$(43)$$

where X_i is an indicator for whether a household possesses a certain asset type, Z_i is an indicator for a household's income quartile, D_i is a vector of controls, and ϵ_i is an error term. The vector of controls D_i includes the sex, education, and a quadratic function of the age of the household head, and controls for the state of residence.

Table 5 displays the regression results for savings and debts. The most salient result is that access to financial markets is increasing in income. This holds for both savings and debts regardless of whether they are formal or informal. This result also holds without controls, documented in Table 13. Table 6 displays the results for illiquid assets. Relative to the lowest income decile, higher income deciles are more likely to possess durable goods and financial assets. The likelihood of owning one's home outright displays small but significant variation with income, and low income households are more likely to own animals.

Tables 5 and 6 consider liquid and illiquid asset holdings from a purely static perspective. However, the observed inequality in access to financial markets has important implications for household responses to aggregate fluctuations of the economy. Following an income shock, we expect unconstrained households to use their liquid assets to maintain their consumption.⁶⁰ Because high income households are more likely to be unconstrained, we therefore expect them to display smaller consumption responses to a decline in income relative to low income households. Conditional on an interest rate shock, unconstrained households have an incentive to change their consumption because its relative price has changed. Following the same line of logic, we expect high income households.⁶¹ Empirically, this is documented by Vissing-Jørgensen (2002) and Havranek et al. (2015) who show that asset holders display larger responses to interest rate

fluctuations.⁶²

owns its house outright, and exclude machinery because it is owned by a small portion of households. I only consider outright home ownership because MFL does not consider net home value for households that are on a mortgage.

 $^{^{60}}$ This assumes that constrained households hold zero liquid assets due to a constraint rather than coincidentally. Kaplan et al. (2018)'s model implements this by introducing a wedge between the lending and deposit rates. While harder to implement in discrete time, this is supported by the wedge between lending and deposit rates we observe in the IMF-IFS interest rate series.

 $^{^{61}}$ An exception to this is if interest rate shocks primarily generate consumption fluctuations through indirect effects such as labor income, like in Kaplan et al. (2018). This will be explicitly allowed and studied in the model.

 $^{^{62}}$ Vissing-Jørgensen (2002) and Havranek et al. (2015) differ in that Vissing-Jørgensen (2002) studies asset holders within the United States whereas Havranek et al. (2015) performs a meta analysis of countries with varying levels of financial development.

6 Conclusion

What drove the Mexican Peso Crisis of 1994? I examine this question using a heterogeneous agent model that characterizes the financial environment of households in emerging markets. I show that a combination of transitory productivity and interest rate shocks captures the Mexican Peso Crisis from the perspective of both the household and aggregate data. A permanent decline in productivity, which generates a permanent decline in income, can capture household consumption responses but fails to account for the responses of aggregates such as investment and the trade balance.

The presence of an illiquid asset allows the model to better capture the household financial environment in Mexico. When calibrated to an emerging market, the single asset model implies an extremely high MPC for the lowest income decile, so that they display a near one to one response for even temporary declines in income. The presence of an illiquid asset weakens this relationship so that low income households do not display a one to one response to income. This reduces the spread between the consumption responses of low and high income households. In terms of the equilibrium dynamics, the contribution of the illiquid asset is conservative in that the general equilibrium movement of the illiquid asset return makes a small contribution to consumption responses. A model that better captures inequality in asset holdings and equilibrium asset price movements could feature a more significant role of the illiquid asset for high income households.

The Mexican Peso Crisis featured a significant increase in the interest rate. This paper takes the increase as given and studies the consequences on the household and production sides of the economy. This abstracts from the policy setter that had a motivation to combat high levels of inflation and bring stability to the exchange rate. While the interest rate increase is purely contractionary in this model, a richer model could introduce the exchange rate devaluation and hyperinflation observed during the Mexican Peso Crisis to develop and study a richer policy tradeoff.

