

# Price Staggering in Cartels

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## Abstract

In this paper we investigate the optimal organization of staggered price increases in cartels. Staggered price increases impose a cost during cartel formation as the price leader initially loses sales. We show that for intermediate discount factors, staggered price increases can only be sustained when the increase is neither too low nor too high. When a cartel executes two consecutive price increases, the choice between using the same leader or alternating leadership depends on the initial price level in the industry. We also discuss the allocation of price leadership in the presence of cost asymmetry and consider the effect of strategic buyers on price staggering.

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# 1 Introduction

Over the last two decades antitrust authorities around the world have stepped up their efforts to prosecute hard-core cartels. Records from recent cartel cases show that companies are well aware of the increased threat. Strategies to avoid detection from buyers and antitrust authorities have become part of the cartel's organization. A particularly well documented practice of cartels to avoid the appearance of collusion is price staggering, the orchestration of sequential rather than simultaneous price increases.<sup>1</sup> Consider, for example, the *Electrical and Mechanical Carbon and Graphite Products (EMCG)* cartel prosecuted by the European Commission (EC):

*For the new prices to take effect, one of the cartel members would circulate its new price list to customers at some time between January and March in the year following the Technical Committee meeting. The other cartel members would follow suit and issue their new price lists over the following weeks or months, thereby trying to create the impression that the companies concerned took their pricing decisions autonomously.*<sup>2</sup>

Similarly, the *Vitamins* cartel used sequential price increases as they "...could be passed off, if challenged, as the result of price leadership in an oligopolistic market."<sup>3</sup> However, staggering a price increase is costly as the price leader risks losing sales before the follower raises the price. This was a clear concern in the *Rubber Chemicals* cartel:

*Bayer was not sure about the success of the increase because it was so large. It needed a three-month waiting period for the adjustment of prices to avoid the appearance of collusion. Flexsys agreed but was worried about losing significant market share. Bayer assured that there was no reason to worry and that it would follow in three months.*<sup>4</sup>

Flexsys' concerns were later attested as it lost significant volumes and key customers.<sup>5</sup>

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<sup>1</sup>For example, staggered pricing is reported in the case documentation of 29 out of the 41 cartel cases of the European Commission during the period 2002-2007.

<sup>2</sup>See EC Case 38.359, 3 December 2003, para 101.

<sup>3</sup>See EC Case 37.512, 21 November 2001, para 203-4.

<sup>4</sup>See EC Case 38.443, 21 December 2005, para 70.

<sup>5</sup>A significant loss of sales also almost caused the *Fine Arts Auction House* cartel to collapse early on. As part of the first price increase agreement, Christie's had already announced its new non-negotiable

In this paper, we develop a theoretical model of staggered pricing in cartels and analyze the optimal organization of collusion in different settings. In our benchmark model, we consider a Bertrand duopoly with homogenous goods when firms compete in prices with an infinite horizon. The firms are interested in raising the industry price from its (exogenously given) current level towards a target price. A staggered price increase involves the cartel specifying a leader who raises the price to the target level in period  $t$ . In period  $t + 1$ , the other cartel member follows suit and raises its price to the same level. We show that for intermediate discount factors the firms' incentive constraints affect the type of price increases that can be implemented. In particular, a staggered price increase can be sustained if and only if the target price is neither too small nor too large relative to the current industry price. Large price increases give the follower a strong incentive to deviate and raise its price to undercut the target price. By contrast, small price increases are unable to satisfy the leader's incentive constraint to raise its price and forego current period sales.

We then explore a situation where the cartel is scheduling two consecutive staggered price increases and can choose between using the same leader or alternating leadership. If the current industry price is relatively low, collusion is easier to sustain with the same price leader. Vice versa, for higher initial price levels, the price increases are easier to implement with alternating leaders. When using the same leader, the binding constraint is the first period incentive constraint of the leading firm. When the initial price is low, the leader's deviation profits are small and the cartel is easier to sustain. With alternating leaders, the more restrictive period  $t$  constraint is the one that controls the incentives of the firm that follows first and leads second. In this case, a high initial market price increases the sustainability of the cartel. Overall, using the optimal choice of leadership pattern allows the cartel to move the industry price from any starting level to the monopoly level under the same condition as a simultaneous price increases is sustainable.

Towards endogenizing the price leader role in the cartel, we consider staggered price schedule with a minimum seller commission of 2 percent. Sotheby's followed three weeks later. In the meantime, Sotheby's won a very significant jewellery consignment from Alghanim worth nearly US\$10 million. Apparently, the CEO of Christie's was furious when he received the news and began to fear that Sotheby's would double-cross him (Mason, 2004, p.166-167).

increases when firms face asymmetric cost of production. Cost asymmetries make it harder to sustain collusion or simultaneous price increases. We show that when firms implement staggered price increases, the cartel can alleviate the burden imposed by asymmetry by selecting the high-cost firm as the leader. As a leader, a low-cost firm stands to gain more from increasing its demand by undercutting the follower. Vice versa, as a follower, the low-cost is less tempted to increase price and reduce its demand level relative to its high-cost competitor. Moreover, the difference in punishment profits between the two firms does not affect the choice of the price leader as the market share has to be adjusted for the stronger incentive of the low-cost firm to deviate independent of its role in the price increase.

We also extend the benchmark model by considering multiple leaders and followers and strategic buyers in a staggered price increase. More firms in the cartel and more leaders in a staggered price increase make collusion harder to sustain. The effect of strategic buyers on the sustainability of the cartel is ambiguous. Strategic buyers are able to bring forward their purchase and buy right at the beginning of the cartel's price move. This reduces future demand and profits of the cartel but also strengthens the incentives of the follower.

The literature on cartels is usually concerned with sustaining collusion at a long-running target price.<sup>6</sup> Very little attention has been given to the gradual process of reaching that target level which seems to be a major impediment to cartel formation in practice. Harrington (2005) characterizes the optimal cartel price path to its steady-state level in the presence of an exogenous buyer detection process and (accumulating) damage awards. The steady-state cartel price is strictly below the monopoly level when penalties include (price dependent) damage awards. When penalties are price independent, the cartel reaches the monopoly price in the long run.<sup>7</sup> Harrington (2004) extends the analysis of this framework by considering lower discount factors in which the cartel stability constraint is binding on the optimal price path. In this case, numerical analysis shows that the optimal cartel price might be first increasing and then decreasing. In both

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<sup>6</sup>See Ivaldi, Jullien, Seabright, and Tirole (2007), Choi and Gerlach (2014), Levenstein and Suslow (2014) and Green, Marshall, and Marx (2014) for recent surveys.

