Monopsony Power, Spatial Equilibrium, and Minimum Wages

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Abstract

What role does labor market competitiveness play in determining the spatial distribution of economic activity? To answer this question, we develop a model of monopsony power in spatial equilibrium. Workers and firms are free to locate in any labor market, and the degree of market power a firm enjoys depends on the number of competing firms in its location. We show the model can rationalize concentrations of economic activity and the city-size wage premium through an endogenous labor market competitiveness channel. Using administrative data from Germany, we estimate that labor markets are more competitive in larger cities. Calibrating the model to match this reduced-form evidence, we find endogenous labor market competitiveness can explain 37% of the city-size wage premium and 14% of all agglomeration. We use our new framework to study the spatial and welfare implications of the 2015 German national minimum wage law. A traditional spatial model that ignores variation in monopsony power across space predicts large unemployment effects in smaller, lower-wage labor markets, contradicting the reduced-form evidence on the law. Accounting for spatially-varying monopsony power, we find the enacted national law outperforms an alternative policy with a lower level of the minimum wage in East Germany, while a law that takes into account variation in productivity and competitiveness significantly outperforms both.

1 Introduction

Disparities in wages and the level of economic activity across space are central concerns in the study of economic geography. Geographic variation in wages and population density are interrelated: typical estimates suggest that doubling population density is associated with a three percent increase in wages (Combes and Gobillon, 2015). Canonical theories of agglomeration propose that areas with dense populations pay higher wages due to the productivity-enhancing effects of density, such as knowledge spillovers or input sharing (Rosenthal and Strange, 2004). In this work, we explore an alternative explanation based on differences in the characteristics of labor markets across space. In particular, if the extent of monopsony power enjoyed by firms varies across labor markets, the spatial wage distribution, and the spatial distribution of economic activity generally, may reflect differing competitiveness rather than only differing productivity. This distinction matters for policy: labor market interventions that attempt to alter the spatial wage distribution, such as national minimum wage laws, will have effects that depend vitally on the underlying determinants of spatial wage differences.

In this project, we ask: how are differences in labor market competitiveness across space generated by the location choices of firms and workers, and how are they sustained in spatial equilibrium? What are the implications of endogenous labor market competitiveness for the spatial distribution of economic activity? Given a landscape of varying competitiveness and productivity, how do minimum wage laws affect outcomes for workers and firms across labor markets in spatial equilibrium?

To answer these questions, we develop, to the best of our knowledge, the first micro-founded model of endogenous labor market competition in a full spatial equilibrium, i.e., an equilibrium in which both workers and firms are free to move across space. In particular, we embed a model of imperfectly competitive local labor markets in a quantitative spatial economics framework (Redding and Rossi-Hansberg, 2017). In the model, workers’ idiosyncratic tastes for the characteristics of firms generate imperfect substitutability between firms in a local labor market. The degree of labor market power a firm enjoys depends on the number of competing firms in its location: with more competitors, each firm has closer substitutes, intensifying competition. In larger labor markets, endogenous firm entry increases labor market competition, decreasing wage markdowns and increasing equilibrium wages. We formalize the sense in which this endogenous labor market competitiveness channel acts as an agglomeration force: compared to the case with per-
fectly competitive labor markets, intensified competition in larger cities increases wage and population differentials across space.

How are differences in labor market competitiveness across space sustained in spatial equilibrium? A framework in which workers and firms are free to move between labor markets allows us to connect to a common critique of the idea that highly monopsonized labor markets can exist: why wouldn’t workers leave or firms enter? Our model formalizes the tradeoffs faced by workers and firms when deciding whether to locate in competitive or less-competitive labor markets. In spatial equilibrium, workers pay for high-competitiveness locations in the form of higher rents or lower amenities. Firms operate at larger scale in high-competitiveness locations, allowing them to produce profitably despite smaller wage markdowns.

To estimate the magnitude of labor market competitiveness differences across space, we utilize matched employer-employee data from Germany. Using a canonical approach from the labor literature on monopsony, we estimate firm monopsony power by measuring the sensitivity of worker turnover to the wage paid (Manning, 2003, Hirsch et al., 2019). We estimate this sensitivity separately in each labor market and find a strong role for market size in determining the degree of monopsony power enjoyed by firms: doubling the number of competing firms in a market is associated with an 11% increase in the labor supply elasticity faced by an individual firm, which translates to a 6.5% decrease in the optimal wage markdown (the gap between firms’ optimal wage and the marginal product of labor). Using this reduced-form evidence, we calibrate the key parameters of the model, fundamentals of the labor allocation process which govern the degree of firm monopsony power and its relationship with the number of competing firms in a local labor market. We calibrate the other model fundamentals by utilizing standard techniques from the quantitative spatial economics literature (Redding and Rossi-Hansberg, 2017).

With our calibrated model in hand, we provide three main results about the quantitative importance of endogenous labor market competitiveness in determining the spatial distribution of economic activity. First, using an exact decomposition of local equilibrium wages, we find that variation in labor market competitiveness can explain 37% of the city-size wage premium. Second, we explore the implications of the new framework for estimated productivity differences across space. While a traditional model assumes that wage differences across space reflect productivity differences alone, our framework suggests a significant proportion of the spatial wage distribution reflects variation in labor
market competitiveness. We find that the productivity differences across space required to rationalize the data are 9% smaller than in a traditional model. Third, we estimate the extent to which endogenous labor market competitiveness acts as an agglomeration force. Taking location-specific fundamentals from the calibrated model, we find turning off labor market competitiveness differences across space reduces agglomeration (as measured by the standard deviation of population across locations) in the resultant spatial equilibrium by 14%.

What are the implications of varying labor market competitiveness across space for policy? Germany introduced a national minimum wage of €8.50 on January 1st, 2015. This law imposed the same minimum wage across all regions in Germany, despite widely varying equilibrium wages across space. This variation in equilibrium wages led commentators to suggest that the law would have disastrous unemployment effects in low-wage labor markets, like those in East Germany (Bosch, 2018). Motivated by this, some commentators suggested alternative policies in which the level of the minimum wage varied across regions with differing wage levels, such as setting a lower level for the minimum wage in the East (Knabe et al., 2014). In the model, the employment effects of a minimum wage in a single local labor market depend on local productivity, local labor market competitiveness, and the distribution of worker-level productivities. A national minimum wage that applies across many local labor markets has effects that depend on the spatial distribution of productivity and labor market competitiveness.

We begin by analyzing the predicted effects of the law in a traditional spatial framework. Taking a special case of the model with perfectly competitive labor markets, calibrating it to data from before the law was enacted, and imposing the minimum wage, we find it predicts large unemployment effects, especially in smaller, lower-wage labor markets. In the perfectly competitive model, wage differences across space reflect differences in productivity — imposing the same level for minimum wage in all labor markets thus induces the largest unemployment effects in the lowest-wage labor markets. This strongly contradicts the effects found in recent empirical work on the law (Ahlfeldt et al., 2018; Dustmann et al., 2021), which found no significant unemployment effects, even in low-wage labor markets.

\footnote{In our framework, fundamentals such as amenities can affect skill shares across locations, and so indirectly determine the productivity of labor and so wages. However, they do not discipline wages directly through firm profit equalization, as in a Rosen-Roback framework (Rosen, 1979; Roback, 1982) in which firms share the market for land with workers. We assume firms use only labor in production, a common assumption in models of economic geography (see, e.g., Allen and Arkolakis, 2014).}
Turning to our monopsony framework, we first note that in the calibrated model, monopsony power is strongest in smaller, lower-wage labor markets: exactly those that the perfectly competitive model predicted would have the largest unemployment effects. Imposing the minimum wage in the calibrated monopsony framework, we find results in line with the reduced-form evidence on the effects of the law — minimal unemployment effects, even in the lowest-wage labor markets, and therefore significant convergence in regional nominal wage inequality.

We next compare the enacted ‘flat’ law to two counterfactual minimum wage policies. First, in the spirit of suggestions above, we implement a ‘targeted’ law with a lower level of the minimum wage in East Germany. In terms of low-skill worker welfare, the proposed ‘targeted’ law performs worse than the enacted ‘flat’ law. In the calibrated model, the market failure is higher on average in smaller, lower-wage labor markets (which are disproportionately in the East), and so lowering the ambition of the minimum wage in these regions reduces welfare gains. Second, we implement a ‘constrained optimal’ law that uses information on local productivity and competitiveness to set a different minimum wage in each labor market. In particular, we choose the minimum wage that maximizes low-skill welfare at the initial allocation of population subject to not inducing unemployment. The ‘constrained optimal’ law outperforms the flat law significantly by increasing low-skill wages more in higher-wage labor markets relatively unaffected by the ‘flat’ law. However, the ‘constrained optimal’ minimum wage does not vary one-to-one with low-skill wages: the ratio of the ‘constrained optimal’ minimum to the average low-skill wage is highest in small, low-wage labor markets in which the market failure is most severe.

**Contribution to the literature.** We develop, to the best of our knowledge, the first micro-founded spatial equilibrium model featuring endogenous labor market competition and mobile firms and workers; provide the first analysis of alternative minimum wage laws in such a framework; and provide the first estimates of the role of labor market competitiveness in agglomeration.

Using a variety of approaches, a collection of recent empirical papers that have estimated the elasticity of labor supply faced by firms have found evidence that labor market competitiveness is higher in larger labor markets (Hirsch et al., 2019; Azar et al., 2019; Lindner et al., 2021). Building on these reduced-form studies, we employ a similar empirical approach to Hirsch et al. (2019), but relate estimates of competitiveness to labor market size using a model-consistent estimating equation, which allows us to use the empirical
relationship to calibrate our microfounded model. With a calibrated model, we can go beyond these reduced-form results, and explore the role of varying labor market competitiveness across space in determining the spatial distribution of economic activity, as well as assess the equilibrium effects of alternative policies.

We contribute to the literature on quantitative spatial equilibrium models (Redding and Rossi-Hansberg, 2017; Allen and Arkolakis, 2014; Behrens et al., 2014; Diamond, 2016) by extending a canonical framework to include imperfectly competitive labor markets. While traditional frameworks have focused on the role of productivity and amenity differences in generating agglomeration, we highlight a new mechanism: endogenous labor market competitiveness. We show that accounting for differences in labor market competitiveness across space has implications for the implied spatial distribution of productivity, and our results imply traditional frameworks that infer productivity directly from wages overestimate productivity differences across space.

Compared to a set of recent papers that study imperfectly competitive labor markets in a spatial framework (Kahn and Tracy, 2019, Berger et al., 2021, MacKenzie, 2019, Azkarate-Askasua and Zerecero, 2020), we consider a model in which the location choices of firms are endogenous. Allowing for mobility of firms is essential in allowing us to rationalize the existence of varying labor market competition in equilibrium and to quantify the role of monopsony power in determining the spatial distribution of economic activity, which depends vitally on the location choices of firms. Compared to Manning (2010), who uses a reduced-form model of labor supply in a stylized model with two locations, we utilize a microfounded framework with many locations, which allows us to assess welfare implications of firm monopsony power and perform policy counterfactuals in a model that can be informed with spatial data.

Lastly, we contribute to a literature on minimum wages, and in particular the German national minimum wage (Ahlfeldt et al., 2018; Lindner et al., 2021; Caliendo et al., 2019; Bossler et al., 2018; Bruttel et al., 2018; Holtemöller and Pohle, 2020) by examining the implications of counterfactual minimum wage laws in a general spatial equilibrium environment in which firms and workers can move following introduction of the minimum wage. Our work complements two recent papers that analyze minimum wages in a spatial framework. Compared to Monras (2019), we utilize a model with imperfectly competitive labor markets, and compared to Ahlfeldt et al. (2019), we utilize a spatial equilibrium framework.
The paper proceeds as follows. In Section 2 we set out the model, in Section 3 we describe the data we employ, in Section 4 we describe our estimation, in Section 5 we describe the calibration of the model, in Section 6 we examine the quantitative importance of the monopsony channel, and in Section 7 we examine the effects of counterfactual minimum wage laws.

2 Model

We develop a spatial general equilibrium model with imperfectly competitive local labor markets. Heterogeneous workers choose in which labor market (‘location’) to live, at which firm to work, and consume housing and the final good. Homogeneous firms choose in which labor market to operate (in the sense that there is free entry into all locations and an inexhaustible latent pool of potential entrants), choose profit maximizing wages given the extent of their local market power, and produce the final good. Local labor markets vary exogenously in their productivity, amenities, and housing supply. We introduce productivity variation across space to allow the model the nest the traditional explanation for wage differences across space, and introduce differences in amenities and housing supply to allow the model to match spatial data on populations and house prices.2

The theory’s basic logic can be sketched as follows. Workers see firms as imperfectly substitutable. In a labor market with more competing firms, workers have better outside options. This intensifies competition in the labor market, decreasing firms’ optimal wage markdowns and increasing equilibrium wages.3 This endogenous labor market competi-

2We specify productivity, amenities, and housing supply as exogenously determined. The model could be extended to allow for endogenous productivity (Allen and Arkolakis, 2014), amenities (Diamond, 2016), and housing supply (Combes et al., 2019), but at the cost of additional complexity. As is standard, specifying these features as exogenous does not matter for results on the static cross-section of cities, such as the decomposition of city-size wage premium in Section 6.1. It will, however, dampen the effect of counterfactual changes, such as removing differences in labor market competitiveness across space in Section 6.3.

3As is standard in models of job-differentiation monopsony (Bhaskar and To, 1999; Azar et al., 2019; Egger et al., 2019), we specify a one-sided model of market power, in which firms post wages and workers are perfectly substitutable from the perspective of firms. This means that only the number of firms (not say, the ratio of the number of firms to the number of workers) matters for firms’ market power. As we will see empirically in Section 4, larger cities (those with more competing firms) are estimated to be more competitive. Large cities have larger firms on average, so the ratio of firms to workers are lowest in large cities. Therefore, our estimates of labor market competitiveness contradict the idea that the ratio of firms to workers is an appropriate measure of firms’ market power, while supporting the model’s prediction.
tion channel increases the attractiveness of large labor markets, driving agglomeration.

We solve the model in two steps. First, taking the number of workers of each skill type that choose each labor market as given, we compute firm entry, equilibrium wages, and equilibrium housing prices in each local labor market. Second, we solve for the allocation of population across labor markets using workers’ optimal location choices.