It is well known that economies can feature permanent declines in productivity and consumption following a crisis, as documented in Cerra and Saxena (2008). This paper provides an explanation for large consumption declines outside of the context of a permanent decline in the economy. The critical feature is the behavior of high income households. If high income households are able to maintain their consumption during a crisis, then we expect the aggregate consumption response to be small because high income households take up a disproportionate share of aggregate consumption. If income is the only driver, and displays a temporary decrease, high income households exploit their asset holdings to maintain their consumption. This paper introduces one of the signature features of a sudden stop, the procyclical increase in the interest rate, which motivates high income households to decrease their consumption and is largely ignored by low income households. This generates consumption responses that align with the data and aggregate responses that align with those observed during the Mexican Peso Crisis.

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A Model

A.1 Impulse Responses



Figure A.1. Impulse Responses to Productivity Shock

Notes: This figure plots aggregate impulse responses to a 1% increase in productivity with a persistence of 0.53.



Figure A.2. Impulse Responses to Interest Rate Shock

Notes: This figure plots aggregate impulse responses to a 1% decrease in the interest rate with a persistence of 0.62.



Figure A.3. Impulse Responses to Sudden Stop

Notes: This figure plots impulses responses to the transitory shocks described in Table 4.



Figure A.4. Impulse Responses to Permanent Shock

Notes: This figure plots impulse responses to the permanent decline in productivity described in Table 4. For each variable, the impulse response is relative to its initial steady value.



Figure A.5. Decomposed Consumption-Income Elasticities: One-Period Elasticity

Notes: This figure decomposes the consumption responses of households to the transitory shocks described in Table 4.



Figure A.6. View II (Permanent): Decomposed Consumption Response

Notes: This figure decomposes the consumption responses of households to the permanent shock described in Table 4.



Figure A.7. View II (Permanent): Long Run Movement of Capital and Labor Income

Notes: This figure plots the responses of capital and labor to the permanent shock described in Table 4.



Figure A.8. Stronger Illiquid Asset Contribution: Drivers

Notes: This figure plots the general equilibrium paths of the illiquid asset return, the firm value, and the interest rate under the transitory sudden stop exercise for three values of capital adjustment costs, $\phi = 2.0, 5.0, 10.0$. For each value of ϕ , I input the calibrated productivity shock of Table 4 and calibrate the increase in the interest rate $d\mu_0$ to target a one to one initial consumption to output response.



Figure A.9. Stronger Illiquid Asset Contribution: Consumption Responses

Notes: This figure plots the one and two period elasticities consumption income elasticities under the transitory sudden stop exercise three values of capital adjustment costs, $\phi = 2.0, 5.0, 10.0$. For each value of ϕ , I input the calibrated productivity shock of Table 4 and calibrate the increase in the interest rate $d\mu_0$ to target a one to one initial consumption to output response.



Figure A.10. Stronger Illiquid Asset Contribution: Decomposed Consumption Responses

Notes: This figure decomposes the two year consumption-income elasticities within each income decile with respect to labor income, the interest rate, the illiquid asset return, and a nonlinearity term, conditional on the sudden stop. The sudden stop is recalibrated to included moderate capital adjustment costs, $\phi = 5.0$. The decomposed responses are computed by separately inputting in the general equilibrium paths of labor income, the interest rate and the illiquid return. The nonlinearity is computed as the difference between the consumption-income elasticity computed using all inputs and the sum of the consumption responses, weighted by the percentage change in income, conditional on each input.



Figure A.11. One and Three Year Consumption-Income Elasticities

Notes: This figure computes consumption-income elasticities while allowing for larger gaps of time between the first and second consumption and income policies that are used to compute the elasticities. For all elasticities, the first observation is given by consumption and income one period before the shock, which is given by the steady state. 'One year' computes the elasticities using the immediate consumption and income responses at time 0. 'Two years' computes elasticities using the consumption policies in periods 1 and -1, respectively. 'Three years' computes elasticities using the consumption policies in periods 2 and -1, respectively.