<sup>7</sup>The reason for this result is that in the steady state, marginal price changes have a negligible effect on detection and only affect penalties through the damage awards. If penalties are not sensitive to the cartel's price, the monopoly price is achieved at no cost.

papers cartel members charge the same price and raise their prices simultaneously. We consider sequential price increases when buyers and antitrust authorities are suspicious of simultaneous price movements of cartel members.

Our paper also relates to the long-standing discussion of price leadership in industries. Going back to the early work by Stigler (1947) and Markham (1951), three (overlapping) forms of price leadership are typically distinguished. Dominant firm price leadership occurs when one large producer sets its price and a smaller competitor or a competitive fringe follows as price takers. Deneckere and Kovenock (1992) show that when two capacity-constrained firms compete in prices, the larger firm emerges as the leader in equilibrium. The reason is that the small firm stands to lose more from being undercut and leads more aggressively. By contrast, the large firm can provide a price umbrella which allows the small firm to undercut and sell its entire capacity.<sup>8</sup> Competitive, barometric price leadership refers to situations where changes in prices reflect market conditions and asymmetrically informed firms.<sup>9</sup> Finally, collusive price leadership occurs when the process of price changes is intended to coordinate prices at the collusive level. Rotemberg and Saloner (1990) consider a repeated game model of a duopoly with differentiated products and asymmetric information. They show that there exist a stationary equilibrium in which the informed firm acts as price leader and the uninformed firm price matches. Although the duopolists achieve a supra-competitive outcome, they are unable to implement the (informationally unconstrained) first-best outcome. More recently, Ishibashi (2008) and Mouraviev and Rey (2011) use a framework that endogenizes the timing of firms' strategy choice in a repeated game. In each period there is an extended game with action commitment in which the firm who wants to lead can commit to its price choice. The waiting firm can observe the choice of the leader and select its price before demand is realized. Mouraviev and Rey (2011) show that price leadership can drastically increase the sustainability of price collusion. A deviation of the leader can be immediately punished and the follower can be assigned a higher market share to prevent undercutting. Ishibashi (2008) demonstrates that the firm with the

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<sup>8</sup>Similar results hold in settings where firm size is measured by the base of loyal customers (Deneckere, Kovenock, and Lee, 1992) and when products are imperfect substitutes (Furth and Kovenock, 1993).

<sup>9</sup>In Cooper (1996), for example, one firm acquires information about the current market conditions and changes its price accordingly while the other firm follows rather than investing in information gathering itself.

larger capacity emerges as price leader in order to demonstrate its commitment not to deviate. Finally, Marshall, Marx, and Raiff (2008) develop a theoretical model of price announcements in a duopoly and use data from the Vitamins cartel to determine the existence of explicit collusion based on the pattern of price announcements. Our paper differs from this literature in two aspects. We do not require explicit action commitment as it arises endogenously in our model. More importantly, we consider situations where demand arises before the cartel's price moves are finished. This introduces an additional cost that the cartel has to bear in order to avoid the appearance of collusion.

The remainder of the paper is organized as follows. In the next section, we set up and analyze the benchmark model. In section 3 we derive the optimal organization of a cartel that implements two staggered price increases. In the following section we introduce cost asymmetry and discuss the optimal allocation of the price leader role. In Section 5, we consider two extensions, cartels with multiple leaders and followers and staggered pricing with strategic buyers. The last section concludes. All proofs are relegated to the Appendix.

## 2 Benchmark Model

Consider a Bertrand duopoly model where firms compete in prices in a homogeneous product market. The firms produce at the same constant marginal cost  $c$  and face a demand of  $D(p)$  with  $D'(p) < 0$ . Total industry profits at price  $p$  are given by  $\pi(p) = (p - c)D(p)$  and assumed to be quasi-concave. Let  $p^m$  be the monopoly price, that is,

$$p^m = \arg \max_p \pi(p),$$

and the corresponding industry profits are  $\pi(p^m)$ .

Firms play a repeated game over an infinite horizon in discrete time. Each firm has a discount factor  $\delta \in [0, 1]$ . Firms set their prices simultaneously in each period. We are interested in coordinated price increases towards the fully collusive level  $p^m$ . Suppose the industry price in the current period  $t$  is  $p_1 \in [c, p^m)$  and firms intend to increase the price to  $p_2 > p_1$ . The initial industry price could be the outcome of a repeated

game in which firms partially collude to sustain a price below the monopoly level.<sup>10</sup> In the benchmark model we consider a single price increase while in Section 3 the cartel organizes two consecutive price increases in the industry.

As discussed in the introduction, firms are aware that simultaneous price increases may raise the suspicion of competition authorities. The firms are thus interested in implementing a staggered price increase. We consider the following strategy in the repeated game. At the beginning of period  $t$ , firms meet to form a cartel agreement specifying their roles. The leader, denoted as firm L, sets price  $p_2$  in the current period. The follower, firm F, keeps its price at the current level  $p_1$ . From period  $t + 1$  onwards, both firms charge the new cartel price  $p_2$ . Deviations from this strategy are punished with reversion to static Nash equilibrium prices and zero continuation profits.<sup>11</sup> We allow firms to assign different market shares for leader and follower from period  $t + 1$  onwards. Let  $s$  be the market share of the price leader. We do not consider monetary transfers between the firms as such payment increases the probability of detection and antitrust prosecution.