2.1 Workers

Workers are either high skill \((\theta = H)\) or low skill \((\theta = L)\). After choosing her local labor market \(c\), worker \(n\) of type \(\theta\) receives a productivity shock \(z_n^\theta \sim G(\cdot)\) such that she supplies \(z_n^\theta\) effective units of labor. We normalize \(\mathbb{E}[z_n^\theta] = 1\). Allowing for ex-ante worker skill heterogeneity allows means the model can incorporate spatial sorting as a key determinant of the spatial wage distribution; allowing for ex-post productivity shocks generates a distribution of wages within each location, essential for thinking about the effects of a minimum wage law.\(^4\) Finally, after choosing her local labor market \(c\), each worker receives a taste shock \(\varepsilon_n \sim U[0, 1]\) that determines her ideal firm characteristic.

The indirect utility of worker \(n\) of skill-type \(\theta\), if she works for firm \(f\) in location \(c\), is given by

\[
\begin{align*}
    u_{\theta,n,c,f} &= \frac{B_c^\theta w_f^\theta z_n^\theta}{P_c^{1-\beta} h(\varepsilon_{nf})} \\
\end{align*}
\]

where \(B_c^\theta\) is skill-type \(\theta\) workers’ valuations of amenities in local labor market \(c\), \(w_f^\theta\) is the wage per effective unit paid by firm \(f\) for skill-type \(\theta\) workers, \(P_c\) is the price of housing in location \(c\), \(\varepsilon_{nf}\) measures how far the characteristic of firm \(f\) is from worker \(n\)’s ideal firm characteristic, and \(h(\cdot)\) measures the strength of worker tastes for non-wage firm characteristics (details to follow in next section).\(^5\) Notice that both the wage per effective unit firm \(f\) pays for skill-type \(\theta\) workers, \(w_f^\theta\), and worker \(n\)’s taste for firm \(f\), \(\varepsilon_{nf}\), enters the worker utility function — this imperfect substitutability will generate monopsony power

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\(^4\)This formulation is inspired by Behrens et al. (2014), who term it ‘talent and luck’. For us, skill \(\theta\) is ‘talent’, and productivity draw \(z_n^\theta\) is ‘luck’.

\(^5\)The consumption problem (Cobb-Douglas utility defined over the final good and housing) for which (1) is the associated indirect utility is given in Appendix A.1.
for firms, as described in the next section.

2.1.1 Worker problem: choosing at which firm to work

Our model of imperfectly competitive local labor markets is based on Bhaskar and To (1999). We set the framework within a spatial equilibrium featuring many heterogeneous labor markets, introduce heterogeneous labor, and modify the form of worker tastes for non-wage characteristics of firms to allow for a flexible effect of entry on competition. The characteristic space of firms is the unit circle. Each firm has a location \(0 \leq \varepsilon_f < 1\), with \(\varepsilon_f\) uniformly spaced, such that the distance between any two firms is \(1/M_c\), where \(M_c\) is the number of firms operating in location \(c\). We denote by \(\varepsilon_{nf}\) the arc length between \(\varepsilon_n\), worker \(n\)'s ideal firm characteristic, and \(\varepsilon_f\), the location of firm \(f\). Worker \(n\) chooses to work for the firm in her location that maximizes her overall utility from employment. That is, she chooses to work for the firm \(f\) that solves

\[
\arg\max_f u_{n,c,f}^\theta = \arg\max_f \frac{w_f^\theta}{h(\varepsilon_{nf})}
\]

We assume \(h > 0, h' \geq 0\), and normalize \(h(0) = 1\). When \(h' > 0\), a utility penalty is applied that increases in the gap between firm \(f\)'s characteristics, \(\varepsilon_f\), and worker \(n\)'s ideal firm characteristic \(\varepsilon_n\), meaning workers are willing to trade off wages \(w_f^\theta\) with their tastes for particular firms, \(\varepsilon_{nf}\). We imagine these tastes as reflecting a bundle of factors that affect the utility of working at a firm beyond the wage, such as how much a worker likes a firm’s culture or how long it takes the worker to commute to the firm.\(^7\) These idiosyncratic tastes act as frictions in the labor allocation process: in their absence, all labor is allocated to the highest paying firm. That is, when \(h' = 0\), the only factor that enters the labor supply decision are wages \(w_f^\theta\), returning the perfectly competitive case.

\(^6\)Notice the terms not specific to firm \(f\) in worker indirect utility (1) do not enter the firm choice decision.

\(^7\)By specifying a flexible multiplicative form for tastes and wages, we depart from the functional form assumption in Bhaskar and To (1999), who specify a linear additive form for tastes and wages. A multiplicative form means common factors in indirect utility other than the wage (like the price of housing) do not enter the firm choice decision, simplifying the structure of the model.
2.2 Firms

Identical firms combine high-skill labor and low-skill labor (both measured in effective units) to produce the freely-traded final good, chosen to be the numeraire. Production takes on a CES form over high- and low-skill labor, and is subject to high- and low-skill location-specific productivity shifters $A_{c}^{\theta}$ (following Diamond, 2016 and others). Firms pay a fixed cost $F$ to produce, measured in terms of the numeraire. Firms choose two wages: a wage per effective unit for high-skill workers $w_{f}^{H}$ and a wage per effective unit for low-skill workers $w_{f}^{L}$. Worker $n$ of skill-type $\theta$, if she works for firm $f$, is thus paid $w_{f}^{\theta}z_{n}^{\theta}$, where $z_{n}^{\theta}$ is worker $n$’s productivity draw.

Firm profits can be written

$$\pi_{f} = Y_{f} - w_{f}^{H}Z_{f}^{H}(w_{f}^{H}) - w_{f}^{L}Z_{f}^{L}(w_{f}^{L}) - F \tag{2}$$

where firm production is given by

$$Y_{f} = \left(\mu^{L}(A_{c}^{L}Z_{f}^{L})^{\frac{\sigma-1}{\sigma}} + \mu^{H}(A_{c}^{H}Z_{f}^{H})^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma}} \tag{3}$$

and $Z_{f}^{\theta}$ is the total amount of effective units of type $\theta$ labor employed by firm $f$, $\mu^{L}$ and $\mu^{H}$ shift the relative importance of high- and low-skill labor in production, and $\sigma > 1$ is the elasticity of substitution between high- and low-skill labor. We write $Z_{f}^{\theta}(w_{f}^{\theta})$ in (2) to highlight that the amount of labor firm $f$ employs depends on the wage it pays. Firms take the total amount of labor in their labor market, $N_{c}^{\theta}$, as given.

Choosing $w_{f}^{H}$ and $w_{f}^{L}$ to maximize profits yields classic monopsony wage-setting expressions

$$w_{f}^{\theta} = \frac{\eta_{f}^{\theta}}{\eta_{f}^{\theta} + 1}MPZ_{f}^{\theta} \tag{4}$$

where $\eta_{f}^{\theta}$ is the elasticity of labor supply (in effective units) faced by firm $f$ for skill-type
\( \theta \) workers,
\[
\eta_f^\theta = \frac{\partial \log(Z^\theta_f)}{\partial \log(w^\theta_f)}
\]
and \( MPZ^\theta_f \) is the marginal product of type-\( \theta \) labor at firm \( f \),
\[
MPZ^\theta_f = Y^\frac{1}{\sigma} (A^\theta_c)^{\frac{1}{\sigma}} (Z^\theta_f)^{-\frac{1}{\sigma}}
\]

### 2.3 Labor allocation to firms

We proceed by determining the labor supply elasticity faced by firms, which governs the gap between the marginal product of skill-type \( \theta \) workers and the optimal choice for their wage, as described in (4).

Within a labor market \( c \), consider a firm \( f \) that pays a wage \( w^\theta_f \), who neighbors firm \( f' \), who pays \( w^\theta_{f'} \) and firm \( f'' \), who pays \( w^\theta_{f''} \). Given \( w^\theta_f \) and \( w^\theta_{f'} \), there is an indifferent worker whose ideal firm characteristic is located \( x^\theta_{ff} \) units from \( \epsilon_f \), where \( x^\theta_{ff} \) solves
\[
\frac{w^\theta_f}{h(x^\theta_{ff})} = \frac{w^\theta_{f'}}{h(\frac{1}{M_c} - x^\theta_{ff'})}
\]

We illustrate this in Figure 1. All workers with ideal characteristics between \( \epsilon_f \) and \( x^\theta_{ff'} \) choose to work for firm \( f \). As firm \( f \) increases its wage \( w^\theta_f \), the location of the indifferent worker \( x^\theta_{ff} \) moves further to the right, increasing the number of workers whose optimal choice is firm \( f \).\(^8\)

We will look for a symmetric equilibrium with \( w^\theta_f = w^\theta_{f'} = w^\theta_c \). In this case, \( x^\theta_{ff'} = x^\theta_{ff''} = x^\theta_f \) and the labor supply in effective units to firm \( f \) is given by
\[
Z^\theta_f = 2N^\theta_c x^\theta_f
\]

\(^8\)Formally, the existence of \( x^\theta_{ff'} \) requires a restriction that \( w^\theta_f \) and \( w^\theta_{f'} \) are not too different. This restriction will be met in the symmetric equilibrium.

\(^9\)Looking forward to our estimation (Section 4), we will examine the quit behaviour of workers as a function of the wage a firm pays. In the model, as a firm reduces its wage, some workers find they have a better option elsewhere. How many such workers quit determines the elasticity of labor supply faced by the firm. Empirically, we will find that the number of competing firms is a strong determinant of this elasticity.
Figure 1: Wage competition in a local labor market

\[ M_c \text{ number of firms operating in labor market } c. \ x^\theta_{ff'} \text{ location of indifferent worker between } f \text{ and } f'. \]

Plotted for \( w^\theta_f > w^\theta_{f'} \) and \( h' > 0 \)

where \( N^\theta_c \) is the number of skill-type \( \theta \) workers in location \( c \) and we have used the fact that individual productivities \( z^\theta_n \) do not enter the firm choice decision. \( x^\theta_f \) solves

\[
\frac{w^\theta_f}{h(x^\theta_f)} = \frac{w^\theta_c}{h(\frac{1}{M_c} - x^\theta_f)}
\]  

(6)

We next derive the elasticity of labor supply faced by firm \( f \). In a symmetric equilibrium with \( w^\theta_f = w^\theta_c \) the elasticity of labor supply is equal across firms in a labor market and across skill types, and is given by (see Appendix A.2),

\[
\eta^\theta_f = \eta_c = \frac{M_c h(\frac{1}{2M_c})}{h'(\frac{1}{2M_c})} = \frac{1}{2\zeta_h(\frac{1}{2M_c})}
\]  

(7)

where \( \zeta_h(\cdot) \) is the elasticity of \( h(\cdot) \) and \( M_c \) is the number of competing firms in location \( c \). We can see from here that the elasticity of labor supply faced by firm \( f \) for both types of labor depends only on the worker taste for non-wage characteristics function \( h(\cdot) \) and the number of competing firms in \( c, M_c \). In particular, the elasticity of labor supply faced
by firm $f$ depends on the elasticity of $h(\cdot)$ evaluated at half the gap between firms, $\frac{1}{2M_c}$.

The intuition is the following: an increase in the wage paid by firm $f$, beginning at the symmetric equilibrium, induces workers whose tastes are closer to $f$’s neighboring firm to consider moving to firm $f$. Whether they do so depends on the tradeoff between the higher wage that $f$ pays and the reduced non-wage utility from working at $f$. How many such workers move to firm $f$ depends on how quickly $h(\cdot)$ changes for marginal workers: that is, it depends on the elasticity of $h(\cdot)$ evaluated at half the gap between firms, $\frac{1}{2M_c}$.

Equation (7) shows how the form of worker tastes for firm characteristics, $h(\cdot)$, governs the level of monopsony power firms enjoy and how it varies with the number of firms that compete in a labor market, $M_c$. As we will see in Section 4, the empirically relevant (and intuitively appealing case) is the one in which the labor supply elasticity is finite and increases in the number of competing firms. We first note two special cases of the model in which this doesn’t hold: perfectly competitive labor markets (Example 1) and constant elasticity (Example 2). Lastly, we describe the functional form that we will implement in the calibration, which features an increasing relationship between the elasticity and the number of competing firms (Example 3). Adapting a term from Arkolakis and Morlacco (2017), we call this case constant super elasticity: the elasticity of $\eta_c$ with respect to $M_c$ is constant.

**Example 1** (Special case: perfectly competitive labor market). If $h'(x) \rightarrow 0$, $\epsilon \rightarrow 0$ and $\eta_c \rightarrow \infty$, returning a perfectly competitive labor market.

**Example 2** (Special case: constant elasticity). If $h(x) = x^{1/\kappa}$ for some $\kappa > 0$, $\eta_c = \kappa$, which is constant in the number of competing firms $M_c$.

**Example 3** (Special case: constant super elasticity). If $h(x) = \exp\left(\frac{\alpha}{\gamma}x^\gamma\right)$ for some $\alpha > 0$, $\gamma > 0$ then $\eta_c = \frac{1}{\alpha}M_c^\gamma$, which is increasing in the number of firms $M_c$.

Proof: Appendix A.3.

As mentioned, for the main exposition, we focus on the case in which the labor supply elasticity is finite and increases in the number of competing firms. This is ensured by the following assumption, which implies that workers care more about a change in match quality when further from their ideal match:

**Assumption 1.** $h(\cdot)$ has a positive and increasing elasticity: that is, $\zeta h > 0$ and $\zeta h' > 0$. 
2.4 Equilibrium in a local labor market

After determining the elasticity of labor supply faced by firms, we can impose the symmetric equilibrium, solve for extent of firm entry $M_c$, determine equilibrium wages $w^\theta_c$, and determine the price of housing $P_c$, each as a function of population $N^\theta_c$, which will be determined endogenously by spatial equilibrium in Section 2.5.

In a symmetric equilibrium, $Y_f = \frac{Y_c}{M_c}$, where

$$Y_c = \left( \mu^L (A^L_c N^L_c)^{\frac{\sigma-1}{\sigma}} + \mu^H (A^H_c N^H_c)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}}$$

The number of competing firms $M_c$ is determined by free entry. Inserting optimal wage setting (4) into firm profits (2) and imposing the symmetric equilibrium yields

$$\pi_f = 0 \implies (\eta_c + 1) M_c F = Y_c$$

(8)

Which under Assumption 1 has a unique solution for $M_c$. The number of competing firms is increasing in local labor market-level production $Y_c$, which in turn depends on local labor market populations $N^\theta_c$ and productivity shifters $A^\theta_c$.

With $M_c$ in hand, effective-unit wages for each skill type are given by

$$w^\theta_c = \frac{\eta_c}{\eta_c + 1} \text{MPZ}^\theta_c$$

(9)

where

$$\text{MPZ}^\theta_c = Y_c^\frac{1}{\sigma} \mu^\theta_c (A^\theta_c)^{\frac{\sigma-1}{\sigma}} (N^\theta_c)^{-\frac{1}{\sigma}}$$

(10)

Housing is in fixed supply $T_c$. The equilibrium price of housing is determined by

$$P_c = \frac{(1 - \beta)(w^L_c N^L_c + w^H_c N^H_c)}{T_c}$$

(11)

Given $N^\theta_c$, the number of competing firms $M_c$ can be determined from (8), equilibrium wages for each skill type can be determined from (9) and the price of housing can be determined from (11). It remains to determine the number of workers of each skill type.
in each location, \( N^\theta_c \), which we do in the next section.