Figure A.12. Robustness Dr0

Notes: This figure plots two year consumption-income elasticities for three values of the initial increase in the interest rate, $dr_0 = 0.06, 0.12, 0.18$. Each elasticity features the same path of productivity as described in Table 4.



Figure A.13. Comparison with Single Asset Model: Steady State

Notes: This figure compares the steady state of the two asset household problem presented in Section 2.1 with three calibrations of the single asset problem presented in Section 4.6. ' β matching' uses the discount factor of the calibrated two asset model. 'Constrained matching' and 'MPC matching' recalibrate β to match the percent of constrained household and average MPC of the two asset model, respectively. The left panel plots the percent of constrained households within each income decile under each model. The right panel plots the average MPC within each income decile.



Figure A.14. Single Asset Model: Consumption Responses

Notes: This figure plots the consumption responses of the single asset model presented in Section 4.6 to transitory shocks in labor income and the interest rate. Labor income wL follows the endogenous labor income path generated by the transitory shocks of Table 4. The interest rate features a variable initial increase and reverts to its steady state with a persistence of 0.62. Darker lines correspond to a higher increase in the interest rate, with an initial increase of 0.01 and maximum increase of 0.15. The left panel plots one year consumption-income elasticities. The right panel plots two year consumption-income elasticities.



Figure A.15. Single Asset Model: The Role of Persistence

Notes: This figure studies how household responses vary with the persistence of aggregate shocks in the single asset household problem. Average income and idiosyncratic income risk match their counterparts in the steady state of the two asset model.

The upper two plots display the one and two period responses to a 5.4% decline in productivity with persistence values of 0.50, 0.70, 0.80, 0.90. For each value of the productivity persistence, Households respond to the general equilibrium path of labor income in the two asset model. The lower two plots display the one and two period responses to a 12% percent increase in the interest rate with persistence values of 0.50, 0.70, 0.80, 0.90.

B Data

B.1 Aggregate



Figure B.1. Growth Rates, Mexican Peso Crisis

Notes: This figure plots the year over year growth rates of consumption, GDP, investment, and the trade balance to GDP ratio. All variables are in real, seasonally adjusted, and in per capita terms. Quarterly population is computed by linearly interpolating the log level of the annual population. Vertical line on 1995:Q2. Source: IMF-IFS.



Figure B.2. Aggregates, Mexican Peso Crisis

Notes: This figure plots the cyclical components of aggregate consumption, GDP, investment, and the trade balance to GDP ratio in Mexico during the mid 1990s. All variables are in real, seasonally adjusted, in per capita terms and detrended using the Hodrick-Prescott filter with a smoothing parameter of 1600. Quarterly population is computed by linearly interpolating the log level of the annual population. Vertical line on 1995:Q2. Source: IMF-IFS



Figure B.3. Annual Cyclical Components, Mexican Peso Crisis

Notes: This figure plots the annual cyclical components of the Mexican Peso Crisis. All variables are in real, per capita log levels and detrended using the HP Filter with a smoothing parameter of 6.25. Vertical line on 1995. Source: WDI

B.2 ENIGH: Heterogeneity in Responses

The model is given by

$$X_{it} = \alpha + Z_{it} + \text{POST}_t D_{it} + \gamma_{it} + \text{POST}_t \beta_{it} + \epsilon_{it}$$
(44)

where Z_{it} is a set of controls that includes the sex, education, and a quadratic function of the age of the household head. POST denotes whether the year is 1996. $\text{POST}_t D_{it}$ includes the sex and education of the household head, interacted with POST_t . γ_{it} denotes the household's income decile, and $\text{POST}_t\beta_{it}$ denotes the household's income decile, interacted with POST_t .



Figure B.4. ENIGH: Consumption and Income Fluctuations, by Decile

Notes: This figure plots the percentage change in consumption and income for each income decile using the 1994 and 1996 data of ENIGH. Consumption and income are residualized by household size, locality size, and the sex, education, and a quadratic function of the age of the household head. The average percentage change of consumption or income for each income decile is obtained by interacting the income decile with an indicator for whether the period is 1996.