In what follows we derive the conditions under which such a staggered price increase strategy is sustainable in a Subgame Perfect Equilibrium of the repeated game. First, consider the incentives in the continuation game from period  $t + 1$  onwards. The leader's incentive constraint is

$$s\pi(p_2)\frac{1}{1-\delta} \geq \pi(p_2) \quad \text{or} \quad s \geq 1 - \delta. \quad (1)$$

Similarly, the follower's incentive constraint is given by

$$(1-s)\pi(p_2)\frac{1}{1-\delta} \geq \pi(p_2) \quad \text{or} \quad s \leq \delta. \quad (2)$$

Undercutting the cartel price allows the deviator to steal the market share of the other firm. The allocated market shares thus have to ensure that the potential gain in market

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<sup>10</sup>Alternatively, with just minor changes to notation, our results would hold in a situation where a cartel faces a cost increase from  $c_1$  to  $c_2 > c_1$  and firms fully collude. In this case the cartel wants to adjust the price from  $p_1 = p^m(c_1)$  to the new monopoly level  $p_2 = p^m(c_2)$ .

<sup>11</sup>This is the maximum punishment that can be imposed. This strategy also ensures that firms always have an incentive to implement the staggered price increase.

share does not exceed the discount factor. It follows that for any  $\delta \geq 1/2$ , any market share allocation  $s \in [1 - \delta, \delta]$  can be supported in the continuation game of the staggered price increase. The range of available market shares thus depends on the discount factor. For high discount factors, it is easier to sustain more asymmetric market shares. For discount factors closer but above  $1/2$ , only cartels with similar market shares are sustainable. Finally, note that these conditions coincide with the usual incentive constraints when firms implement a simultaneous price increase from  $p_1$  to  $p_2$ . That is, a simultaneous price increase is sustainable if and only if  $\delta \geq 1/2$ .

Let us now look at the incentives to implement the staggered price increase in period  $t$ . If the leader increases the price to  $p_2$ , it loses sales in period  $t$  but receives the assigned continuation profit from period  $t + 1$  onwards. Alternatively, the leader can undercut the follower and capture the entire market at price  $p_1$ . The leader's constraint in period  $t$  is then

$$\frac{\delta}{1 - \delta} s \pi(p_2) \geq \pi(p_1). \quad (3)$$

Ceteris paribus, the leader's constraint is easier to satisfy the larger the difference between  $\pi(p_2)$  and  $\pi(p_1)$ , that is, the steeper the price increase is. A large price increase implies that a deviating leader has to cut its price strongly in order to attract demand. This makes a cartel easier to sustain. If the current price is equal to the firms' marginal cost, then the leader has no incentive to deviate. Similarly, if the leader's future market share is sufficiently close to one and the discount factor greater than  $1/2$ , this condition is always satisfied.

If the follower adheres to the collusive agreement, it will serve the entire market and earn  $\pi(p_1)$  in period  $t$  before receiving its continuation value from period  $t + 1$  onwards. The best deviation for the follower is to shave the price of the leader and serve the entire market. The follower's incentive constraint in period  $t$  is therefore

$$\pi(p_1) + \frac{\delta}{1 - \delta} (1 - s) \pi(p_2) \geq \pi(p_2) \quad (4)$$

This condition is satisfied for any price increase if the discount factor and the market share of the follower are sufficiently high. Otherwise, the condition is easier to satisfy if the difference between  $\pi(p_2)$  and  $\pi(p_1)$  is small, that is, when the price increase is not

too steep. A large price increase raises the price a deviating follower can charge. This makes the cartel harder to sustain.

The cartel chooses the market share  $s$  to increase the sustainability of the cartel. We call a cartel organization optimal when the choice of the market share minimizes the discount factor threshold above which collusion is sustainable. Our first result explores which of the four constraints are binding in an optimal cartel organization.

**Lemma 1** *Consider the lowest possible discount factor such that conditions (1) to (4) are jointly satisfied. If  $\pi(p_1) \leq \pi(p_2)/2$ , then the follower's period  $t$  and the leader's period  $t + 1$  constraints are strictly binding. Otherwise, the leader's period  $t$  and the follower's period  $t + 1$  are strictly binding.*

When firms intend to implement a large price increase, the follower has a strong incentive to raise the price just below the leader's price and serve the entire market. This implies that the period  $t$  constraint of the follower is more restrictive and the cartel has to assign more market share to the follower. An increase in the follower's market share is, however, limited by the period  $t + 1$  incentive constraint of the leader. Thus, for large price increases, conditions (1) and (4) are strictly binding. Vice versa, for small price increases, the leader has a strong incentive to lower the price and undercut the follower. Hence, the period  $t$  constraint of the leader is more restrictive. Thus, for price increases where  $\pi(p_1) > \pi(p_2)/2$ , the cartel assigns more market share to the leader and conditions (2) and (3) are strictly binding.

This leads us to the conditions under which a staggered price increase is sustainable.

**Lemma 2** *There exists a  $\delta'$  with  $1/2 \leq \delta' < 1$  such that if  $\delta > \delta'$ , then a staggered price increase is sustainable. It holds that  $\delta' = 1/2$  if and only if  $\pi(p_1) = \pi(p_2)/2$ .*

First, reflecting the usual Folk Theorem type result, if firms are sufficiently patient, any staggered price increase is sustainable. Since  $\delta' \geq 1/2$ , staggered price increases are weakly harder to sustain as simultaneous price increases. A staggered price increase reduces the cartel's profit on the equilibrium path due to the follower's undercutting of the leader. The cartel's ability to adjust market shares to satisfy the more restrictive

period  $t$  constraint is limited by the fact that the continuation of the cartel beyond period  $t + 1$  is only feasible if the market shares are not too asymmetric. The leader and follower's period  $t$  constraints coincide when the staggered price increase doubles industry profit. In this case symmetric market shares are optimal. A staggered and a simultaneous price increase can be implemented under the same condition.

In what follows we focus on the case of intermediate discount factors and explore the optimal organization of staggered price increases when deviation incentives are strictly binding. To further illustrate the constraints of the cartel, consider FIGURE 1 below. The figure uses a  $\pi(p_1) - \pi(p_2)$  diagram for intermediate discount factors. First, consider values such that  $\pi(p_1) = \pi(p_2)/2$ . We know from Lemma 2 that along those values, a price increase can be sustained for symmetric market shares and any  $\delta \geq 1/2$ . Now fix a target price  $p_2$  and reduce the initial industry price  $p_1$ . This makes the period  $t$  constraint of the leader easier to sustain and that of the follower harder to sustain. At some point, the follower's constraint is strictly binding for symmetric market shares and the cartel needs to assign more market share to the follower in order to sustain the price increase. Eventually, this increase in market share is limited by the  $t + 1$  constraint of the leader. The line denoted (1) and (4) depicts the pairs of industry profits before and after the price increase at which conditions (1) and (4) are both strictly binding. For lower  $\pi(p_1)$ , the cartel is unable to satisfy the follower's period  $t$  constraint without violating the leader's  $t + 1$  constraint, and the price increase is not implementable.