### 2.5 Spatial equilibrium

Workers make optimal location decisions given their skill type, but before knowing the realizations of their productivity shock \( z^\theta_n \) and taste shock \( \epsilon_n \). That is, workers make their location decision based on expected utility

\[
 u^\theta_c = \mathbb{E}[u^\theta_{n,c,f}] = \frac{B^\theta_c w^\theta_c \chi_c}{p^{1-\beta}}
\]

where we have used the fact that \( z^\theta_n \) and \( \epsilon_n \) are independent and \( \mathbb{E}[z^\theta_n] = 1 \), and expected match quality \( \chi_c \) is given by

\[
\chi_c = \mathbb{E} \left[ \frac{1}{h(\epsilon_{nf})} \right] = 2M_c \int_0^{\chi_c} h(x) x^{\beta} \, dx
\]

An interior spatial equilibrium is a vector of populations \( \{N^\theta_c\} \) such that

- Within-location equilibrium conditions (8), (9) and (11) hold in each location
- \( u^\theta_c = u^\theta \) \( \forall c \) and \( \theta \in \{H, L\} \)
- \( \sum_c N^\theta_c = N^\theta \) for \( \theta \in \{H, L\} \)

As described in Appendix A.4, we compute interior spatial equilibrium using a procedure in the spirit of Allen and Arkolakis (2014), as implemented in Redding and Rossi-Hansberg (2017).

**Monopsonized labor markets in spatial equilibrium.** The model can rationalize the existence of varying labor market competitiveness across labor markets as an equilibrium outcome consistent with the optimal location choices of both firms and workers. While a set of recent papers on monopsony and space leave firm location decisions exogenous (Kahn and Tracy, 2019; Berger et al., 2021; MacKenzie, 2019; Azkarate-Askasua and Zerecero, 2020), we allow both firms and workers to be freely mobile across space. Allowing
for firms to be mobile allows us to (i) understand how differences in monopsony power across labor markets emerges endogenously through the location choices of firms (ii) allows us to understand how variation in monopsony power is sustained in spatial equilibrium and (iii) allows us to examine counterfactuals (such as minimum wage laws) that may induce changes in the location decisions of firms.

Our framework formalizes the tradeoffs faced by both workers and firms when choosing within which labor market to operate. Workers make location decisions according to expected utility (12). Using wages (9), we can rewrite (12) as

$$u_c^{\theta} = \frac{B_c^{\theta} \eta_c \chi_c MPZ_c^{\theta} \chi_c}{P_c^{1-\beta}}$$

From here we can see, holding other factors equal, workers prefer a more competitive labor market in which $\eta_c$ is higher — this means workers get a larger share of their marginal product as wages. In an interior spatial equilibrium, workers must be indifferent across locations, meaning that if $\eta_c < \eta_{c'}$ for some $c'$, this difference must be compensated by some combination of either higher amenities $B_c^{\theta}$, a higher marginal product of their labor $MPZ_c^{\theta}$, or lower house prices $P_c$.

Firms must also be indifferent across labor markets in spatial equilibrium. From free entry (8), we note that

$$\frac{1}{1+\eta_c} Y_f = F$$

where $Y_f = \frac{Y_c}{M_c}$ is total firm production. While workers, other things equal, prefer more competitive labor markets, the opposite is true for firms. Given (13), the scale at which firms operate, $Y_f$, must balance their monopsony power, which increases in $\frac{1}{1+\eta_c}$ such that profits are equalized to zero. In larger labor markets (those with a higher number of competing firms $M_c$ and so competitiveness $\eta_c$), firms must operate at larger scale. This prediction is consistent with the data in Germany: average firm size (as measured by workers per firm) is higher in larger cities, as shown in Appendix Figure B.1. It is worth noting that the largest firms in the model (as measured by $Y_f$) therefore have the least monopsony power, as they are located in the largest and so most competitive labor markets. As one would expect, however, the firms with the largest relative scale $\frac{Y_f}{\eta_c}$ have the most monopsony power: given a symmetric equilibrium in each location, the firms
with the largest relative scale are in the smallest, least competitive labor markets.

**Endogenous labor market competition as an agglomeration force.** Our main result compares the distribution of economic activity when labor markets are imperfectly competitive to the case when labor markets are perfectly competitive, as in traditional spatial models (Redding and Rossi-Hansberg, 2017). Perfect competition is returned as a limiting special case of the model when the fixed cost of firm entry approaches zero: $F \to 0$.

Our main result is described in Proposition 1.

**Proposition 1 (Monopsony magnifies population differences).** Consider two locations $c$ and $c'$ in a many-location spatial equilibrium. Denote vector of fundamentals $X_c = (A^\theta_c; B^\theta_c; T_c)'$. If $X_c > X_{c'}$ (that is, all elements are weakly larger and one element is strictly larger), in any spatial equilibrium without reversals (i.e., $N_c > N_{c'}$), it must be the case that

$$\frac{w_c^\theta}{w_{c'}^\theta} > \frac{w_c^{\theta, pcomp}}{w_{c'}^{\theta, pcomp}} > 1$$

and

$$\frac{N_c}{N_{c'}} > \frac{N_c^{pcomp}}{N_{c'}^{pcomp}} > 1$$

where $w_c^{\theta, pcomp}$ and $N_c^{pcomp}$ denote the distributions of wages and population under perfect competition ($F \to 0$).


Proposition 1 formalizes the sense in which endogenous labor market competitiveness acts as an agglomeration force: given an initial fundamental advantage of a location $c$ over a location $c'$, an equilibrium allocation with imperfectly competitive labor markets must feature a more unequal population distribution between them than under perfectly competitive labor markets. With endogenous labor market competition, larger populations have two additional benefits compared to the case of perfectly competitive labor markets: monopsony power is weakened, increasing wages, and the larger number of firms results in better average matches. These benefits cause more crowding into larger

---

10 In this case, $M_c \to \infty$, $\eta_c \to \infty$, and $\chi_c \to 1$.

11 As noted in the proposition, this formally requires ruling out extreme (‘reversal’) equilibria in which so few workers choose the “better” location that an insufficient number of firms operate, resulting in extremely strong monopsony power.
Following Proposition 1, we want to quantify the importance of varying labor market competitiveness in generating differences in wages and population across space. We proceed by estimating labor market competitiveness differences across space and calibrating the model.

3 Data

We employ several data sources. We use employer-employee data to estimate monopsony power across space, and employ several data sources aggregated to regional levels to calibrate other features of the model.

Throughout, we define labor markets (the locations in the model) as Kreis (districts). There are 401 Kreis in Germany. Employment in each district/Kreis is mapped in Appendix Figure B.2.

Our main data source is the Sample of Integrated Labor Market Biographies and Establishment History Panel, provided by the Institute for Employment Research (IAB) through the Research Data Centre (FDZ) of the German Federal Employment Agency (BA). The Sample of Integrated Labour Market Biographies (SIAB) covers a 2% sample of workers in Germany, including a Worker ID, firm ID, and information on pay, occupation, industry, geography (district/Kreis by home location and workplace location); the Establishment History Panel is a 50% sample of firms, including information on firm exit, employment, wages, and geography (district/Kreis identifiers). Firms are defined in the data as consisting of one or more branch offices or workplaces belonging to one company in one district/Kreis. We provide information on our estimation sample in Section 4.

We employ several data sources aggregated to the district/Kreis level in calibrating the model. We utilize house price indexes at the district/Kreis level from RWI-GEO-REDX, which is based on asking price data from the website immobilienscout24.de. In particular, we use the 2014 house price index, which is constructed from coefficients on Kreis indicators in a regression of prices on property characteristics. We map this price index in Appendix Figure B.4. We utilize aggregate compensation and employment data from 2014, disaggregated by Kreis, from the National Accounts of the Federal States (VGRdL). We map employment in Appendix Figure B.2 and compensation in Appendix Figure B.3.
We utilize census information on education counts across Kreis from the 2011 census, accessed through zensusDATENBANK. We plot the high-skill share (share of residents with University or above education) in Appendix Figure B.5.

4 Estimation

We estimate variation in labor market competitiveness across space and relate our estimates of competitiveness to market size. Using a canonical approach from the labor literature on monopsony, we estimate the elasticity of labor supply faced by firms (the key measure of firm monopsony power) by measuring the sensitivity of worker turnover to the wage paid (Manning, 2003, Hirsch et al., 2019).

The key mechanism in the model is that in large cities, firm entry increases competition in the labor market, increasing the elasticity of labor supply faced by firms, and pushing equilibrium wages closer to marginal products. To estimate this force empirically, we will, in a first step, estimate the elasticity of labor supply faced by firms in each Kreis, yielding 401 labor supply elasticities. We will then, in a second step, relate the estimated labor supply elasticities to the number of competing firms in each Kreis. We will use the estimated second-step relationship to calibrate key parameters of the model.

In particular, imposing in the model the functional form for worker tastes for non-wage characteristics from Example 3, \( h(x) = \exp(\frac{\alpha \gamma}{2^{1-\gamma}}) \) yields a model-implied log-log relationship between the labor supply elasticity and the number of competing firms:

\[
\log \eta_c = \log \frac{1}{\alpha} + \gamma \log M_c \tag{14}
\]

We proceed by estimating labor supply elasticities \( \log \eta_c \) in each Kreis (first step) and relating these to the number of competing firms in each Kreis (second step) using the empirical counterpart of (14).

First step estimation. We estimate the labor supply elasticity faced by firms in a location using an elasticity of separations approach, the canonical methodology from the labor economics literature (Manning, 2003, Sokolova and Sorensen, 2021, Bassier et al., 2021). This observational approach involves relating variation in the wage a worker is paid to the probability there is an employment separation (for example, the worker quitting to
work for another firm). When this estimated sensitivity is high, we infer a high labor supply elasticity (a small decrease in the wage a firm pays would result in many separations) and if it is low we infer a low labor supply elasticity.\textsuperscript{12} Exploiting the rich structure of our panel data here, we condition the analysis on worker-region fixed effects – that is, we allow each worker in the sample to have different baseline separations behavior, and exploit only variation in the wage the same worker is paid over time and across different firms to inform the elasticity. Following Bassier et al. (2021), we use a linear specification for baseline results, and a hazard rate specification for robustness.\textsuperscript{13}

The linear specification is given by

\[
\text{sep}_{kq} = \delta_{nc} + \tilde{\beta}_c \log(w_{kq}) + x'_{kq} \gamma + \epsilon_{kq}
\]

where \(\text{sep}_{kq}\) is an indicator for whether job \(k\) ended in a separation in quarter \(q\), \(\delta_{nc}\) is a worker-region fixed effect, \(\log(w_{kq})\) is the (log) wage paid in job \(k\) in quarter \(q\), and \(x_{kq}\) is a vector of controls, which will include industry and occupation fixed effects. Following Bassier et al. (2021), we translate \(\tilde{\beta}_c\) to an elasticity \(\beta_c\) by dividing the regression coefficient by the corresponding mean of the outcome.

The hazard-rate specification is given by

\[
s_k(\tau) = s^0(\tau, n, c) \exp(\beta_c \log(w_k(\tau)) + x_k(\tau)'\gamma)
\]

where \(s_k(\tau)\) denotes the hazard rate of job \(k\) at duration \(\tau\) (measured in days), \(s^0(\tau, n, c)\) denotes the baseline hazard, which is an arbitrary function of duration \(\tau\) and allowed to be worker-region specific, \(\log(w_k(\tau))\) is the (log) wage paid in job \(k\) at duration \(\tau\), and \(x_k(\tau)\) is a vector of controls, which will include industry and occupation fixed effects.

For both the linear and hazard rate specification, we translate the estimated elasticity of separations to an estimated labor supply elasticity by setting \(\eta_c = -2\beta_c\) (following Hirsch et al., 2019).

\textbf{Second step estimation}. We relate our estimated elasticities to market size using the empirical counterpart to (14),

\textsuperscript{12}In the model, a reduction in the wage a firm pays would result in some workers leaving to a competitor firm. How many such workers leave determines the elasticity of labor supply faced by a firm.

\textsuperscript{13}We follow Hirsch et al. (2019) and use only separations to estimate the labor supply elasticity, rather than incorporating e.g., the share of recruits from unemployment, that would be motivated by a model with explicit unemployment in laissez-faire (Manning, 2003).
\[
\log \eta_c = \delta_0 + \delta_1 \log(M_c) + X'_c \delta_x + \epsilon_c
\]

where \(X_c\) is a vector of district/Kreis-level controls, specified below.

**Sample selection.** Hours worked are not directly reported in the data, so to infer wages from total compensation we restrict to full-time jobs only. Data from East Germany is not available before 1991 and is incomplete in immediately subsequent years, so we restrict the analysis to jobs beginning 1994 and onwards. The minimum wage is introduced in 2015, so we restrict our analysis to job spells occurring in 2014 and before. We drop jobs with missing firm information, missing regional information or implausibly low or missing wages (<€12 equivalent per day, similar restrictions are imposed by, for example, Card et al., 2013 and Blömer et al., 2018). This yields a sample of 2.12 million jobs held by 803,000 workers across 820,000 firms. Sample statistics for jobs in the sample are given in Table B.1. We define regions in the data to be 401 districts/Kreis. Sample statistics for these regions are given in Table B.2.

**First step estimation results.** Table 1 summarizes the estimated labor supply elasticities. The estimated labor supply elasticities have a magnitude similar to that found in other observational studies (Sokolova and Sorensen, 2021), and feature significant variation across districts. In column (1) of Table 1, we report the elasticities from the linear specification without controls, and column (2) we report the elasticities from the hazard model without controls, both yielding labor supply elasticities of a similar magnitude. In column (3) we add 1st-step controls (industry and occupation fixed effects), which does not alter much the average size of the estimated elasticities. In column (4), rather than examine all separations, we restrict the definition of separations to those in which the worker is in full-time employment at another firm after separating from the current firm. This attenuates the magnitude of estimated elasticities somewhat, but yields elasticities broadly in line with the other approaches.