Variable	Code				
Household ID	folio				
Household Size	tamhog				
Panel Weight	factor				
Location Size	estrato				
Consumption					
Consumption: Total	gastot				
Consumption: Current	gascor				
Consumption: Food	alimentos				
Consumption: Clothing	vestido				
Consumption: Health	salud				
Consumption: Education	educacion				
Consumption: Personal	personal				
Income					
Income: Total	ingtot				
Income: Current	ingcor				
Income: Salary	trabajo				
Income: Business	negocio				
Income: Rental	rentas				
Income: Transfers	transfer				
Income: Other	otros				

Table 7. ENIGH: Variables

Notes: This table displays variables from the ENIGH dataset and their database names.

B.3 MFL: Heterogeneity in Asset Holdings

Code	Variable
Income	$tb36a_2$
Wages	$tb36aa_2$
Piecework	$tb36ab_2$
Tips	$tb36ac_2$
Extra Hours	$tb36ad_2$
Christmas Bonus	$tb36ae_2$
Annual Bonus	$tb36af_2$
Profit Distribution	$tb36ah_2$
Other	$tb36am_21$
Second Job	$tb36b_2$
Net Income Main Business	$tb36p2_2$
Net Income Second Business	$tb36s2_2$

Table 8. MFL: Income Sources

Notes: This table documents income sources in MFL. For the majority of households, income is documented under 'income'. Otherwise, income is decomposed under wages, piecework, tips, etc.

Asset	Possession	Value	Type
House	ah03a	$ah04a_2$	Property
Other House	ah03b	$ah04b_2$	Property
Bicycle	ah03c	$ah04c_2$	Durable
Vehicle	ah03d	$ah04d_2$	Durable
Electronics	ah03e	$ah04e_2$	Durable
Washing Machine/ Stove	ah03f	$ah04f_2$	Durable
Domestic Appliance	ah03g	$ah04g_2$	Durable
Financial Asset	ah03h	$ah04h_2$	Financial
Machinery	ah03i	$ah04i_2$	Durable
Bull/Cow	ah03j	$ah04j_2$	Animal
Horses/Mules	ah03k	$ah04k_2$	Animal
Pigs/Goats	ah03l	$ah04l_2$	Animal
Poultry	ah03m	$ah04m_2$	Animal
Other	ah03n	$ah04n_2$	

Table 9. MFL: Illiquid Assets

Notes: This table documents how illiquid assets are classified. 'Possession' is an indicator for whether a household possesses a type of illiquid asset. 'Value' lists the estimated value of the asset. 'Type' refers to the asset's classification.

Savings Location	Variable	Type
No Savings	$cr29_1a$	
Bank	$cr29_{-}1b$	Formal
Coop	$cr29_{-1}c$	Formal
Savings Bank	$cr29_1d$	Formal
Friend (not household head)	$cr29_1e$	Informal
a fores	$cr29_1f$	Formal
caja solidaria	$cr29_1g$	Formal
At House	$cr29_1h$	Informal
Work	$cr29_1i$	Informal
Other	$cr29_1j$	Informal

Table 10. MFL: Has Savings

Notes: This table documents savings types in MFL and whether they are classified as formal or informal.

Code: cr14_1					
Loan Location	Value	Type			
Bank	1	Formal			
Savings Fund	2	Formal			
Moneylender	3	Formal			
Relative	4	Informal			
Friends	5	Informal			
Work	6	Informal			
Pawn Shop	7	Formal			
Verbal Agreement Credit Program	8	Informal			
Other Government	9	Informal			
Other	10	Informal			

Table 11. MFL: Loan Sources

Notes: This table documents loan types in MFL and whether they are classified as formal or informal.