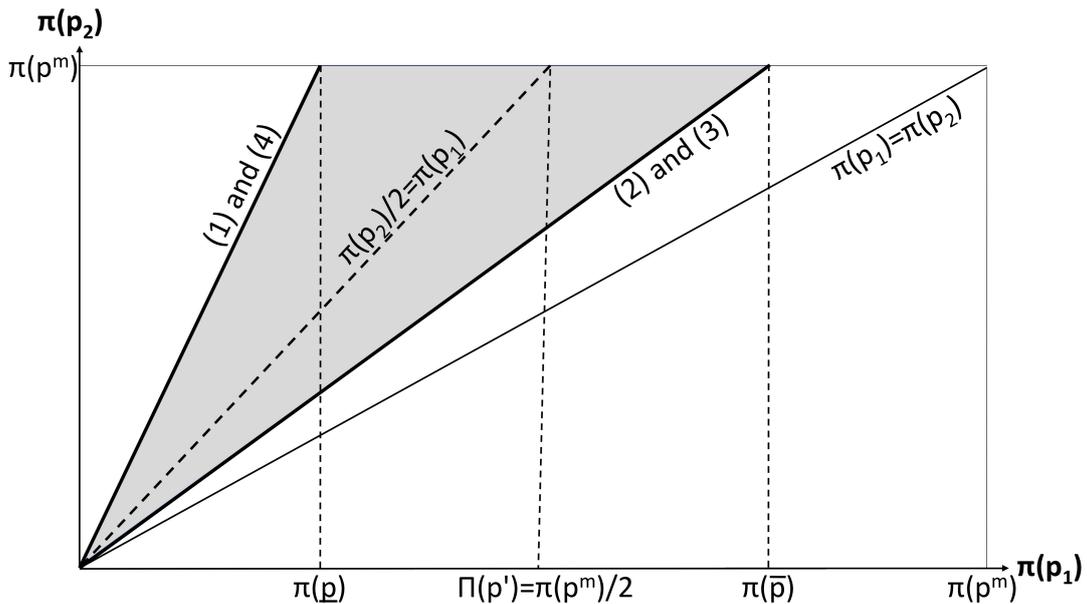


Figure 1: *Implementable staggered price increases for intermediate discount factors*

Vice versa, increasing the initial industry price beyond the level where  $\pi(p_1) = \pi(p_2)/2$  relaxes the follower's and tightens the leader's period  $t$  constraint. Satisfying the leader's constraint with a higher market share is limited by (2). Any prices yielding profit pairs below the line denoted (2) and (3) are not implementable in a staggered price increase. We can thus conclude the following.

**Proposition 1** *Consider intermediate discount factors and a cartel that intends to implement a single staggered increase to a target price  $p_2$ . The staggered price increase is sustainable if and only if the initial industry price  $p_1$  is neither too low nor too high.*

Another way of stating this result is to say that if the current industry price is low, then only price increases of intermediate size can be sustained. If the current industry price is high, then a price increase can only be sustained if it is sufficiently large. In particular, our result implies the following corollary. Let  $p'$ , with  $p' < p^m$ , be the price such that

$$\pi(p') = \pi(p^m)/2.$$

**Corollary 1** *A staggered price increase to the monopoly price is sustainable if and only if the current industry price satisfies  $p_1 \in [\underline{p}, \bar{p}]$ , with  $c < \underline{p} \leq p'$  and  $p' \leq \bar{p} < p^m$ .*

A cartel is not able to implement a single staggered price increase from a sufficiently competitive industry level to the monopoly price due to the follower's incentive to undercut the leader. The cartel is also not able to raise the price to the monopoly level if the current price is too high as the leader's constraint in period  $t$  would be binding. The price thresholds  $\underline{p}$  and  $\bar{p}$  are both equal to  $p'$  at  $\delta = 1/2$ , but they diverge as  $\delta$  increases further.

### 3 Two Staggered Price Increases

There is evidence that in some instances cartels plan more than one staggered price increase at a time. For example, the *Rubber Chemicals* cartel scheduled two consecutive

price increases in anti-degradants and primary accelerators in meetings in early 1999. The first increase was led by Bayer, effective on 1 October 1999 for non-tyre customers and on 1 January 2000 for tyre customers. The second increase on 1 July 2000 was led by Flexsys and implemented on a global scale.<sup>12</sup> In this section we analyze the optimal organization and sustainability of two consecutive staggered price increases. We investigate whether collusion is easier to sustain when using the same price leader in both increases or when alternating price leadership like in the *Rubber Chemicals* cartel.

We consider the following set-up. In period  $t$ , the follower charges the current industry price  $p_0$  while the leader sets a price  $p_1 > p_0$ . In period  $t + 1$ , the leading firm sets  $p_2$  while the follower charges  $p_1$ . From period  $t + 2$  onwards, both firms charge the new cartel price  $p_2$ . We treat the initial industry price  $p_0$  and the target price  $p_2$  as parameters. The cartel can choose the intermediate price level to increase the sustainability of the cartel. In the previous section we showed that a single staggered price increase might not be able to raise the cartel's price to the monopoly level. Hence, when considering two price increases we are most interested in the case where the cartel would not be able to sustain a single staggered price increase to the target monopoly price. That is, the current industry price satisfies  $p_0 \leq \underline{p}$  and the target price is  $p_2 = p^m$ . For notational convenience, let  $s$  be the market share of the leader of the second price increase from period  $t + 2$  onwards. We first consider the case where the same firm leads both price increases and then consider alternating price leadership.

### 3.1 Same Price Leader

Suppose the cartel decides that the same firm leads both staggered price increases. In period  $t + 2$ , the cartel has reached its target price  $p_2 = p^m$  and the incentive constraints for leader and follower are given by (1) and (2). In period  $t + 1$ , after the first price increase from  $p_0$  to  $p_1$ , the same incentive constraints as in the single price increase have to hold. That is, the cartel needs to satisfy condition (3) and (4) for the leader and follower, respectively.