As is well understood in papers that use a similar observational methodology to estimate labor supply elasticities, the estimates may be biased by omitted factors that are correlated with wages and separations (Manning, 2003, Bassier et al., 2021). Here, we exploit the richness of our employer-employee data to allow for worker fixed effects in the estimation of our elasticities, allowing us to control for unobserved time-invariant worker characteristics, typically not feasible with survey data. Omitted time-varying worker characteristics or other characteristics of jobs may still affect our estimates, motivating the inclusion of occupation and industry fixed effects described above.
Table 1: First-step results: estimated elasticity of labor supply

<table>
<thead>
<tr>
<th></th>
<th>Linear spec.</th>
<th>Hazard spec.</th>
<th>Linear, 1st step controls</th>
<th>Exits to employment only</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>Mean</td>
<td>1.77</td>
<td>1.79</td>
<td>1.67</td>
<td>1.45</td>
</tr>
<tr>
<td>p10</td>
<td>0.98</td>
<td>1.08</td>
<td>0.96</td>
<td>0.36</td>
</tr>
<tr>
<td>p25</td>
<td>1.37</td>
<td>1.40</td>
<td>1.29</td>
<td>0.96</td>
</tr>
<tr>
<td>p50</td>
<td>1.78</td>
<td>1.82</td>
<td>1.70</td>
<td>1.46</td>
</tr>
<tr>
<td>p75</td>
<td>2.13</td>
<td>2.13</td>
<td>2.01</td>
<td>2.02</td>
</tr>
<tr>
<td>p90</td>
<td>2.51</td>
<td>2.45</td>
<td>2.38</td>
<td>2.42</td>
</tr>
<tr>
<td>N</td>
<td>401</td>
<td>401</td>
<td>401</td>
<td>401</td>
</tr>
</tbody>
</table>

Distribution of estimated labor supply elasticities across 401 districts. p10 refers to the 10th percentile of estimated elasticities. Column (1) reports elasticities from the linear specification, column (2) from the hazard specification, column (3) adds 1st-step controls, a full set of industry and occupation fixed effects, and column (4) considers only separations to employment.

**Second step estimation results.** Table 2 summarizes the relationship between the (log) estimated elasticity of labor supply facing the firm, log \( \hat{\eta}_c \), and district/Kreis size, as measured by the number of competing firms log \( M_c \). In column (1) we report the unconditional correlation between the number of competing firms and the estimated labor supply elasticities from the linear specification. The coefficient on log number of firms of 0.11 implies doubling the number of firms is associated with an increase in the labor supply elasticity of 11%. Moving from a district of the median size to the largest size is thus associated with an increase in the labor supply elasticity of 37%. In column (2) we use labor supply elasticities estimated using the hazard formulation yielding a relationship of similar magnitude. In columns (3) and (4), we add first step and both first and second step controls (dummy for East, share of workers in manual occupations, share of workers in professional occupations, and share of workers who are German), resulting in a relationship of a similar magnitude. While other omitted factors could still bias the estimated relationship between the labor supply elasticity and market size, its magnitude is not significantly altered by the inclusion of these relevant (as illustrated by the increase in \( R^2 \) between column (3) and (4)) controls, giving us some confidence in the robustness of the relationship. Finally we consider labor supply elasticities estimated using exits to employment only, resulting in an estimated relationship of slightly stronger magnitude but with a larger standard error.\(^{14}\)

\(^{14}\)The number of observations here is smaller as some estimated labor supply elasticities (fewer than 10%) are negative, which may be the result of sampling variation.
Table 2: Second-step results: elasticity of labor supply and district/Kreis size

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>(Log) estimated elasticity of labor supply</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>(Log) number of firms</td>
<td>0.11***</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
</tr>
<tr>
<td>1st-step worker-region FE</td>
<td>Yes</td>
</tr>
<tr>
<td>1st-step controls</td>
<td>Yes</td>
</tr>
<tr>
<td>2nd-step controls</td>
<td>Yes</td>
</tr>
<tr>
<td>Hazard specification</td>
<td>Yes</td>
</tr>
<tr>
<td>Only exits to employment</td>
<td>Yes</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.03</td>
</tr>
<tr>
<td>$N$</td>
<td>401</td>
</tr>
</tbody>
</table>

Standard errors in parentheses. Significance levels: * 10%, ** 5%, ***1%. 1st-step controls are full set of occupation and industry fixed effects. 2nd-step controls are dummy for East, share of workers in manual occupations, share of workers in professional occupations, and share of workers who are German. Regressions are linear specification except column (2), which reports a hazard specification.

We present further robustness of the results in Table B.3. We examine a sample only including workers with less than University education and find a positive relationship between the number of competing firms and the elasticity of labor supply faced by a particular firm which is only slightly smaller than that for the full sample, motivating the simplifying restriction that the function $h(\cdot)$ is the same for both skill types that we made in the model. We also run the analysis at the commuting zone rather than district/Kreis level, again finding a positive relationship between the number of competing firms and the elasticity of labor supply, though with a smaller magnitude. In Table B.4 we include share of workers with a university degree as a 2nd-step control, again finding a significant positive relationship.

We next use this reduced form evidence to calibrate the key parameters of the model, and use our aggregate data sources to inform other quantities in the model.

5 Calibration

There are both global and local (location-specific) parameters in the model. The set of global parameters is $\beta$, the share of income spent on housing, $F$, the fixed cost of firm
entry, σ, the elasticity of substitution between the two factors of production, the function \( h(\cdot) \), and the functions \( G^\theta(\cdot) \). The set of local parameters are exogenous housing stocks \( T_c \), local productivities \( A^\theta_c \) and amenities \( B^\theta_c \).

### 5.1 Global parameters

To calibrate the worker taste for non-wage characteristics function \( h(\cdot) \) we choose (as already mentioned) \( h(x) = \exp\left(\frac{\alpha}{\gamma x^{\gamma}}\right) \), yielding a relationship between the labor supply elasticity and the number of competing firms identical to that estimated in Section 4

\[
\log(\eta_c) = \log\left(\frac{1}{\alpha}\right) + \gamma \log(M_c)
\]

We therefore calibrate \( \gamma = 0.11 \) and \( \alpha = 1.25 \) using the coefficients in our preferred specification (the first column of Table 2).

We set \( \beta = 0.73 \), based on share of expenditure on housing in Germany (Eurostat). We set \( F = 200,000 \) so that the calibrated model matches the average number of firms across regions. We set \( \sigma = 1.6 \), following Diamond (2016) and others.

We specify \( G^L(\cdot) \) as a truncated log normal distribution, with bounds \([z^L, z^L] \). Taking low-skill workers (those with less than University education) in the estimation sample in Section 4, retaining data from 2010-2014, we construct the ratio of a workers wage to the average wage in her city. With this ratio in hand for each worker, we set \( z^L \) to the 10th percentile of the distribution, \( z^L \) to the 90th percentile, and fit the standard deviation of the log normal to the truncated data, yielding a standard deviation of 0.27. Given \( z^L, z^L \) and the standard deviation, we choose the mean to ensure \( E[z^\theta_n] = 1 \). Without loss of generality for all results in the main text (we assume the minimum wage does not bind for high-skill workers), we set \( G^H(\cdot) \) to be deterministic.

### 5.2 Local parameters

Given global parameters, we show that data \( \{w^H_c\}, \{w^L_c\}, \{N^L_c\}, \{N^H_c\} \) can be used to find unique values for local parameters \( \{A^H_c\}, \{A^L_c\}, \{B^L_c\}, \{B^H_c\} \).

\[^{15}\text{without loss of generality, we set } \mu^L \text{ and } \mu^H \text{ to 1.}\]
To construct \( \{w^H_c\} \) and \( \{w^L_c\} \), we begin with compensation per employee (across all education types) in each location from the National Accounts of the Federal States (VGRdL). To construct skill premia, using the SIAB, we regress daily wages on a high-skill indicator interacted with a dummy for East and log district/Kreis population, allowing a skill premium that varies by East and district/Kreis population. We take the predicted skill premia from this regression for each district/Kreis, combined with average compensation from VGRdL, to compute projected high-skill and low-skill wages \( \{w^H_c\}, \{w^L_c\} \).

We first show that given \( \{w^H_c\}, \{w^L_c\}, \{N^L_c\}, \{N^H_c\} \), productivities \( \{A^H_c\}, \{A^L_c\} \) can be computed exactly. From optimal wage setting we can write

\[
\frac{\eta_c}{\eta_c + 1} Y_c = w_c^L N^L_c + w_c^H N^H_c
\]

where we have used the fact that \( Y_c = \left( MPZ_c^L N^L_c + MPZ_c^H N^H_c \right) \). Therefore \( M_c \) can be computed using

\[
\eta_c M_c F = w_c^L N^L_c + w_c^H N^H_c
\]

The calibrated number of firms in each labor market \( M_c \), is mapped in Appendix Figure B.7 and can be compared to the number of firms in each region in the data in appendix figure B.6. We plot the relationship in Appendix Figure B.8.

With \( M_c \) (and so \( \eta_c \) and \( Y_c \)) in hand, inverting equilibrium wage setting (9) yields productivities as

\[
A^\theta_c = \left( \frac{w_c^\theta}{\frac{\eta_c}{\eta_c + 1} Y_c^\theta \mu^\theta (N_c^\theta)^{-\frac{1}{\sigma}}} \right)^{\frac{\sigma}{\sigma-1}}
\]

We next calibrate local housing stocks \( \{T_c\} \) using market clearing for housing and data on house prices \( \{P_c\} \)
Lastly, we infer amenities $B^θ_c$ using a revealed preference procedure in the spirit of Allen and Arkolakis (2014), as implemented in Redding and Rossi-Hansberg (2017). We describe this in Appendix A.5.

6 Quantitative importance of varying labor market competitiveness

Using the calibrated model, we examine the quantitative importance of endogenous labor market competitiveness in the spatial distribution of economic activity. We analyze the role of labor market competitiveness in three foundational features of the literature on the spatial economy: the city-size wage premium, productivity differences across space, and agglomeration. In Section 6.1, we decompose the city-size wage premium into the contributions of varying competitiveness and varying productivity and sorting. Next, in Section 6.2, we examine the implications of accounting for competitiveness differences across space in the implied spatial distribution of productivity. Finally, in Section 6.3, we examine the role of varying labor market competitiveness in agglomeration.

6.1 Decomposition of the city-size wage premium

We decompose the city-size wage premium, the empirical relationship between the (log) population of a region and its (log) average wage, into competitiveness (‘monopsony markdown factor’), and productivity and sorting factors.

The empirical relationship between the (log) population of a region and its (log) average wage can be expressed as

$$\log(\mathbb{E}[W_{nc}]) = \beta_0 + \beta_1 \log(N_c) + \epsilon_c$$

(15)

where $\mathbb{E}[W_{nc}]$ is the average per-worker wage in location $c$ (across all skill types) and $N_c$ is total population in location $c$ (across all skill types). As described in Section 5, the cali-
brated model exactly fits the data on average wages and population, and so fits this empirical relationship exactly. We use the model to decompose the relationship between average wage and population into two components: variation in monopsony power across space and variation in productivity and sorting. In particular, we will decompose \( \beta_1 \) into two components, \( \beta_{\text{monopsony}}^1 \) and \( \beta_{\text{prod sort}}^1 \) such that

\[
\beta_1 = \beta_{\text{monopsony}}^1 + \beta_{\text{prod sort}}^1
\]

We first note that average wages can be written as a function of a monopsony markdown factor and a productivity and sorting term,

\[
\log(\mathbb{E}[W_{nc}]) = \log\left(\frac{\eta_c}{\eta_c + 1}\right) + \log\left(\frac{\text{MPZ}_c^L N_c^L + \text{MPZ}_c^H N_c^H}{N_c^L + N_c^H}\right)
\]

(16)

We project the monopsony factor and productivity and sorting on population as follows

\[
\log\left(\frac{\eta_c}{\eta_c + 1}\right) = \beta_{\text{monopsony}}^0 + \beta_{\text{monopsony}}^1 \log(N_c) + \epsilon_{\text{monopsony}}
\]

(17)

\[
\log\left(\frac{\text{MPZ}_c^L N_c^L + \text{MPZ}_c^H N_c^H}{N_c^L + N_c^H}\right) = \beta_{\text{prod sort}}^0 + \beta_{\text{prod sort}}^1 \log(N_c) + \epsilon_{\text{prod sort}}
\]

(18)

Notice by (16), \( \beta_1 = \beta_{\text{monopsony}}^1 + \beta_{\text{prod sort}}^1 \). The results of this decomposition are given in Table 3. From Table 3, \( \beta_1 = 0.094 \), \( \beta_{\text{monopsony}}^1 = 0.035 \) and \( \beta_{\text{prod sort}}^1 = 0.059 \), implying that variation in monopsony power across space accounts for 37% of the city-size wage premium. This striking result follows closely from the reduced form results in Section 4: firms face significantly higher elasticities of labor supply in larger cities, which pushes up wages through optimal monopsony wage setting.

### 6.2 Spatial distribution of productivity

A large set of papers in economic geography infer productivity differences across space from differences in wages across space (Allen and Arkolakis, 2014, Balboni, 2021, Redding and Rossi-Hansberg, 2017). Comparing the calibrated monopsony model with a traditional perfectly competitive model, we wish to understand whether the overall spatial
distribution of implied productivities is substantially different after having accounted for the estimated magnitude of the endogenous labor market competitiveness mechanism. To do this, we compare the coefficient of variation (standard deviation divided by the mean) of low skill and high skill productivity ($\text{MPZ}^\theta_c$) in the calibrated monopsony model vs calibrated perfectly competitive model. We find the coefficient of variation of low-skill (high-skill) productivity is 9.5% lower (9.4% lower) in the monopsony model. This result suggests standard exercises in economic geography that ignore monopsony power overestimate productivity differences across space.

### 6.3 Agglomeration

Finally, we explore how much of observed agglomeration can be explained by our labor market competitiveness channel. Beginning with the calibrated monopsony framework, we compute a counterfactual in which we turn off labor market competitiveness differences across space (by setting $\eta_c \to \infty$ in all locations; holding $\chi_c$ at initial values). We allow population to reallocate until a new spatial equilibrium is found. As predicted by Proposition 1, the resultant distribution of population is less varied across space. In the resultant spatial equilibrium, we find the distribution of population is significantly less agglomerated, with the standard deviation of population across the 401 labor markets falling by 14%.

Taken together, the results in this section portray an important role for labor market com-
petitiveness differences across space in determining the spatial distribution of economic activity. We next turn to the implications of this insight for policy.

7 Minimum wages

We now explore the implications of minimum wage laws. A minimum wage that applies across many local labor markets has effects that depend on the spatial distribution of productivity and market power. Minimum wages are motivated by the observation that the marginal product of labor exceeds the wage paid. In this case, as we discuss below, a minimum wage can increase wages without necessarily inducing unemployment. Through the lens of the calibrated model, there is variation in the ratio of the marginal product of labor to the low-skill wage across space. In Figure 2 we plot the model-implied value for this ratio across the 401 districts in our data. Firm monopsony power is non-zero everywhere, so the marginal product of labor always exceeds the low-skill wage. The marginal product most exceeds the wage paid in (i) smaller labor markets with lower populations $N^\theta_c$ and (ii) lower-wage labor markets with lower low-skill wages $w^L_c$.