Loon Turno	Percent of Loans		
Loan Type	that Charge Interest		
Bank	62		
Savings Fund	82		
Money Lender	75		
Relative	10		
Friends	36		
Work	22		
Pawnshop	66		
Other Government	32		
Other	27		

Table 12. MFL: Percent of Loans that Charge Interest, by Type

Notes: This table documents the percentage of loans that charge interest for each loan type. Loans are taken from individual level data in MFL. Computations are weighted using the panel weights provided in the data.

	Liquid: Formal	Savings: Formal	Debt: Formal	Liquid: Informal	Savings:	Debt: Informal
	Formar	Formar	Formar	mormai	mormai	mormar
2nd Income Quartile	0.079^{***}	-0.036***	0.264^{***}	0.090***	0.123^{***}	0.025^{***}
	(0.001)	(0.002)	(0.002)	(0.001)	(0.002)	(0.002)
3rd Income Quartile	0.286^{***}	0.275^{***}	0.106^{***}	0.112^{***}	0.063^{***}	0.114^{***}
	(0.001)	(0.002)	(0.002)	(0.001)	(0.002)	(0.002)
4th Income Quartile	0.837***	0.831***	0.596***	0.362***	0.173***	0.375***
	(0.001)	(0.002)	(0.002)	(0.001)	(0.002)	(0.002)
Controls	Ν	Ν	Ν	Ν	Ν	Ν
Observations	2946	2946	2946	2946	2946	2946

Note:

p<0.1; p<0.05; p<0.05; p<0.01Standard Errors in Parentheses

Table 13. MFL: Income and Financial Inclusion, No Controls

Notes: This table repeats the probit regression of Table 5 without controls.

	Durable	Property	Animal	Financial
2nd Income Quartile	0.311***	-0.076***	-0.400***	-0.064***
	(0.002)	(0.001)	(0.001)	(0.002)
3rd Income Quartile	0.273***	-0.002**	-0.529***	0.395***
	(0.002)	(0.001)	(0.001)	(0.001)
4th Income Quartile	0.465^{***}	0.109***	-0.632***	0.869***
	(0.003)	(0.001)	(0.001)	(0.001)
Controls	Ν	N	N	N
Observations	2946	2946	2946	2946

Note:

*p<0.1; **p<0.05; ***p<0.01

Standard Errors in Parentheses

Table 14. MFL: Income and Illiquid Assets

Notes: This table repeats the probit regression of Table 6 without controls.

C Conditions

C.1 Steady State Conditions

$$z_{ss} = 1 \tag{C.1.45}$$

$$\mu_{ss} = 1 \tag{C.1.46}$$

$$r_{ss} = r^* \tag{C.1.47}$$

$$r_{ss}^b = r^* \tag{C.1.48}$$

$$\log(e') = \rho_e \log e + \sigma_e \epsilon_e, \epsilon^e \sim \mathcal{N}(0, 1) \tag{C.1.49}$$

$$u(c_{ss}(e,b,a)) = \mu_{ss}^{b}(e,b,a) + \beta(1+r^{*})Eu(c_{ss}(e',b',a')) \quad (C.1.50)$$

$$u(c_{ss}(e,b,a))(1+\chi_1(a_{ss}(e,b,a),a)) = \mu_{ss}^a(e,b,a) + \beta E(1+r^a - \chi_2(a_{ss}(e,b,a),a))u(c_{ss}(e',b',a'))$$

(C.1.51)

$$c_{ss}(e,b,a) + b_{ss}(e,b,a) + a_{ss}(e,b,a) + \chi(a_{ss}(e,b,a),a) = (1+r^*)b + (1+r^a)a + ewL$$
(C.1.52)