Now consider the leader's incentives to start the first price increase in period  $t$ .

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<sup>12</sup>See EC Case 38.443, 21 December 2005, para 105-107, 121.

Leading two consecutive price increases means that the firm is losing sales in period  $t$  and  $t + 1$  but then it receives its share of the continuation profits in period  $t + 2$ . Alternatively, the leader could undercut the follower's price and obtain  $\pi(p_0)$ . Incentive compatibility requires

$$\frac{\delta^2}{1 - \delta} s \pi(p^m) \geq \pi(p_0). \quad (5)$$

This condition is easier to satisfy if the current industry price  $p_0$  is relatively low compared to the monopoly price level. Similarly, consider the incentives of the follower. The follower makes strictly positive profits in periods  $t$  and  $t + 1$  before obtaining its share of the continuation profits. The best deviation is to shave the price of the leader. It thus has to hold that

$$\pi(p_0) + \delta \pi(p_1) + \frac{\delta^2}{1 - \delta} (1 - s) \pi(p^m) \geq \pi(p_1). \quad (6)$$

The follower's incentives are easier to satisfy if the initial price level is high relative to the intermediate price. The intermediate price enters the period  $t$  deviation profits and the period  $t + 1$  cartel profits. Hence, a higher  $p_1$  makes the constraint harder to sustain. Put together, it is easier to satisfy the follower's incentive when a small profit increase in period  $t + 1$  is followed by a large profit increase in  $t + 2$ .

The cartel chooses the firms' market shares and the intermediate price  $p_1$  to minimize the discount factor threshold above which the cartel is sustainable subject to the constraints (1) to (6).<sup>13</sup> Before solving for the optimal cartel organization let us analyze how the constraints in period  $t$  relate to the constraints in period  $t + 1$  and  $t + 2$ . Let  $p'' < p'$  be the price such that

$$\pi(p'') = \pi(p^m)/4.$$

We then get the following result.

**Lemma 3** *If conditions (1) and (2) hold and  $p_0 \leq p''$ , then the leader's constraint (5) is satisfied. If conditions (2) and (3) hold, then the follower's constraint (6) is satisfied.*

The first point states that if the cartel's incentives from period  $t + 2$  are satisfied and the

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<sup>13</sup>If a set of prices  $p_1$  is able to sustain the minimum discount factor, then we select the highest intermediate price to maximize cartel profits.

current industry price is sufficiently low, then the leader would not deviate in period  $t$ . The lowest discounted profit for the leader that are consistent with the cartel operating after period  $t + 2$  occur at  $s = 1 - \delta$  and  $\delta = 1/2$ . Hence, if the minimum cartel profit of  $\pi(p^m)/4$  exceeds the deviation profits in period  $t$ , the leader will never undercut the follower. This implies that constraint (5) can only be binding for the cartel when the current industry price is sufficiently large such that  $p_0 > p''$ .

The second point states that the follower's constraint in period  $t$  is always satisfied if there is no profitable deviation for the leader in  $t + 1$  and the follower in  $t + 2$ . The follower's gain from deviating in period  $t + 2$ ,  $(1 - \delta)\pi(p_1)$ , is bounded from above by the leader's incentive constraint in  $t + 1$ . At the same time, incentive compatibility from  $t + 2$  requires a minimum market share of  $1 - \delta$  for the follower. Thus, overall, cartel profits exceed deviation profits in  $t$  and we can drop condition (6) from the cartel's problem.

**Proposition 2** *Consider two consecutive staggered price increases with the same price leader and  $\delta \geq 1/2$ . (i) If  $p_0 \leq p''$ , then the cartel is always able to raise the industry price to the monopoly level. Otherwise, there exists a  $\delta_S$  with  $\delta_S > 1/2$  such that if  $\delta \geq \delta_S$ , then the staggered price increases are sustainable. (ii) The maximum intermediate price satisfies  $p_S^* = \bar{p}$  and the market share of the leader is  $s = \delta$ .*

First consider the case where the initial price level in the industry is sufficiently small and the overall price increase of the cartel is large. In this case, as discussed above, the leader's incentive to undercut the follower in period  $t$  is low and the first staggered price increase from  $p_0$  to  $p_1$  does not impose any additional constraint. Moreover, following our analysis in the previous section, the second price increase can be sustained for any  $\delta \geq 1/2$  by choosing  $\pi(p_1) = \pi(p^m)/2$ . This implies that, for values of  $p_0 \leq p''$ , two staggered price increases with the same leader are able to implement a raise to the monopoly price under the same condition as a single, simultaneous price increase.

If the current industry price level is sufficiently high, the cartel needs to take the leader's period  $t$  constraint into account and allocate a higher market share to the leader. Any increase in  $s$ , however, is limited by the follower's period  $t + 1$  or  $t + 2$  constraints. Note that conditions (2) and (5) are independent of  $p_1$ . We show in the

appendix that the lowest discount factor at which collusion is sustainable is determined by (2) and (5) as long as  $p_1$  is chosen such that (3) is satisfied. The highest sustainable intermediate price is then the highest price that does not interfere with the period  $t + 1$  and  $t + 2$  constraints. From our analysis in the previous section, we know that for a given discount factor, the highest starting price to reach the monopoly level is given by  $p_1 = \bar{p}$  where  $\pi(\bar{p}) > \pi(p^m)/2$ .

### 3.2 Alternating Leadership

Now consider a situation where one firm leads the first price increase and the other firm leads the second price increase. Again, the incentive constraints (1) to (4) ensure that there is no deviation in periods  $t + 1$  and  $t + 2$ .

The first price leader is the follower in period  $t + 1$  and receives a share  $1 - s$  of the continuation profit in  $t + 2$ . In period  $t$  the first leader prefers to set  $p_1$  rather than to deviate and shave  $p_0$  if

$$\delta\pi(p_1) + \frac{\delta^2}{1 - \delta}(1 - s)\pi(p^m) \geq \pi(p_0). \quad (7)$$

This constraint is satisfied if the current industry price is sufficiently low relative to the intermediate price  $p_1$ . This is more likely to hold for a large profit increase in period  $t + 1$  followed by a small profit increase in  $t + 2$ .