To analyze a minimum wage in the model, we begin by noting there are two effective-unit wages in each location $c$, $w^*_{cL}$, $w^*_{cH}$, but a distribution of worker-wages $w^*_{\theta z\theta}^\theta$, with $z^\theta_n \sim G^\theta(\cdot)$. (We denote laissez-faire quantities – those that attain in the absence of a minimum wage – with $^\ast$.) A minimum wage $\bar{W}_c$ imposes that all worker-wages paid in location $c$ must exceed $\bar{W}_c$. We study the implication of such a policy for a single labor market in Section 7.1 and then the implications of a vector of minimum wages $\{\bar{W}_c\}$ for overall spatial equilibrium and welfare in Section 7.2.

We assume throughout that a minimum wage only binds for low-skill workers and that the density corresponding to $G^L(\cdot)$, $g^L(\cdot)$, is bounded below by $\bar{z}^L > 0$, bounded above by $\bar{z}^L > 0$, is continuous, and has $g^L(z) > 0 \forall z \in [\bar{z}^L, \bar{z}^L]$.

7.1 Equilibrium determination within a location

Given populations $N^\theta_c$ in location $c$, a minimum wage $\bar{W}_c$ can be (i) non binding (ii) binding without inducing unemployment (iii) binding with inducing unemployment. The minimum wage will bind if it exceeds the lowest worker-wage paid under laissez-faire,
Figure 2: Model-implied \( \frac{MPL_c}{w_c} \) (ratio of marginal product of labor to wage (low skill)) on log \( w_c \) (log average wage (low-skill)). Circle size proportional to population \( N_c \), with East German districts in blue; West German districts in orange.

and it will induce unemployment if it exceeds the marginal product of the lowest productivity worker when all workers are employed. In laissez-faire, the lowest worker-wage paid in \( c \) is \( w^{*L_z}_c \) and the marginal product of the lowest productivity worker is \( MPZ^{*L_z}_c \).

In laissez-faire, all workers are employed.

The range of minimum wages that bind but do not induce unemployment is

\[
w^{*L_z}_c < \overline{W}_c < MPZ^{*L_z}_c
\]

which is increasing in the gap between the low-skill effective-unit wage \( w^{*L}_c \) and the marginal product of low-skill labor when all workers are employed, \( MPZ^{*L}_c \), the key measure of monopsony power. We illustrate this in Figure 3.

As before, each firm posts a wage per effective unit for low-skill workers, \( w^L_f \), and high-skill workers, \( w^H_f \). It considers all applicants. Depending on an applicant’s skill \( z^n_f \) and the level of the minimum wage, the firm either does or does not offer employment. If the firm offers employment, and the firm’s choice for the effective-unit wage implies a total payment to the worker that exceeds the minimum wage, the firm pays the worker the
effective-unit wage. If the total payment under the effective-unit wage does not exceed the minimum wage, the firms pays the worker the minimum wage instead. We first consider the problem of optimally choosing effective-unit wages and which applicants to employ.

Given a choice of effective-unit wages \( w_j^\theta \), the firm will only employ applicants whose marginal product exceeds the minimum wage: that is, it will employ applicants with \( z_n^L \) such that

\[
\text{MPZ}_f z_n^L \geq W_c
\]

Note that \( \text{MPZ}_f \) will depend on labor employed, and so the choices for effective-unit wages \( w_j^\theta \). For employed workers, it pays the minimum wage to all workers with \( z_n^L \) such that

\[
w_f^L z_n^L \leq W_c
\]

and pays the effective-unit wage to all workers with \( z_n^L \) such that

\[
w_f^L z_n^L > W_c
\]

Given \( w_j^\theta \), firm \( f \) employs \( N_f^{L,\text{min}} \) workers (a total of \( Z_f^{L,\text{min}} \) effective units) at the minimum wage and \( N_f^{L,\text{eff}} \) workers (a total of \( Z_f^{L,\text{eff}} \) effective units) at the effective-unit wage.

The firm’s profit function is
\[
\pi_f = \left( \mu^L (A^L_c Z_f^{\text{L,min}} + Z_f^{\text{L,eff}}) \right)^{\sigma^L} + \mu^H (A^H_c Z_f^{\text{H}}) \right)^{\sigma^H} - \bar{W}_c N_f^{\text{L,min}}
- w_f^L Z_f^{\text{L,eff}} - w_f^H Z_f^{\text{H}} - F
\]

The firm chooses \( w_f^L \) and \( w_f^H \) to maximize profits. In a symmetric equilibrium where \( w_f^L = w_c^L \) \( \forall f \), optimal wage setting implies (see Appendix A.7)

\[
w_c^L = \frac{\eta_c}{\eta_c + 1} \text{MPZ}_c^L \tag{19}
\]

\[
w_c^H = \frac{\eta_c}{\eta_c + 1} \text{MPZ}_c^H \tag{20}
\]

In the case of a minimum wage that induces unemployment, the lowest productivity worker employed has \( \hat{z}_c \) effective units, where \( \hat{z}_c \) solves

\[
\text{MPZ}_c^L \hat{z}_c = \bar{W}_c \tag{21}
\]

where

\[
\text{MPZ}_c^L = \left( \mu_c^L (A^L_c N_c^L \int_{2z_c}^{\hat{z}_c} zg^L(z)dz)^{\sigma_c^L} + \mu^H (A^H_c N_c^H) \right)^{\sigma_c^H} \mu_c^L (A^L_c)^{-1} \int_{2z_c}^{\hat{z}_c} zg^L(z)dz
\]

In the case of a minimum wage that does not induce unemployment, \( \hat{z}_c = z_c^L \). With \( \hat{z}_c \) in hand, we can write the total effective units of low-skill labor employed in \( c \) as

\[
Z_c^L = \int_{2z_c}^{z_c^L} zg^L(z)dz
\]

The number of operating firms \( M_c \) is determined by free entry,

\[
\pi_f = 0 \implies Y_c = M_c F + \bar{W}_c N_c^L (G_c^L (\frac{\bar{W}_c}{w_c^L}) - G_c^L (\hat{z}_c)) - w_c^L N_c^L \int_{\frac{w_c^L}{\bar{W}_c}}^{\hat{z}_c} zg^L(z)dz - w_c^H N_c^H \tag{22}
\]
With $M_c$ in hand, all other equilibrium quantities in $c$ can be computed. A low skill worker’s expected wage (integrating over $z^n_L$) is

$$
\mathbb{E}[W_{n,c}^L] = 0 \cdot G^L(\hat{z}_c) + \overline{W}_c \cdot \left( G^L\left(\frac{W_c}{w_c^L}\right) - G^L(\hat{z}_c) \right) + w_c^L \cdot \int_{\hat{z}_c}^{z_c^L} g^L(z)dz
$$

The price of housing is determined by

$$
P_c = (1 - \beta) \left( \mathbb{E}[W_{n,c}^L] N_c^L + w_c^H N_c^H \right) T_c
$$

Perfect competition special case. In perfect competition, with $F \to 0$, a binding minimum wage $\overline{W}_c$ always induces unemployment: in this case, $\text{MPZ}_c^L = w_c^L$. In this case, the lowest productivity worker employed is determined by (21), $w_c^L = \text{MPZ}_c^L$, and the average low-skill wage (integrating over $z^n_L$) is given by

$$
\mathbb{E}[W_{n,c}^L] = 0 \cdot G^L(\hat{z}_c) + \text{MPZ}_c^L \cdot \int_{\hat{z}_c}^{z_c^L} g^L(z)dz
$$

For a given minimum wage $\overline{W}_c$, we plot the low-skill per-worker wage as a function of worker-level productivity $z^n_\theta$ for the case of the perfectly competitive model in Figure 4 and the case of the monopsony model in Figure 5. The laissez-faire per-worker wage is given by $W_{nc}^L$ in both plots; the per-worker wage in the presence of the minimum wage is given by $W_{nc}^L$. We consider a $\overline{W}_c$ such that the minimum wage does not induce unemployment in the monopsony model. (The case of a higher minimum wage, that does induce unemployment in the monopsony model, is shown in Appendix Figure A.1.)

In the perfectly competitive model (Figure 4), the minimum wage leads to unemployment for the lowest productivity workers (unemployment is represented as $W_{nc}^L = 0$). The remaining low-skill workers (those with productivities such that their marginal product is above the minimum wage) are paid more than under laissez-faire. This follows from the increase in the marginal product of low-skill workers as they are now relatively scarce in production.

In the monopsony model (Figure 5), the minimum wage does not induce unemployment, and increases the take-home pay of workers with the lowest productivities, who are now

33
paid exactly the minimum. The remaining low-skill workers (those with productivities such that their marginal product is above the minimum wage) are paid less than under laissez-faire. This follows from the anticompetitive effect of firm exit – with fewer competing firms, there is less competition in setting the effective-unit wage. As described in Result 1, the overall effect on average low-skill pay is necessarily positive.

The effect of a minimum wage that does not induce unemployment, holding population fixed, is summarized in Result 1.

**Result 1** (Non-unemployment inducing minimum wage in a local labor market). Consider a local labor market, taking populations $N_c^L$ and $N_c^H$ as given (partial equilibrium). The marginal product of low-skill labor under full employment, $MPZ^*_L$, is given by (10) and the low-skill effective-unit wage that would attain under laissez-faire, $w^*_L$, is given by (9). There exist a range of minimum wages $w^*_L < \bar{W}_c \leq MPZ^*_L$ that do not induce unemployment. Such a minimum wage, relative to laissez-faire,

- Increases average income for low skill workers, $\mathbb{E}[W_{nc}^L]$. A mass of low-skill workers are paid exactly the minimum.

- Decreases the number of operating firms, $M_c$

- Decreases average income for high skill workers, $\mathbb{E}[W_{nc}^H]$

- Increases the price of housing, $P_c$

Proof: Appendix A.10.

The case of a minimum wage that does induce unemployment is given in Appendix Result A.1. In this case, while the number of operating firms and the number of operating firms again necessarily decline, the effect on average income for low skill workers and the price of housing is ambiguous, and depends on how high the level of the minimum is set.

### 7.2 Equilibrium determination across locations

After imposition of a vector of minimum wages across locations $\bar{W}_c$, the allocation of population across locations is determined using a procedure similar to that in the baseline case without a minimum wage: workers choose the location with the highest expected utility and in spatial equilibrium utility is equalized across space. Worker expected utility
Figure 4: Perfectly competitive model: effect of a minimum wage on low-skill pay

Figure: Effect of a minimum wage \( W_c \) on low-skill per-worker wages \( W_{nc}^L \) in perfectly competitive model as a function of worker-level productivity \( z_n^L \). \( W_{nc}^{*L} \) denotes the low-skill per-worker wage in laissez-faire.

Figure 5: Monopsony model: effect of a minimum wage on low-skill pay

Figure: Effect of a minimum wage \( W_c \) on low-skill per-worker wages \( W_{nc}^L \) in monopsony model as a function of worker-level productivity \( z_n^L \). \( W_{nc}^{*L} \) denotes the low-skill per-worker wage in laissez-faire. Shown for a minimum wage \( W_c \) that does not induce unemployment.
from locating in $c$ is now, for low skill workers,

$$u^L_c = \frac{B^L_c E[W^L_{nc}] \chi_c}{p^1-\beta}$$  \hspace{1cm} (26)$$

where $E[W^L_{nc}]$ is determined by (23). For high skill workers,

$$u^H_c = \frac{B^H_c w^H_c \chi_c}{p^1-\beta}$$  \hspace{1cm} (27)$$

We compute equilibrium using a similar procedure to that under laissez faire, as described in Appendix A.8.

7.3 Alternative minimum wage laws

We consider three minimum wage laws: (i) a ‘flat’ national law, based on the law actually adopted in Germany in 2015 (ii) a proposed ‘targeted’ law that sets the minimum to a lower level in East Germany based on proposals made before 2015 (iii) a ‘constrained optimal’ law that sets a different level of the minimum in each labor market based on the initial distribution of population.

For the ‘flat’ national law, we impose $W_c = \bar{W}$ and choose $\bar{W}$ such that average exposure (the share of full-time workers earning less than the minimum at the point of introduction) across regions in the model matches the data at the point of introduction of the real minimum wage law (as reported in Garloff, 2016).

For the proposed ‘targeted’ law we note that several authors suggested setting the minimum wage at a lower level in East Germany due to lower equilibrium wages in East Germany and fears of induced unemployment from a universal minimum wage (Knabe et al., 2014). In particular, Knabe et al. (2014) notes one suggestion of €8.15 in West Germany and €7.50 in East Germany and another of €7.50 in West Germany and €6.50 in East Germany (Moller, 2013 and Rürup, 2013, as cited by Knabe et al., 2014). To capture the spirit of these proposals, we consider a targeted law in which the level of the minimum wage is unchanged from the flat law in West Germany but set to 88% of the flat level in East Germany.
For the ‘constrained optimal’ law, we begin at the laissez-faire equilibrium. We find for each location the minimum wage that maximizes low-skill welfare, subject to not inducing any unemployment at the initial distribution of population.\textsuperscript{16} To maximize low-skill welfare in a location, the ‘constrained optimal’ law takes into account local productivity and local competitiveness, not only local equilibrium wages. It thus differs from proposals that ignore variation in competitiveness across space, such as Dube (2014), that suggest e.g., setting the minimum wage at one-half of local median wages. We show in Section 7.4 the constrained optimal minimum wage varies less than one-to-one with average low-skill wages due to variation in competitiveness across space.

\textbf{7.4 Alternative minimum wage laws: results}

We impose the three laws above, each characterized by a vector $\overrightarrow{W}_c$, and solve for the resultant long-run spatial equilibrium as described in Section 7.2. Throughout, we compare effects in the calibrated perfectly competitive model to the calibrated monopsony model.

\textit{‘Flat’ law.} We first examine the enacted ‘flat’ law. The recent reduced-form evidence on the effects of the law across labor markets has found no significant induced unemployment, even in initially low-wage labor markets (Ahlfeldt et al., 2018; Dustmann et al., 2021). As the minimum wage bound more in those initially low-wage labor markets, the policy led to nominal wage convergence across regions (Ahlfeldt et al., 2018).