$$\mu^{b}_{ss}(e,b,a) \ge 0 \tag{C.1.53}$$

$$\mu_{ss}^a(e,b,a) \ge 0 \tag{C.1.54}$$

$$\mu_{ss}^{b}(e,b,a)b_{ss}(e,b,a) = 0 \tag{C.1.55}$$

$$\mu_{ss}^{a}(e,b,a)a_{ss}(e,b,a) = 0 \tag{C.1.56}$$

$$\Psi_{ss}(e, b, a) = Pr(e \le a, a \le a, b \le b) \tag{C.1.57}$$

$$C_{ss} = \int_{e,b,a} c_{ss}(e,b,a) d\Psi_{ss}(e,b,a)$$
(C.1.58)

$$B_{ss} = \int_{e,b,a} b_{ss}(e,b,a) d\Psi_{ss}(e,b,a)$$
(C.1.59)

$$A_{ss} = \int_{e,b,a} a_{ss}(e,b,a) d\Psi_{ss}(e,b,a)$$
(C.1.60)

$$\chi_{ss} = \int_{e,b,a} \chi(a_{ss}(e,b,a),a) d\Psi_{ss}(e,b,a)$$
(C.1.61)

$$Y_{ss} = z_{ss} K^{\alpha}_{ss} L^{1-\alpha}_{ss} \tag{C.1.62}$$

$$\pi_{ss} + K_{ss} = z_{ss} K_{ss}^{\alpha} L_{ss}^{1-\alpha} + (1-\delta) K_{ss} - w_{ss} L_{ss}$$
(C.1.63)

$$(1 + r_{ss}^{a}) = z_{ss} \alpha K_{ss}^{\alpha - 1} L_{ss}^{1 - \alpha} + 1 - \delta$$
(C.1.64)

$$w_{ss} = z_{ss}(1-\alpha)K_{ss}^{\alpha}L_{ss}^{-\alpha} \tag{C.1.65}$$

$$L_{ss} = 1 \tag{C.1.66}$$

$$w_{ss} = \kappa (L_{ss})^{\omega} \tag{C.1.67}$$

$$I_{ss} = \delta K_{ss} \tag{C.1.68}$$

$$1 + r_{ss}^{a} = \frac{q_{ss} + \pi_{ss} + \chi_{ss}}{q_{ss}}$$
(C.1.69)
$$A_{ss} = q_{ss}$$
(C.1.70)

$$A_{ss} = q_{ss} \tag{C.1.70}$$

$$TB_{ss} = B_{ss} - (1 + r_{ss})B_{ss}$$
(C.1.71)

$$TBY_{ss} = TB_{ss}/Y_{ss} \tag{C.1.72}$$

C.2 List of Equilibrium Conditions

$$\log e_t = \rho_e \log e_{t-1} + \sigma_e \epsilon_t^e, \epsilon_t^e \sim \mathcal{N}(0, 1)$$
(C.2.1)

$$u(c_t(e, b, a)) = \mu_t^b + \beta E_t(1 + r_t^b)u(c_{t+1}(e, b, a))$$
(C.2.2)

$$u(c_t(e, b, a))(1 + \chi_1(a_t(e, b, a), a)) =$$
(C.2.3)

$$\mu_t^a + \beta E_t (1 + r_{t+1}^a - \chi_2(a_{t+1}(e', b', a'), a_t(e, b, a))) u(c_{t+1}(e', b_t(e, b, a), a_t(e, b, a)))$$

$$c_t(e, b, a) + b_t(e, b, a) + a_t(e, b, a) + \chi(a_t(e, b, a), a) = (1 + r_{t+1}^b)b + (1 + r_t^a)a + ew_t L_t$$
(C.2.4)

$$\mu_t^b(e,b,a) \ge 0 \tag{C.2.5}$$

$$\mu_t^a(e,b,a) \ge 0 \tag{C.2.6}$$

$$\mu_t^b(e, b, a)b_t(e, b, a) = 0 \tag{C.2.7}$$

$$\mu_t^a(e, b, a)a_t(e, b, a) = 0 \tag{C.2.8}$$

$$\Psi_t(e, a, b) = Pr(e_t \le e, a_{t-1} \le a, b_{t-1} \le b)$$
(C.2.9)