Vice versa, the follower of the price increase in  $t$  is the leader in  $t + 1$ . Hence, this firm receives full industry profits at the initial price level, makes no profits as the leader in  $t + 1$  and receives a share  $s$  of the continuation profits. A deviation in period  $t$  to undercut the leader at  $p_1$  is not profitable if

$$\pi(p_0) + \frac{\delta^2}{1 - \delta}s\pi(p^m) \geq \pi(p_1). \quad (8)$$

This condition requires  $p_0$  to be sufficiently large relative to the intermediate price  $p_1$ . This is more conducive to a small profit increase followed by a larger one.

Again we first compare the two period  $t$  constraints with the constraints from periods  $t + 1$  and  $t + 2$ .

**Lemma 4** *Assume  $\delta \geq 1/2$ . If conditions (4) hold, then the first leader's constraint (7) is always satisfied. If conditions (2) and (3) hold and  $\pi(p_0) \geq \delta^2\pi(p^m)$ , then the first follower's constraint (8) is satisfied.*

The first part establishes that if the firm who leads first has no incentive to deviate in period  $t + 1$  when it is the follower, then it also has no incentive to deviate in period  $t$  as leader. In both cases, the firm gets  $\pi(p_1)$  on the equilibrium path in period  $t + 1$ . The difference is that as follower in  $t + 1$ , deviation yields monopoly profits whereas as leader in  $t$  the firm has to wait longer for its share of monopoly profits but the deviation profits are just  $\pi(p_0)$ . Hence, since  $\pi(p_0) < \delta\pi(p^m)$  for  $\delta \geq 1/2$ , condition (4) is harder to satisfy for any  $\pi(p_1)$  and  $s$ .

The follower in period  $t$  is the leader in period  $t + 1$  and has the same deviation profits on both occasions. As follower in period  $t$ , the firm receives current period profits of  $\pi(p_0)$  while as leader in period  $t + 1$  it gets an additional  $\delta s\pi(p^m)$ . To satisfy (3), the maximum market share for the firm is  $\delta$ . Hence, if and only if  $\pi(p_0) \geq \delta^2\pi(p^m)$ , that is if the initial industry price is sufficiently large, this constraint can be ignored in the cartel's problem.

Under alternating price leadership, the cartel chooses the market share  $s$  and intermediate price  $p_1$  to maximize sustainability subject to the conditions (1) to (4) and (8).

**Proposition 3** *Consider two consecutive staggered price increases with alternating price leadership and  $\delta \geq 1/2$ . (i) If  $p_0 \geq p''$ , then a cartel is always able to raise the industry price to the monopoly level. Otherwise, there exists a  $\delta_A$  with  $\delta_A > 1/2$  such that if  $\delta \geq \delta_A$ , then the staggered price increases are sustainable. (ii) The maximum intermediate price is weakly increasing in  $p_0$  and satisfies  $p_A^* \leq \bar{p}$ . (iii) The market share of the second leader at the maximum price  $p_A^*$  increases in  $p_0$  from  $\delta$  to  $1 - \delta$ .*

To implement staggered price increases with alternating leadership, the cartel needs to satisfy the period  $t + 1$  and  $t + 2$  constraints and the period  $t$  incentives of the first follower. The latter constraint is easier to satisfy if the current price is high while the cartel can strengthen the incentives by reducing the intermediate price  $p_1$  and increasing the

long run market share  $s$ . There are three different regimes for the cartel's optimal organization. From Lemma 4 follows that for  $\pi(p_0) \geq \delta^2 \pi(p^m)$ , the first follower's constraint can be dropped from the problem and the cartel can implement the price increases for any  $\delta \geq 1/2$  and a maximum price of  $p_A^* = \bar{p}$ . If the initial industry price is intermediate, the first follower's constraint restricts the cartel's problem at a maximum price of  $\bar{p}$  but the cartel can adjust its organization by reducing the intermediate price to make collusion sustainable for any  $\delta \geq 1/2$ . In particular, as the discount factor approaches  $1/2$ , the cartel chooses an intermediate price  $p_1 = p'$  to implement the price increases. This is no longer possible for lower values such that  $p_0 < p''$  as the first follower's constraint is strictly binding in the cartel's optimal organization for  $\delta$  sufficiently close to  $1/2$ . In this case the maximum sustainable intermediate price is when either (2) and the first follower's constraint or (4) and (8) hold with equality. As the market share affects the first follower's period  $t + 1$  constraint (4) more than its period  $t$  constraint, the optimal  $s$  is decreasing in  $p_0$  when both conditions are binding. Ultimately, the cartel is able to sustain collusion by adjusting its organization as long as  $(s, \pi(p_1))$  can satisfy conditions (1) and (4). At the minimum discount threshold  $\delta_A$  these two constraints and (8) hold with equality.

Let us summarize and compare the analysis of staggered price increases with the same leader and alternating leadership.

**Proposition 4** *Consider two consecutive staggered price increases and  $\delta \geq 1/2$ . (i) A cartel can always implement a move from  $p_0 \leq \underline{p}$  to the monopoly price. (ii) If the initial price is relatively low,  $p_0 \leq p''$ , the staggered price increases are easier to sustain with the same price leader. Otherwise, collusion is easier to sustain with alternating price leadership. (iii) Staggered price increases with the same leader allow for a (weakly) higher intermediate price and require a (weakly) higher market share for the leader of the second increase.*

Our analysis implies that a cartel can implement two staggered price increases under the same condition as it can implement one simultaneous price increase. This is achieved by selecting the same leader in situations where the initial price is sufficiently low and

alternating leadership, otherwise. With alternating leaders, the more restrictive period  $t$  constraint is the one that controls the incentives of the firm that follows first and leads second. In this case, a high initial market price increases the sustainability of the cartel. When using the same leader, the potentially binding constraint is the period  $t$  constraint of the leading firm. When the initial price is low, the leader's deviation profits are small and the cartel is easier to sustain.

A major difference in the optimal cartel organization is that, with the same leader, firms' incentives during the first increase does not impose additional constraints on the maximum intermediate price. By contrast, with alternating leaders, the corresponding incentives become weakly easier to satisfied when the intermediate price is lower. This means that when both types of increases are sustainable, using the same price leader yields weakly higher industry profits. Finally, since alternating leadership requires each firm to suffer the loss of sales only once, the implied market share of the second leader is lower under this cartel organization.