Taking the calibrated perfectly competitive model and calibrated monopsony model, we impose the ‘flat’ law and solve for the resultant spatial equilibrium. The models have sharply divergent predictions about the effects of the law. As shown in Table 4, the perfectly competitive model predicts extensive unemployment in the long run spatial equilibrium, with an economy-wide average unemployment rate of 5.11%, rising to 18% at the 90th percentile across labor markets, sharply contradicting the reduced-form evidence. The calibrated monopsony model, however predicts very minimal equilibrium unemployment, with an economy-wide average unemployment rate of 0.07%, rising only to 0.2% at the 90th percentile. While in the perfectly competitive model, workers are always paid their marginal product, and so no workers are paid exactly the minimum, in the monopsony model, about 6% of workers are paid exactly the minimum in equilibrium.

\textsuperscript{16}By taking this location-by-location approach, we do not attempt to find the globally-optimal minimum wage. In spatial equilibrium after population reallocation, it is not guaranteed that there will be no unemployment, but we find the induced unemployment to be very small (less than 0.01%).

37
(Consistent with this idea, Cengiz et al. (2017) use the extent of bunching at the minimum wage to make inferences about the effect of minimum wages on jobs and and Dube et al. (2018) use the extent of bunching at round numbers, like $10/hour, to make inferences about the strength of monopsony power.)

These differences in employment and bunching effects translate to changes in the average income of low-skill workers across space compared to laissez-faire. While the perfectly competitive model predicts a large increase in regional low-skill income inequality (+27%), the monopsony model predicts a decline (−3.55%), consistent with the reduced-form evidence. The overall welfare effect of the law for low-skill workers is significantly negative according to the perfectly competitive model (-2.97%), while modest and positive (+0.21%) according to the monopsony model. In the monopsony model the welfare effect for low skill workers is less than the effect on their nominal incomes. This follows from the reduction in match qualities and competitive pressure on the effective-unit wage after firms exit due to the squeeze on the profit margins, increases in house prices, and movements of high-skill labor across space in response to the minimum.

Table 4: ‘Flat’ law in perfectly competitive model vs monopsony model.

<table>
<thead>
<tr>
<th></th>
<th>Calibrated perfectly competitive model</th>
<th>Calibrated monopsony model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>p10</td>
<td>p90</td>
</tr>
<tr>
<td>Unemployment</td>
<td>0.80</td>
<td>18.01</td>
</tr>
<tr>
<td>Paid exactly minimum</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Avg low-skill wage</td>
<td>-12.36</td>
<td>-0.91</td>
</tr>
<tr>
<td>Avg high-skill wage</td>
<td>-10.30</td>
<td>1.43</td>
</tr>
<tr>
<td>Low-skill welfare</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>High-skill welfare</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>Overall worker welfare</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>Regional low-skill nominal income inequality</td>
<td>/</td>
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</tr>
</tbody>
</table>

Long run spatial equilibrium effects of ‘flat’ law in calibrated monopsony model vs perfectly competitive model. Figures are in percent (unemployment, workers paid exactly minimum) and percentage change vs laissez-faire (all other variables). p10 denotes the 10th percentile across districts; p90 denotes the 90th percentile across districts; national denotes the national value.

‘Targeted’ law. We next turn to the proposed ‘targeted’ law, which set a lower level for the minimum wage in East Germany. As shown in Appendix Table B.6, in the perfectly competitive model, the ‘targeted’ law has a smaller negative effect on low-skill welfare
than the ‘flat’ law – targeting in this manner is beneficial according to the perfectly competitive model. As show in Appendix Table B.7, the monopsony model has the opposite implication: the ‘targeted’ law has a smaller positive effect on low-skill welfare than the ‘flat’ law – targeting in this manner is detrimental. This is borne out in the effect on low-skill nominal income across space. As shown in Figure 6, in the perfectly competitive model, labor markets in East Germany (that had the largest reductions according to the ‘flat’ law) fair less poorly. In the monopsony model, as shown in Figure 7, those same labor markets fair less well. This follows from the fact that although equilibrium wages are indeed lower on average in East Germany, in the calibrated monopsony model, the market failure is greater on average (recalling Figure 2). The ‘flat’ law is relatively ambitious in East Germany: it bites the most in Eastern labor markets — through the lens of the monopsony model, this is appropriate, however, as the market failure is greatest there.

‘Constrained optimal’ law. Finally, we turn to the constrained optimal law, which we examine in the monopsony model. The ‘constrained optimal’ law sets a different level for the minimum in each district: we plot the value of the ‘constrained optimal’ minimum in each district in Appendix Figure B.9. Due to the correlation between average wages and competitiveness, the ‘constrained optimal’ law varies less than one-to-one with average low-skill wages (that is, it is set at a higher relative level in lower-wage markets on average). As shown in Appendix Table B.5, the minimum wage has an elasticity with respect to the average low-skill wage of 0.90, reflecting the fact the market failure in the calibrated model is on average higher in low-wage labor markets. The constrained optimal minimum wage is also relatively high in smaller markets: while the average low-skill wage has an elasticity with respect to district employment of 0.08, the ‘constrained optimal’ minimum wage has an elasticity with respect to district employment of 0.05. This concurs with the fact the market failure is higher in smaller markets.

As shown in Appendix Table B.7, the constrained optimal law outperforms the ‘flat’ law by a factor of three in terms of the increase in low-skill welfare. As shown in Figure 8, this follows from the fact the constrained optimal law, which has freedom to set a higher level of the minimum wage in more productive labor markets, has a much more even effect across labor markers on low-skill nominal income increases than the ‘flat’ law.
Figure 6: Perfectly competitive model: ‘Flat’ vs Targeted’ law, % change in low-skill average income

(a) ‘Flat’ law  
(b) ‘Targeted’ law

Figure 7: Monopsony model: ‘Flat’ vs ‘Targeted’ law, % change in low-skill average income

(a) ‘Flat’ law  
(b) ‘Targeted’ law
These results illustrate the importance in accounting for variation in labor market competitiveness across labor markets in imposing a spatially differentiated minimum wage: while having more policy instruments (a different level for the minimum in each labor market) will always allow the policy maker to do better (as illustrated by the ‘constrained optimal’ law), intuitive targeting, like taking the adopted law and imposing a lower level for the minimum in East Germany, forgets that the market failure varies across space, and so variation in the relative ambition of the minimum wage is desirable.

Figure 8: Monopsony model: ‘flat’ law vs ‘Constrained optimal’ law, % change in low-skill average income

8 Conclusion

In this paper, we argue that imperfectly competitive labor markets are an important determinant of the spatial distribution of economic activity. We first develop a spatial general equilibrium system-of-cities model featuring firm monopsony power. The model implies that larger cities, by providing improved worker outside options, have more competitive labor markets, as measured by the elasticity of labor supply faced by firms. We take this prediction to data, and find robust support for the theory: larger cities (as measured by the number of competing firms) have higher estimated labor supply elasticities. Using
this evidence to calibrate the model, we find that endogenous labor market competition can explain 37% of the city-size wage premium and 14% of agglomeration.

As an application of the model, we examine the spatial equilibrium effects of alternative minimum wage laws. First, we argue a traditional framework fails to match the reduced form evidence on the employment effects of the German national minimum wage law: while a traditional framework predicts very large unemployment effects, especially in initially low-wage labor markets, our monopsony framework has predictions in line with the reduce-form evidence. We next examine the implications of alternative minimum wage laws, and find a particular proposal that set a lower value for the minimum wage in East Germany (while leaving it unchanged in West Germany) performs slightly worse than the national law, while an ‘optimally’ targeted law performs significantly better.

Spatial productivity differences are a foundational idea in regional economics. While productivity differences are undoubtedly important, we see our paper as part of a set of recent works that attempt to explain spatial wage differences by the differing optimal behaviour of firms and workers given their differing environments (see, e.g., Tian, 2021).

We also see our work as contributing to the analysis of the spatial effects of policy interventions. If spatial wage differences are partially the result of market failures, interventions (such as minimum wage laws) can reduce nominal income disparities across space and increase welfare. Place-based policy is usually conceived as a system of taxes and subsidies; minimum wages have place-based effects but involve no direct subsidies to low-wage areas. Such policies, that alter the ‘rules of the game’, will have different effects across space even if they are not explicitly spatially targeted – understanding the determinants of these effects is an important path towards better policy making.

References


Garloff, A. (2016), Side effects of the new German minimum wage on (un-) employment: First evidence from regional data, Technical report, IAB-discussion paper.


A Theory appendix

A.1 Consumption problem

Worker utility is given by

\[ u_{ncf}^{\theta}(C_n, T_n) = \frac{B_i^\theta}{h(e_{nf})} \left( \frac{C_n}{\beta} \right)^{\beta} \left( \frac{T_n}{1 - \beta} \right)^{1-\beta} \]  

(A.1)

where \( C_n \) denotes consumption of the freely-traded final good and \( T_n \) denotes consumption of housing. Workers choose \( C_n \) and \( T_n \) to maximize (A.1), subject to

\[ C_n + P_c T_n \leq w_f^\theta z_n^\theta \]

Solving this optimization problem yields the standard Cobb-Douglas expressions for consumption of the two goods

\[ C_n = \beta w_f^\theta z_n^\theta \]

\[ T_n = (1 - \beta) \frac{w_f^\theta z_n^\theta}{P_c} \]

Inserting into utility (A.1) yields indirect utility (1).

A.2 Elasticity of labor supply faced by the firm

Starting with equation (5),

\[ Z_f^\theta = 2N_c^\theta x_f^\theta \]

The elasticity of labor supply faced by firm \( f \) is thus

\[ \frac{dZ_f^\theta w_f^\theta}{dw_f^\theta Z_f^\theta} = \frac{dx_f^\theta w_f^\theta}{dw_f^\theta x_f^\theta} \]
Totally differentiating (6) yields,

\[
\frac{h(x_f^\theta) - w_f^\theta h'(x_f^\theta) \frac{dx_f^\theta}{dw_f^\theta}}{(h(x_f^\theta))^2} = \frac{w_c}{(h(\frac{1}{M_c} - x_f^\theta))^2} h'(\frac{1}{M_c} - x_f^\theta) \frac{dx_f^\theta}{dw_f^\theta}
\]

Imposing symmetry \((w_f^\theta = w_c^\theta, \text{ and so } x_f^\theta = \frac{1}{2M_c})\) and rearranging yields

\[
\frac{dx_f^\theta}{dw_f^\theta} \bigg|_{x_f^\theta = x_c^\theta} = \frac{M_c h(\frac{1}{2M_c})}{h'(\frac{1}{2M_c})}
\]

yielding the expression in the text.

A.3 Derivations for elasticity examples

Example 1. Notice that, given \(h > 0 \text{ and } M_c > 0, h' \to 0 \) implies

\[
\eta_c = \frac{M_c h(\frac{1}{2M_c})}{h'(\frac{1}{2M_c})} \to \infty
\]

Example 2. If \(h(x) = x^{\frac{1}{2\kappa}}, h'(x) = \frac{1}{2\kappa} x^{\frac{1}{2\kappa}-1} \) and so

\[
\eta_c = \frac{M_c(\frac{1}{2M_c})^{\frac{1}{2\kappa}}}{\frac{1}{2\kappa}(\frac{1}{2M_c})^{\frac{1}{2\kappa}-1}}
\]

\[= \kappa \]

Example 3. If \(h(x) = \exp(\frac{\alpha}{\gamma^{2\gamma-1}} x^{\gamma}) \) for some \(\alpha > 0, \gamma > 0, h'(x) = \frac{\alpha}{2\gamma^{2\gamma-1}} x^{\gamma-1} \exp(\frac{\alpha}{\gamma^{2\gamma-1}} x^{\gamma}) \) and so
\[
\eta_c = \frac{M_c \exp\left(\frac{\alpha}{\gamma^{2(1-\gamma)} / (2M_c)^\gamma}\right)}{\gamma^{2-\gamma}(2M_c)^\gamma - 1} \exp\left(\frac{\alpha}{\gamma^{2(1-\gamma)} / (2M_c)^\gamma}\right)
= \frac{1}{\alpha} \frac{M_c}{c}
\]

### A.4 Computing spatial equilibrium

First choose a functional form for \(h(\cdot)\). Given a guess of \(N_c^\theta\), expected utility (12) can be computed using the within-location equilibrium conditions (8), (9), and (11). In an interior spatial equilibrium,

\[
u_c^\theta = \frac{1}{C} \sum_c u_c^\theta
\]

Rearranging and using the identity \(N_c^\theta = N_c^{\theta}\),

\[
N_c^\theta = \left(\frac{1}{C} \sum_c u_c^\theta\right)^\sigma
\]

(A.2)

We construct a mapping from a guess of population in each location \(N_c^\theta\), \(\tau\) to next round guess \(N_c, \tau + 1\) using (A.2),

\[
N_c^\theta, \tau + 1 = \left(\frac{1}{C} \sum_c u_c^\theta, \tau\right)^\sigma
\]

(A.3)

Beginning with a guess \(N_c^\theta, 0\), we compute \(u_c^\theta, 0\) and then compute \(N_c^\theta, 1\) using (A.3), rescaling all populations such that \(\sum_c N_c^\theta, 1 = N^\theta\). We proceed in this manner until utility converges: \(u_c, \tau + 1 \rightarrow u_c, \tau\).

### A.5 Inferring amenities

In an interior spatial equilibrium,
\[ u^\theta_c = \frac{1}{C} \sum_c u^\theta_c \]

Using the form for utility we can write

\[ u^\theta_c = \frac{B^\theta_c w^\theta_c}{p_c^{1-\beta}} 2M_c \int_0^{\frac{1}{2M_c}} h(x)^{-1} dx \]

Rearranging and using the identity \( B^\theta_c = B^\theta_c \),

\[ B^\theta_c = \frac{1}{C} \sum_c u^\theta_c \]

We construct a mapping from a guess of amenities in each location \( B^\theta_{c,t} \) to next round guess \( B^\theta_{c,t+1} \) using (A.4),

\[ B^\theta_{c,t+1} = \frac{1}{C} \sum_c u^\theta_{c,t} \]

Beginning with a guess \( B^\theta_{c,0} \), we compute \( u^\theta_{c,0} \) and then compute \( B^\theta_{c,1} \) using (A.5). We proceed in this manner until utility converges: \( u_{c,t+1} \rightarrow u_{c,t} \).