$$\Psi_{t+1}(e',b',a') = \tag{C.2.10}$$

$$\int_{e,b,a} \Pr(e_{t+1} \le e'|e_t = e) \mathcal{I}\left[a_t(e,b,a;\Gamma) \le a', b_t(e,b,a;\Gamma) \le b'\right] d\Psi_t(e,b,a)$$

$$C_{t} = \int_{e,b,a} c_{t}(e,b,a) d\Psi_{t}(e,b,a)$$
(C.2.11)

$$B_{t} = \int_{e,b,a} b_{t}(e,b,a) d\Psi_{t}(e,b,a)$$
(C.2.12)

$$A_{t} = \int_{e,b,a} a_{t}(e,b,a) d\Psi_{t}(e,b,a)$$
(C.2.13)

$$\chi_t = \int_{e,b,a} \chi_t(a_t(e,b,a),a) d\Psi_t(e,b,a)$$
(C.2.14)

$$Y_t = z_t K_{t-1}^{\alpha} L_t^{1-\alpha}$$
 (C.2.15)

$$\pi_t + K_t + \Phi(K_t, K_{t-1}) = z_t K_{t-1}^{\alpha} L_t^{1-\alpha} + (1-\delta)K_{t-1} - w_t L_t$$
(C.2.16)

$$(1 + r_{t+1}^{a})(1 + \Phi_1(K_t, K_{t-1})) = E_t \left(z_{t+1} \alpha K_t^{\alpha} L_{t+1}^{1-\alpha} + 1 - \delta - \Phi_2(K_{t+1}, K_t) \right) \quad (C.2.17)$$

$$w_t = z_t (1 - \alpha) K_{t-1}^{\alpha} L_t^{-\alpha}$$
(C.2.18)

$$w_t = \kappa(L_t)^{\omega} \tag{C.2.19}$$

$$I_t = K_t - (1 - \delta)K_{t-1}$$
(C.2.20)

$$1 + r_t^a = \frac{q_t + \pi_t + \chi_t}{q_{t-1}} \tag{C.2.21}$$

$$r_t = r^* + \mu_t - 1 \tag{C.2.22}$$

$$r_t^b = r_{t-1}$$
 (C.2.23)

$$A_t = q_t \tag{C.2.24}$$

$$TB_t = B_t - (1+r_t)B_{t-1} \tag{C.2.25}$$

$$TBY_t = TB_t/Y_t \tag{C.2.26}$$
D Decomposition Methodology

Elasticities As discussed in Section 2, the path of aggregate shocks $z = \{z_t\}_{t=0}^T$, $\mu = \{\mu_t\}_{t=0}^T$ produces the general equilibrium path of households inputs $\Gamma = \{w_t L_t, r_t^b, r_t^a\}_{t=0}^T$. Given Γ , I compute the path of the distribution $\{\Psi_t(e, b, a; \Gamma)\}_{t=0}^T$, and policies $\{c_t(e, b, a; \Gamma)\}_{t=0}^T$, $\{b_t(e, b, a; \Gamma)\}_{t=0}^T$, $\{a_t(e, b, a; \Gamma)\}_{t=0}^T$. I then integrate over the liquid and illiquid assets to form the marginal distributions with respect to income and aggregates policies for each level of income: $\{\Psi_t(e; \Gamma)\}_{t=0}^T$, and $\{C_t(e; \Gamma), B_t(e; \Gamma), A_t(e; \Gamma)\}_{t=0}^T$ where

$$\Psi_t(e;\Gamma) = \int_b \int_a d\Psi_t(e,b,a;\Gamma)$$
(D.1)

$$C_t(e;\Gamma) = \int_b \int_a c_t(e,b,a;\Gamma) d\Psi_t(e,b,a;\Gamma)$$
(D.2)

and similarly for $B_t(e;\Gamma)$, $A_t(e;\Gamma)$.⁶³ Finally, I interpolate along the distribution of $e \ \psi_e(e;\Gamma)$ to compute the average policy of the *j*th income decile, $\{C_t(j;\Gamma), B_t(j;\Gamma), A_t(j;\Gamma)\}_{t=0}^T$ for j = 1, ..., 10 at each time *t*.