Let us return to the *Rubber Chemicals* cartel from the beginning of this section. In line with our result that increases are easier to sustain with a lower intermediate price, there is evidence suggesting the cartel did not want the resulting price from the first increase to be too high. During implementation of the first increase, the cartel decided to lower the increase in Europe from 7% to 5% while the second move was scheduled to be a 10% increase worldwide.<sup>14</sup>

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<sup>14</sup>See EC Case 38.443, 21 December 2005, para 118 and 125.

## 4 Cost Asymmetries and Price Leadership

In this section we explore the effect of cost asymmetries on the sustainability of staggered price increases and the optimal allocation of the leadership role within the cartel. To fix ideas, suppose the cartel members face different constant marginal cost of production such that firm 1 is the low-cost firm with  $c_1$  and firm 2 faces a cost of  $c_2 \geq c_1$ . Suppose that the cost advantage of firm 1 is non-drastring, that is,  $c_2 < p^m(c_1)$ . Furthermore, adjust notation and let  $\pi(p, c) = (p - c)D(p)$  be the industry profit at price  $p$  when producing with a marginal cost level of  $c$ .

Suppose the cartel intends to implement a staggered price increase from the competitive level  $p_1 = c_2$  to  $p_2 > c_2$  using trigger strategies with reversion to the static Nash equilibrium.<sup>15</sup> This means that, after a deviation, firm 2 receives zero continuation profit while the low-cost firm gets  $\pi(c_2, c_1) > 0$ . In what follows, we consider the sustainability of a staggered price increase with a low-cost and high-cost leader, respectively. Then, we compare and derive the optimal leadership allocation and discuss the role of asymmetry.

Consider the sustainability of the cartel at the new industry price  $p_2$  from period  $t + 1$  onwards. Suppose firm  $i \in \{1, 2\}$  is the leader and firm  $j \in \{1, 2\}$ ,  $j \neq i$ , is the follower of the staggered price increase. A firm has no incentive to deviate if its stream of future cartel profits (given its market share) exceeds the profits from shaving the collusive price and earning punishment profits thereafter. The leader's constraint is thus

$$\frac{1}{1 - \delta} s \pi(p_2, c_i) \geq \pi(p_2, c_i) + (2 - i) \frac{\delta}{1 - \delta} \pi(c_2, c_1) \quad (9-i)$$

while for the follower it has to hold that

$$\frac{1}{1 - \delta} (1 - s) \pi(p_2, c_j) \geq \pi(p_2, c_j) + (2 - j) \frac{\delta}{1 - \delta} \pi(c_2, c_1). \quad (10-i)$$

Note that without any further restrictions from the incentives in period  $t$ , the sustainability of collusion at price  $p_2$  is the same independent of which firm started the price

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<sup>15</sup>The qualitative nature of the results would not change if firms would use continuation strategies which impose a harsher punishment on the more efficient firm. Note that the analysis focuses on intermediate discount factors and optimal punishment strategies as in Abreu (1988) might not be able to implement the minmax continuation value of zero.

increase. The RHS is the total gain from deviation which is equal for both firms independent of whether they are leader or follower. In both scenarios, collusion is sustainable as long as there exist a market share such that the leader and follower constraint hold with equality. This implies that firm 1 is allocated the same market share as leader or follower. Moreover, the low-cost firm 1 has more to gain from increasing its market share by deviation and it faces a lower punishment. Hence, optimal collusion entails that the low-cost firm gets a higher market share than the high-cost firm. It is easy to check that collusion with asymmetric capacities at a price  $p_2$  is sustainable if and only if

$$\frac{\pi(c_2, c_1)}{\pi(p_2, c_1)} \leq \frac{2\delta - 1}{\delta}.$$

More cost asymmetry means the LHS increases and collusion from period  $t + 1$  is harder to sustain relative to the previous sections.

We are now in a position to analyze the two price leadership scenarios in turn. First assume the low-cost firm 1 is the price leader. In the first period of the price increase, firm 1 has an incentive to raise the price to the new collusive level  $p_2$  rather than undercutting the current price level at  $p_1 = c_2$  and triggering punishment if and only if

$$\frac{\delta}{1 - \delta} s \pi(p_2, c_1) \geq \pi(c_2, c_1) + \frac{\delta}{1 - \delta} \pi(c_2, c_1). \quad (11)$$

Firm 1 thus requires future cartel profits that exceed the long-term profits it would get in a competitive equilibrium. The higher the new collusive price, the easier it is to sustain this constraint. The high-cost firm follows by charging the competitive price in period  $t$  and not making any profits. This is sustainable if the future cartel profits exceed the gains from raising the price just below the level  $p_2$  charged by firm 1. It thus has to hold that

$$\frac{\delta}{1 - \delta} (1 - s) \pi(p_2, c_2) \geq \pi(p_2, c_2). \quad (12)$$

It is clear that if firm 2 has no incentive to deviate in period  $t$ , it will not deviate in period  $t + 1$  either. Hence, collusion is sustainable as long as there exists a market share that jointly satisfies (9-1), (11) and (12). The two constraints of the leader have the

the same future value on the equilibrium path and for deviation. The current gains are higher in (9-1) if the price increase is sufficiently high. Hence, this constraint is binding for high values of  $p_2$  while (11) is more restrictive for smaller price increases. We thus get the following result.

**Lemma 5** *A staggered price increase with the low-cost firm leading is sustainable if and only if*

$$\frac{\pi(c_2, c_1)}{\pi(p_2, c_1)} \leq \min\left\{\frac{\delta^2 + \delta - 1}{\delta^2}, 2\delta - 1\right\}.$$

*The binding constraints are (i) the follower's period  $t$ , and (ii) the leader's period  $t$  or  $t + 1$  condition.*

A staggered price increase with the low-cost firm leading is more likely to be sustainable if the price increase is sufficiently high and the cost asymmetry small.