### A.6 Proof of Proposition 1

In perfect competition, with \( F \rightarrow 0 \), in spatial equilibrium it must be the case that, for \( \theta \in \{ H, L \} \),

\[ \frac{T_c^{1-\beta} B_c^\theta \text{MPZ}^\theta_{c,p\text{comp}}}{(Y_{p\text{comp}}^{\text{pcomp}})^{1-\beta}} = \frac{T_c^{1-\beta} B_{c'}^\theta \text{MPZ}^\theta_{c',p\text{comp}}}{(Y_{p\text{comp}}^{\text{pcomp}})^{1-\beta}} \tag{A.6} \]

With imperfectly competitive labor markets, it must be the case, in a spatial equilibrium, that
\[
\frac{T_c^{1-\beta}B_c^\theta MPZ_c^\theta}{Y_c^{1-\beta} \left( \frac{\eta_c}{\eta_c + 1} \right)^\beta} 2M_c \int_0^{\frac{1}{2}M_c} h(x)^{-1} dx = \frac{T_c^{1-\beta}B_c^\theta MPZ_c^\theta}{Y_c^{1-\beta} \left( \frac{\eta_c'}{\eta_c' + 1} \right)^\beta} 2M_c' \int_0^{\frac{1}{2}M_c'} h(x)^{-1} dx
\]

We first derive a result on relative skill shares across locations. Taking (A.7) for \(\theta = H\) and \(\theta = L\), and taking the ratio yields

\[
\frac{B_c^L MPZ_c^L}{B_c^L' MPZ_c^L'} = \frac{B_c^H MPZ_c^H}{B_c^H' MPZ_c^H'}
\]

Using the form of marginal product of labor (10), we note that relative skill shares across locations depend only on fundamentals

\[
\frac{N_c^L}{N_c^H} = \left( \frac{B_c^L}{B_c^L'} \left( \frac{A_c^L}{A_c^L'} \frac{\sigma-1}{\sigma} \right) \right)^\sigma
\]

We next note that match quality is increasing in the number of competing firms \(M_c\),

\[
\frac{d2M_c}{dM_c} \int_0^{\frac{1}{2}M_c} h(x)^{-1} dx = 2 \int_0^{\frac{1}{2}M_c} h(x)^{-1} dx - \frac{1}{M_c h(\frac{1}{2M_c})} \]

\[
> 2 \int_0^{\frac{1}{2}M_c} \frac{1}{h(\frac{1}{2M_c})} dx - \frac{1}{M_c h(\frac{1}{2M_c})} \]

\[
= 0
\]

where we used, following Assumption 1, \(h'(\cdot) > 0\). Finally we note that the monopsony markdown is decreasing in the number of competing firms \(M_c\),

\[
\frac{d\eta_c}{dM_c} = \left( \frac{\sigma}{2M_c \eta_c h(\frac{1}{2M_c})} \right)^2 > 0
\]

where the last inequality follows from Assumption 1.

Begin by considering
\[
\begin{align*}
\frac{u_c^\theta}{u_{c'}^\theta} &= \frac{T_c^{1-\beta} B^\theta \text{MPZ}_c^\theta}{Y_c^{1-\beta}} \left( \frac{\eta_c}{\eta_c + 1} \right) \beta \frac{2M_c \int_0^{\frac{1}{M_c}} h(x)^{-1} dx}{T_{c'}^{1-\beta} B^\theta \text{MPZ}_{c'}^\theta} \left( \frac{\eta_{c'}}{\eta_{c'} + 1} \right) \beta \frac{2M_{c'} \int_0^{\frac{1}{M_{c'}}} h(x)^{-1} dx}{\text{MPZ}_c^\theta} \frac{1}{\text{MPZ}_{c'}^\theta} \\
\text{By } \frac{N_c^\theta}{N_{c'}^\theta} > 1 \text{ (no reversals), } \frac{Y_c^c}{Y_c^{c'}} > 1 \text{ and so } \frac{M_c}{M_{c'}} > 1, \text{ the second and third terms in this ratio exceed 1, by (A.9) and (A.12).}
\end{align*}
\]

If \( \frac{N_c^\theta}{N_{c'}^\theta} = \frac{N_c^\theta}{N_{c'}^\theta} \), the first term equals 1. If \( \frac{N_c^\theta}{N_{c'}^\theta} < \frac{N_c^\theta}{N_{c'}^\theta} \), the first term exceeds 1.

Therefore, in any spatial equilibrium, it must be the case that \( \frac{N_c^\theta}{N_{c'}^\theta} > \frac{N_c^\theta}{N_{c'}^\theta} \). Lastly, note that by (A.8),

\[
\frac{\text{MPZ}_c^\theta}{\text{MPZ}_{c'}^\theta} = \frac{\text{MPZ}_c^\theta}{\text{MPZ}_{c'}^\theta}
\]

and so it must be the case that \( \frac{w_c^\theta}{w_{c'}^\theta} > \frac{w_c^\theta}{w_{c'}^\theta} \).

### A.7 Optimal wage setting in the presence of a minimum wage

Notice first that the form of the first-order condition for the high-skill wage is unchanged compared to laissez-faire. Beginning with firm profits,

\[
\pi_f = \left( \mu_L (A_c Z_{L,\min} + Z_{L,\text{eff}}) \frac{\sigma-1}{\sigma} + \mu_H (A_H Z_{H}^{H} \frac{\sigma-1}{\sigma}) \frac{\sigma}{\sigma-1} \right) - W_c N_f^{L,\min} - w_f^L Z_{L,\text{eff}} - w_f^H Z_{f}^{H} - F
\]

The first-order condition for \( w_f^H \) yields, as before,

\[
w_f^H = \frac{\eta_f}{\eta_f + 1} \text{MPZ}_f^H
\]

Turning to the low skill wage, we search for a symmetric equilibrium in which \( w_f^L = \)}
Consider a choice of \( w_f^L > w_c^L \) (symmetric results attain for \( w_f^L < w_c^L \)). In this case, we can consider three groups of low skill workers that firm \( f \) employs:

- **Group 1**: offered minimum by firm \( f \); offered minimum at all other firms
- **Group 2**: offered effective-unit wage by firm \( f \); offered minimum at all other firms
- **Group 3**: offered effective-unit wage by firm \( f \); offered effective-unit wage elsewhere

Profits can then be written

\[
\pi_f = \left( \mu^L \left( A_c^L Z_f^L, \text{group 1} + Z_f^L, \text{group 2} + Z_f^L, \text{group 3} \right) \right)^{\frac{\sigma-1}{\sigma}} - \bar{W} c N_f^L, \text{group 1} - w_f^L Z_f^L, \text{group 2} - w_f^L Z_f^L, \text{group 3} - w_f^H Z_f^H - F
\]

Taking the first-order condition with respect to \( w_f^L \) yields

\[
\gamma_f^L \mu^L \left( A_c^L \right)^{\frac{\sigma-1}{\sigma}} \left( Z_f^L, \text{group 1} + Z_f^L, \text{group 2} + Z_f^L, \text{group 3} \right)^{-\frac{1}{\sigma}} \left( \frac{\partial Z_f^L, \text{group 1}}{\partial w_f^L} + \frac{\partial Z_f^L, \text{group 2}}{\partial w_f^L} + \frac{\partial Z_f^L, \text{group 3}}{\partial w_f^L} \right) = 0
\]

(A.13)

where

\[
Z_f^L, \text{group 1} = \frac{N_c^L}{M_c} \int_{\frac{w_c^L}{MPZ_f^L}}^{\frac{w_f^L}{MPZ_f^L}} \frac{W_c}{w_f^L} \frac{w_c^L}{(w_f^L)^3} \frac{z g^L(z)}{z} \, dz
\]

\[
\frac{\partial Z_f^L, \text{group 1}}{\partial w_f^L} = -\frac{N_c^L}{M_c} \left( \frac{W_c^2}{(w_f^L)^3} g^L \left( \frac{W_c}{w_f^L} \right) + \frac{W_c^2}{(MPZ_f^L)^3} g^L \left( \frac{W_c}{MPZ_f^L} \right) \frac{\partial MPZ_f^L}{\partial w_f^L} \right)
\]
We look for a symmetric equilibrium in which \( w^L_f = w^L_c \). The first-order condition (A.13) becomes
\[
\begin{align*}
\text{MPZ}^L_c \frac{\partial Z_f^{L, \text{group 1}}}{\partial w_f^l} \bigg|_{w_f^l = w_c^l} + \text{MPZ}^L_c \frac{\partial Z_f^{L, \text{group 2}}}{\partial w_f^l} \bigg|_{w_f^l = w_c^l} + \text{MPZ}^L_c \frac{\partial Z_f^{L, \text{group 3}}}{\partial w_f^l} \bigg|_{w_f^l = w_c^l} \\
- \bar{W}_c \frac{\partial N_f^{L, \text{group 1}}}{\partial w_f^l} \bigg|_{w_f^l = w_c^l} - \left( Z_f^{L, \text{group 2}} \bigg|_{w_f^l = w_c^l} + w_f^l \frac{\partial Z_f^{L, \text{group 2}}}{\partial w_f^l} \bigg|_{w_f^l = w_c^l} \right) \\
- \left( Z_f^{L, \text{group 3}} \bigg|_{w_f^l = w_c^l} + w_f^l \frac{\partial Z_f^{L, \text{group 3}}}{\partial w_f^l} \bigg|_{w_f^l = w_c^l} \right) &= 0 \quad (A.14)
\end{align*}
\]

where

\[
\frac{\partial Z_f^{L, \text{group 1}}}{\partial w_f^l} \bigg|_{w_f^l = w_c^l} = -\frac{N_c^L}{M_c} \frac{\bar{W}_c^2}{(w_c^l)^3} \delta^L \left( \frac{\bar{W}_c^l}{w_c^l} \right) + \frac{\bar{W}_c^2}{(\text{MPZ}_c^L)^3} \delta^L \left( \frac{\bar{W}_c}{\text{MPZ}_c^L} \right) \frac{\partial \text{MPZ}_c^L}{\partial w_c^l}
\]

\[
\frac{\partial Z_f^{L, \text{group 2}}}{\partial w_f^l} \bigg|_{w_f^l = w_c^l} = \frac{N_c^L (\bar{W}_c)^2}{M_c (w_c^l)^3} \delta^L \left( \frac{\bar{W}_c^l}{w_c^l} \right)
\]

\[
\frac{\partial Z_f^{L, \text{group 3}}}{\partial w_f^l} \bigg|_{w_f^l = w_c^l} = N_c^L \text{Pr} \left( z_n^L > \frac{\bar{W}_c^l}{w_c^l} \right) \mathbb{E} \left[ z_n^L > \frac{\bar{W}_c^l}{w_c^l} \right] \frac{\partial \text{Pr}(n \text{ choose } f \mid w_f^l)}{\partial w_f^l} \bigg|_{w_f^l = w_c^l}
\]

\[
\frac{\partial N_f^{L, \text{group 1}}}{\partial w_f^l} \bigg|_{w_f^l = w_c^l} = -\frac{N_c^L}{M_c} \frac{\bar{W}_c^2}{(w_c^l)^3} \delta^L \left( \frac{\bar{W}_c^l}{w_c^l} \right) + \frac{\bar{W}_c}{(\text{MPZ}_c^L)^2} \delta^L \left( \frac{\bar{W}_c}{\text{MPZ}_c^L} \right) \frac{\partial \text{MPZ}_c^L}{\partial w_c^l}
\]
\[ Z_{f}^{L_{\text{group}2}}^{2} \bigg|_{w_{f}^{l}=w_{c}^{l}} = 0 \]

\[ Z_{f}^{L_{\text{group}3}} \bigg|_{w_{f}^{l}=w_{c}^{l}} = \frac{N_{c}^{L}}{M_{c}} \Pr \left( z_{n}^{L} > \frac{W_{c}}{w_{c}^{l}} \right) \mathbb{E} \left[ z | z_{n}^{L} > \frac{W_{c}}{w_{c}^{l}} \right] \]

Inserting these expressions into (A.14) and noticing several terms cancel, yields

\[ \text{MPZ}_{c}^{L} \frac{\partial \Pr(n \text{ choose } f | w_{f}^{l})}{\partial w_{f}^{l}} \bigg|_{w_{f}^{l}=w_{c}^{l}} - \frac{1}{M_{c}} - \left( \frac{w_{c}^{l}}{M_{c}} \right) \frac{\partial \Pr(n \text{ choose } f | w_{f}^{l})}{\partial w_{f}^{l}} \bigg|_{w_{f}^{l}=w_{c}^{l}} = 0 \]

Rearranging yields,

\[ w_{c}^{l} = \frac{\frac{\partial \Pr(n \text{ choose } f | w_{f}^{l})}{\partial w_{f}^{l}} \bigg|_{w_{f}^{l}=w_{c}^{l}}}{\frac{\partial \Pr(n \text{ choose } f | w_{f}^{l})}{\partial w_{f}^{l}} \bigg|_{w_{f}^{l}=w_{c}^{l}} \Pr(n \text{ choose } f | w_{f}^{l}=w_{c}^{l})} + 1 \]

\[ = \frac{\eta_{c}}{\eta_{c} + 1} \text{MPZ}_{c}^{L} \]

yielding the expression in the text.

### A.8 Computing spatial equilibrium with a minimum wage

Expected utility for low-skill workers is now

\[ u_{c}^{L} = \mathbb{E}[u_{n,c,f}^{L}] = \frac{B_{c}^{L} \mathbb{E}[u_{n,c}^{L}] \chi_{c}}{P_{c}^{1-\beta}} \quad \text{(A.15)} \]

while expected utility for high-skill workers remains as
\[ u^H_c = \mathbb{E}[u^H_{n_c,f}] = \frac{B^H_c u^H_c X_c}{P_c^{1-\beta}} \]  

(A.16)

We proceed using an algorithm similar to that in Section A.4. Given a guess of \( N^\theta_c \), expected utility (A.15) and (A.16) can be computed using the within-location equilibrium conditions entry (22), expected wages (19) and (20), and price of housing (24). In an interior spatial equilibrium,

\[ u^\theta_c = \frac{1}{C} \sum_c u^\theta_c \]

Rearranging and using the identity \( N^\theta_c = N^\theta_c \),

\[ N^\theta_c = \left( \frac{u^\theta_c (N^\theta_c)^{1}}{\frac{1}{C} \sum_c u^\theta_c} \right)^{\sigma} \]  

(A.17)

We construct a mapping from a guess of population in each location \( N^\theta_{c,\tau} \) to next round guess \( N^\theta_{c,\tau+1} \) using (A.17),

\[ N^\theta_{c,\tau+1} = \left( \frac{u^\theta_{c,\tau} (N^\theta_{c,\tau})^{1}}{\frac{1}{C} \sum_c u^\theta_{c,\tau}} \right)^{\sigma} \]  

(A.18)

Beginning with a guess \( N^\theta_{c,0} \), we compute \( u^\theta_{c,0} \) and then compute \( N^\theta_{c,1} \) using (A.18), rescaling all populations such that \( \sum_c N^\theta_{c,1} = N^\theta \). We proceed in this manner until utility converges: \( u_{c,\tau+1} \rightarrow u_{c,\tau} \).
A.9 Effect of minimum wage on low-skill pay: case of an unemployment-inducing minimum wage

Figure A.1: Monopsony model: effect of a unemployment-inducing minimum wage on low-skill pay

![Graph showing the effect of a minimum wage on low-skill per-worker wages](image)

Figure: Effect of a minimum wage $W_c^*$ on low-skill per-worker wages $W_{nc}$ in monopsony model as a function of worker-level productivity $z_n^L$. $W_{nc}^*$ denotes the low-skill per-worker wage in laissez-faire. Shown for a minimum wage $W_c$ that does induce unemployment

A.10 Proof of Result 1

Begin by noting that all output is distributed to fixed costs and labor

$$Y_c = M_c F + \mathbb{E}[W_{nc}^L] N_c^L + \mathbb{E}[W_{nc}^H] N_c^H \quad (A.19)$$

By full employment, $Y_c = Y_c^*$. First, note that it must be the case that $M_c$ strictly declines: if $M_c$ increases, by (19) and (20), then total payments to labor and fixed costs both increase, despite unchanged $Y_c$, contradicting (A.19). If $M_c$ remains unchanged, total payments to
labor increase (by the fact the minimum wage binds), again contradicting (A.19). Therefore, $M_c$ strictly declines. Given this, average payment to high-skill labor $\mathbb{E}[W^H_{nc}]$ must decline (by (20)) and average payments to low-skill labor $\mathbb{E}[W^L_{nc}]$ must increase; total payments to labor must increase by (A.19). Therefore, the price of housing must increase, by (24).