Dropping Γ , the percentage change in consumption of decile j after t periods is given by

$$\mathcal{E}_{C,t}(j) = \frac{C_t(j) - C_{-1}(j)}{C_{-1}(j)} \tag{D.3}$$

where $C_{-1}(j)$ is the consumption of decile j before impact.⁶⁴ Given $\{w_t, L_t\}_{t=0}^T$, the percentage change in labor income of decile j is

$$\mathcal{E}_{wL,t}(j) = \frac{e_t(j)w_tL_t - e_{-1}(j)w_{-1}L_{-1}}{e_{-1}(j)w_{-1}L_{-1}} = \frac{w_tL_t - w_{-1}L_{-1}}{w_{-1}L_{-1}}$$
(D.4)

where I've applied that the distribution of idiosyncratic income is exogenous and static, e.g. $e_t(j) = e_{-1}(j)$ for each income decile j and time period t. This implies all income deciles experience the same percentage change in income. Given the percentage change in consumption over two years $\mathcal{E}_{C,1}(j)$ and the percentage change in income over two years $\mathcal{E}_{wL,1}(j)$, I can compute the two-year consumption-income elasticity for decile j as $\mathcal{E}_1(j) = \mathcal{E}_{C,1}(j)/\mathcal{E}_{wL,1}(j)$. Similarly, we can compute the one-year elasticity as $\mathcal{E}_0(j) = \mathcal{E}_{C,0}(j)/\mathcal{E}_{wL,0}(j)$.

Decomposition Given $\{w_t L_t, r_t^b, r_t^a\}_{t=0}^T$, the consumption response of household (e, b, a) at time t can be decomposed as

$$c_t(e, b, a; wL, r^b, r^a) = c_t(e, b, a; wL) + c_t(e, b, a; r^b) + c_t(e, b, a; r^a) + \epsilon(e, b, a; wL, r^b, r^a)$$
(D.5)

⁶³After integrating over b and a, the distribution $\Psi_t(e)$ is static and given by the stationary distribution of equation (5).

⁶⁴In the basic exercise, $C_{-1}(j)$ coincides with the steady state $C_{ss}(j)$.

where $c_t(e, b, a; wL)$ denotes the consumption response when only the path of $\{w_t L_t\}_{t=0}^T$ deviates from the steady state and similarly for r_t and r_t^a . As before, I integrate with respect to b and a and interpolate over e to decompose the average consumption within each income decile j as

$$C_t(j;wL,r^b,r^a) = C_t(j;wL) + C_t(j;r^b) + C_t(j;r^a) + \epsilon_t(j;wL,r^b,r^a).$$
(D.6)

We can then compute the percentage change in consumption

$$\mathcal{E}_{C,t}(j|X) = \frac{C_t(j;X) - C_{-1}(j)}{C_{-1}(j)}$$
(D.7)

for $X = wL, r, r^a$. Given the percentage change in labor income given in equation the time t consumptionincome elasticity for income decile j is decomposed as

$$\mathcal{E}_{t}(j;\Gamma) = \underbrace{\frac{\mathcal{E}_{C,t}(j;wL)}{\mathcal{E}_{wL,t}(j)}}_{\text{Contribution of }wL} + \underbrace{\frac{\mathcal{E}_{C,t}(j;r^{b})}{\mathcal{E}_{wL,t}(j)}}_{\text{Contribution of }r^{b}} + \underbrace{\frac{\mathcal{E}_{C,t}(j;r^{a})}{\mathcal{E}_{wL,t}(j)}}_{\text{Contribution of }r^{a}} + \underbrace{\frac{\mathcal{E}_{t}(j;\Gamma)}{\mathcal{E}_{wL,t}(j)}}_{\text{Nonlinearity}}$$
(D.8)