A.11 Unemployment inducing minimum wage in a local labor market in partial equilibrium

Result A.1 (Unemployment inducing minimum wage in a local labor market in partial equilibrium). There exist a range of minimum wages $W_c > MPZ^L_c \geq L$ that do induce unemployment. Such a minimum wage, relative to laissez-faire,

- **Ambiguous effect on income for low skill workers, $\mathbb{E}[W^L_{nc}]$.** A mass of low-skill workers are unemployed; a mass of low-skill workers are paid exactly the minimum.
- **Decreases the number of operating firms, $M_c$**
- **Decreases average income for high skill workers, $\mathbb{E}[W^H_{nc}]$**
- **Ambiguous effect the price of housing, $P_c$**

Proof: Begin by noting again that all output is distributed to fixed costs and labor

$$Y_c = M_c F + \mathbb{E}[W^L_{nc}] N^L_c + \mathbb{E}[W^H_{nc}] N^H_c$$  \hspace{1cm} (A.20)

First note it must be the case that $M_c$ strictly declines: if $M_c$ increases, by (A.20), payments to low-skill labor fell by more than output, contradicting wage-setting (19). Therefore, by (20), average payment to high-skill workers declines.
### B Empirics appendix

#### Table B.1: Sample statistics: jobs

<table>
<thead>
<tr>
<th>Industry</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log daily wage</td>
<td>4.24</td>
</tr>
<tr>
<td>Below university</td>
<td>0.85</td>
</tr>
<tr>
<td>University and above</td>
<td>0.13</td>
</tr>
<tr>
<td>Agriculture, hunting and forestry, fishing</td>
<td>0.04</td>
</tr>
<tr>
<td>Manufacture of food products beverages and tobacco</td>
<td>0.02</td>
</tr>
<tr>
<td>Manufacture of consumer products</td>
<td>0.02</td>
</tr>
<tr>
<td>Manufacture of industrial goods</td>
<td>0.07</td>
</tr>
<tr>
<td>Manufacture of capital and consumer goods</td>
<td>0.07</td>
</tr>
<tr>
<td>Construction</td>
<td>0.10</td>
</tr>
<tr>
<td>Hotels and restaurants</td>
<td>0.21</td>
</tr>
<tr>
<td>Transport, storage, o. serv</td>
<td>0.29</td>
</tr>
<tr>
<td>Education</td>
<td>0.15</td>
</tr>
<tr>
<td>Observations</td>
<td>2123000</td>
</tr>
</tbody>
</table>

Sample statistics for (log) daily wages, worker education, and industries across jobs in the estimation sample.

#### Table B.2: Sample statistics: regions

<table>
<thead>
<tr>
<th></th>
<th>Count</th>
<th>Mean</th>
<th>p10</th>
<th>p25</th>
<th>p50</th>
<th>p75</th>
<th>p90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log daily wage</td>
<td>401</td>
<td>4.42</td>
<td>4.21</td>
<td>4.35</td>
<td>4.44</td>
<td>4.51</td>
<td>4.57</td>
</tr>
<tr>
<td>Log worker count (sample)</td>
<td>401</td>
<td>6.08</td>
<td>5.29</td>
<td>5.58</td>
<td>5.98</td>
<td>6.51</td>
<td>6.98</td>
</tr>
<tr>
<td>Log firm count (sample)</td>
<td>401</td>
<td>5.64</td>
<td>4.87</td>
<td>5.17</td>
<td>5.55</td>
<td>6.06</td>
<td>6.49</td>
</tr>
</tbody>
</table>

Sample statistics for regions in SIAB, based on the estimation sample. p10 refers to the value at the 10th percentile across the 401 districts/Kreis.
Table B.3: Second-step results robustness: elasticity of labor supply and size

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>(Log) estimated elasticity of labor supply</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Log) number of firms</td>
<td></td>
<td>0.11**</td>
<td>0.17***</td>
<td>0.08**</td>
<td>0.08**</td>
<td>0.08***</td>
<td>0.06**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.054)</td>
<td>(0.058)</td>
<td>(0.030)</td>
<td>(0.032)</td>
<td>(0.026)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>1st-step worker-region FE</td>
<td></td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Only exits to employment</td>
<td></td>
<td>Yes</td>
<td>Yes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low education only</td>
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<td></td>
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<td>Yes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commuting zones</td>
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<td></td>
<td></td>
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<td></td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>2nd-step controls</td>
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<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>R^2</td>
<td>0.01</td>
<td>0.05</td>
<td>0.02</td>
<td>0.14</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td>N</td>
<td>380</td>
<td>380</td>
<td>400</td>
<td>400</td>
<td>136</td>
</tr>
</tbody>
</table>

Standard errors in parentheses. Significance levels: * 10%, ** 5%, ***1%. 2nd-step controls are dummy for East, share of workers in manual occupations, share of workers in professional occupations, and share of workers who are German. Regressions are all linear specification. Columns (1) and (2) use only separations to employment in defining separations. Columns (3) and (4) use only workers with less than a university degree. Columns (5) and (6) are specified at the labor market region level, from Kosfeld and Werner (2012).
Table B.4: Second-step results robustness: education in 2nd-step controls

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>(Log) estimated elasticity of labor supply</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>(Log) number of firms</td>
<td>0.17***</td>
</tr>
<tr>
<td></td>
<td>(0.033)</td>
</tr>
<tr>
<td>1st-step worker-region FE</td>
<td>Yes</td>
</tr>
<tr>
<td>2nd-step educ + occ controls</td>
<td>Yes</td>
</tr>
<tr>
<td>1st-step controls</td>
<td>Yes</td>
</tr>
<tr>
<td>N</td>
<td>401</td>
</tr>
</tbody>
</table>

Standard errors in parentheses. Significance levels: * 10%, ** 5%, ***1%. 2nd-step controls are share of workers with University and above education and share of workers in manual occupations, share of workers in professional occupations. 1st-step controls are industry and occupation fixed effects. Regressions are linear specification.
Figure B.1: Employees per firm vs district size. Employees from VGRdL, number of firms per district from BHP metadata.
Table B.5: ‘Constrained optimal’ minimum wage: variation with average wages and size

<table>
<thead>
<tr>
<th></th>
<th>(log) ‘constrained optimal’ MW</th>
<th>(log) avg low-skill wage</th>
</tr>
</thead>
<tbody>
<tr>
<td>(log) avg low-skill wage</td>
<td>0.90 (0.007)</td>
<td></td>
</tr>
<tr>
<td>(log) employees</td>
<td>0.05 (0.009)</td>
<td>0.08 (0.009)</td>
</tr>
<tr>
<td>$N$</td>
<td>401</td>
<td>401</td>
</tr>
</tbody>
</table>

Standard errors in parentheses. Elasticities of ‘Constrained optimal’ minimum wage with average low-skill wage and employees. Last column: elasticity of average low-skill wage and employees. Constant terms included in regressions but omitted.
Table B.6: ‘Flat’ law vs ‘targeted’ law in perfectly competitive model.

<table>
<thead>
<tr>
<th></th>
<th>‘Flat’ law</th>
<th></th>
<th>‘Targeted’ law</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>p10</td>
<td>p90</td>
<td>national</td>
<td>p10</td>
</tr>
<tr>
<td>Unemployment</td>
<td>0.80</td>
<td>18.01</td>
<td>5.11</td>
<td>0.80</td>
</tr>
<tr>
<td>Paid exactly minimum</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Avg low-skill wage</td>
<td>-12.36</td>
<td>-0.91</td>
<td>-1.91</td>
<td>-7.95</td>
</tr>
<tr>
<td>Avg low-skill wage</td>
<td>-12.36</td>
<td>-0.91</td>
<td>-1.91</td>
<td>0.09</td>
</tr>
<tr>
<td>Avg high-skill wage</td>
<td>-10.30</td>
<td>1.43</td>
<td>0.27</td>
<td>-6.11</td>
</tr>
<tr>
<td>Low-skill welfare</td>
<td>/</td>
<td>/</td>
<td>-2.97</td>
<td>/</td>
</tr>
<tr>
<td>High-skill welfare</td>
<td>/</td>
<td>/</td>
<td>-0.69</td>
<td>/</td>
</tr>
<tr>
<td>Overall worker welfare</td>
<td>/</td>
<td>/</td>
<td>-2.48</td>
<td>/</td>
</tr>
<tr>
<td>Regional low-skill nominal income inequality</td>
<td>/</td>
<td>/</td>
<td>27.76</td>
<td>/</td>
</tr>
</tbody>
</table>

Long run spatial equilibrium effects of ‘flat’ law vs ‘targeted’ law in perfectly competitive model. Figures are in percent (unemployment, workers paid exactly minimum) and percentage change vs laissez-faire (all other variables). p10 denotes the 10th percentile across districts; p90 denotes the 90th percentile across districts; national denotes the national value.
Table B.7: ‘Flat’ law vs ‘targeted’ law in monopsony model.

<table>
<thead>
<tr>
<th></th>
<th>‘Flat’ law</th>
<th></th>
<th>‘Targeted’ law</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>p10</td>
<td>p90</td>
<td>national</td>
<td>p10</td>
</tr>
<tr>
<td>Unemployment</td>
<td>0.00</td>
<td>0.20</td>
<td>0.07</td>
<td>0.00</td>
</tr>
<tr>
<td>Paid exactly minimum</td>
<td>0.75</td>
<td>20.93</td>
<td>6.26</td>
<td>0.75</td>
</tr>
<tr>
<td>Avg low-skill wage</td>
<td>0.09</td>
<td>1.73</td>
<td>0.31</td>
<td>0.07</td>
</tr>
<tr>
<td>Avg low-skill wage</td>
<td>-0.39</td>
<td>1.25</td>
<td>-0.07</td>
<td>-0.26</td>
</tr>
<tr>
<td>Low-skill welfare</td>
<td>/</td>
<td>/</td>
<td>0.21</td>
<td>/</td>
</tr>
<tr>
<td>High-skill welfare</td>
<td>/</td>
<td>/</td>
<td>-0.27</td>
<td>/</td>
</tr>
<tr>
<td>Overall worker welfare</td>
<td>/</td>
<td>/</td>
<td>0.10</td>
<td>/</td>
</tr>
<tr>
<td>Regional low-skill nominal income inequality</td>
<td>/</td>
<td>/</td>
<td>-3.55</td>
<td>/</td>
</tr>
</tbody>
</table>

Long run spatial equilibrium effects of ‘flat’ law vs ‘targeted’ law in monopsony model. Figures are in percent (unemployment, workers paid exactly minimum) and percentage change vs laissez-faire (all other variables). p10 denotes the 10th percentile across districts; p90 denotes the 90th percentile across districts; national denotes the national value.
Table B.8: ‘Flat’ law vs ‘constrained optimal’ law in monopsony model.

<table>
<thead>
<tr>
<th></th>
<th>‘Flat’ law</th>
<th></th>
<th>‘Constrained optimal’ law</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>p10</td>
<td>p90</td>
<td>national</td>
<td>p10</td>
</tr>
<tr>
<td>Unemployment</td>
<td>0.00</td>
<td>0.20</td>
<td>0.07</td>
<td>0.00</td>
</tr>
<tr>
<td>Paid exactly minimum</td>
<td>0.75</td>
<td>20.93</td>
<td>6.26</td>
<td>13.79</td>
</tr>
<tr>
<td>Avg low-skill wage</td>
<td>0.09</td>
<td>1.73</td>
<td>0.31</td>
<td>1.31</td>
</tr>
<tr>
<td>Avg low-skill wage</td>
<td>-0.39</td>
<td>1.25</td>
<td>-0.07</td>
<td>-0.10</td>
</tr>
<tr>
<td>Low-skill welfare</td>
<td>/</td>
<td>/</td>
<td>0.21</td>
<td>/</td>
</tr>
<tr>
<td>High-skill welfare</td>
<td>/</td>
<td>/</td>
<td>-0.27</td>
<td>/</td>
</tr>
<tr>
<td>Overall worker welfare</td>
<td>/</td>
<td>/</td>
<td>0.10</td>
<td>/</td>
</tr>
<tr>
<td>Regional low-skill nominal income inequality</td>
<td>/</td>
<td>/</td>
<td>-3.55</td>
<td>/</td>
</tr>
</tbody>
</table>

Long-run spatial equilibrium effects of ‘flat’ law vs ‘constrained optimal’ law in monopsony model. Figures are percent (unemployment, workers paid exactly minimum) and percentage change vs laissez-faire (all other variables).
Figure B.2: Employees in each district/Kreis in 2014, from VGRdL
Figure B.3: Compensation per employee in each Kreis in 2014, from VGRdL
Figure B.4: Price index 2014, from RWI-GEO-REDX
Figure B.5: Price index 2014, from RWI-GEO-REDX
Figure B.6: Number of firms in each district/Kreis in 2005 (data)
Figure B.7: Number of firms in each district/Kreis (calibrated model)
Figure B.8: Number of firms in each district in calibrated model on number of firms in the data

Figure: Log number of firms $\log M_c$ in calibrated model vs log number of firms in the data. Coefficient from regression 0.98 [0.024], $R^2 = 0.80$. 
Figure B.9: ‘Constrained optimal’ law vs ‘flat’ law and ‘targeted’ law

(a) East Germany

(b) West Germany

Figure: The value of the ‘constrained optimal’ law in each district. Circe size proportional to district employment.