Now consider the high-cost firm 2 as the price leader. In the first period of the price increase, firm 2 always has an incentive to raise the price to  $p_2$  rather than to undercut the current price level of  $p_1 = c_2$  since

$$\frac{\delta}{1 - \delta} s\pi(p_2, c_2) \geq 0. \quad (13)$$

By contrast, the low-cost firm follows by maintaining the current competitive price level. The best deviation is to undercut firm 2 at the collusive price level and trigger punishment. It has to hold that

$$\pi(c_2, c_1) + \frac{\delta}{1 - \delta}(1 - s)\pi(p_2, c_1) \geq \pi(p_2, c_1) + \frac{\delta}{1 - \delta}\pi(c_2, c_1). \quad (14)$$

Looking at the low-cost firm's constraint for period  $t$  and  $t + 1$ , it shows that the future profit from collusion and deviation are the same in both cases. In period  $t$ , the current period net gain from deviating is  $\pi(p_2, c_1) - \pi(c_2, c_1)$  whereas in period  $t + 1$  the gain is  $\pi(p_2, c_1) - (1 - s)\pi(p_2, c_1)$ . For the cartel to work, the future profit from collusion must exceed the punishment profit for firm 1, that is,  $(1 - s)\pi(p_2, c_1) > \pi(c_2, c_1)$ . This implies that firm 1 stands to gain more deviating in period  $t$ . The staggered price increase is thus sustainable if there exist market shares that jointly satisfy (9-2) and (14).

**Lemma 6** *A staggered price increase with the high-cost firm leading is sustainable if and only if*

$$\frac{\pi(c_2, c_1)}{\pi(p_2, c_1)} \leq \frac{\delta^2 + \delta - 1}{2\delta - 1}.$$

*The binding constraints are the low-cost firm's period  $t$  and the high-cost firm's period  $t + 1$  condition.*

Upon inspection, the condition in Lemma 6 is easier to sustain than the one in Lemma 5. As a consequence, a staggered price increase is easier to implement when the leader is the high-cost firm. As a leader, a low-cost firm stands to gain more from increasing its demand by undercutting the follower. Vice versa, as a follower, the low-cost is less tempted to increase price and reduce its demand level relative to its high-cost competitor. The difference in punishment profits between the two firms does not affect the choice of the price leader as the market share has to be adjusted for the stronger incentive of the low-cost firm to deviate independent of its role in the price increase. We can thus get the following result.

**Proposition 5** *A staggered price increase is (weakly) easier to sustain if the firm with the higher marginal cost is the leader.*

For confidentiality reasons, market share information - which could be used to proxy cost asymmetric cost positions - is not always available in published cartel case decisions. However, in the *EMCG* cartel mentioned in the introduction, there was at least one staggered price increase that was led by a smaller competitor. On December 14, 1999, the cartel decided that Schunk would lead a price increase on March 10, 2000 and Morgan would follow one month later. According to the market share ranges in the case documentation, Morgan was a significantly larger competitor.<sup>16</sup>

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<sup>16</sup>See EC Case 38.359 - Electrical and mechanical carbon and graphite products, 3 December 2003, para 37, 107.

## 5 Extensions

### 5.1 Price Staggering with Multiple Firms

In this section we analyze the optimal organization of a staggered price increase from  $p_1$  to  $p_2$  with  $n \geq 2$  firms. Let  $k \in \{1, 2, \dots, n\}$  be the number of firms who lead the price increase and charge price  $p_2$  in the first period  $t$  while the remaining  $n - k$  firms charge price  $p_1 < p_2$ . From period  $t + 1$  onwards, all firms charge the new cartel price  $p_2$ . Suppose that the firms are ordered such that firms  $i \in \{1, \dots, k\}$  are leaders while firms  $i \in \{k + 1, \dots, n\}$  are followers. Let  $s_i$  be the market share of firm  $i$  such that  $\sum_{i=1}^n s_i = 1$ .

In period  $t + 1$ , the cartel is sustainable if, for each firm  $i$ , it holds that

$$\frac{1}{1 - \delta} s_i \pi(p_2) \geq \pi(p_2) \quad \text{or} \quad s_i \geq 1 - \delta \quad (15)$$

In the first period of the staggered price increase, there are two types of constraints. First, consider a price leader firm  $i \in \{1, \dots, k\}$ . This firm has no incentive to lower its price and shave  $p_1$  if

$$\frac{\delta}{1 - \delta} s_i \pi(p_2) \geq \pi(p_1). \quad (16)$$

Then consider a price follower  $i \in \{k + 1, \dots, n\}$  who has been assigned a market share  $\sigma_i$  in period  $t$ . where  $\sum_{i=k+1}^n \sigma_i = 1$ . The deviation profits of a follower depend on the number of co-followers. If there is only one follower, the optimal deviation is to a price that shaves  $p_2$ . With more followers, deviation requires undercutting co-followers at  $p_1$ . The incentive constraint for the follower is then

$$\sigma_i \pi(p_1) + \frac{\delta}{1 - \delta} s_i \pi(p_2) \geq \begin{cases} \pi(p_2) & \text{if } k = n - 1, \\ \pi(p_1) & \text{otherwise.} \end{cases} \quad (17)$$

We will restrict attention to symmetric market shares for followers in period  $t$ , that is,  $\sigma_i = 1/(n - k)$ . This assumption ensures that if the leaders and/or the followers have a binding constraint in an optimal market share allocation, then all firms in the same group must have the same market share. This means all leader firms  $i \in \{1..k\}$  each obtain a market share of  $s_i = s/k$  while followers  $i \in \{k + 1, \dots, n\}$  get  $s_i = (1 - s)/(n - k)$

where  $s$  is the total market share of the leaders from period  $t + 1$ . In an optimal cartel organization, firms thus adjust the overall market share of the leaders in order to maximize sustainability of the cartel. The analysis is similar to the benchmark model. We thus focus on the comparative statics with respect to the number of leaders and firms.

**Proposition 6** *Consider a staggered price increase with  $k \in [1, n - 1]$  leaders. There exists a  $\delta'_n$  with  $(n - 1)/n \leq \delta'_n < 1$  such that if  $\delta \geq \delta'_n$ , then a staggered price increase from  $p_1$  to  $p_2$  is sustainable. The threshold value  $\delta'_n$  (weakly) increases in  $n$  and  $k$ .*

Increasing the number of cartel firms makes collusion harder to sustain as the potential market share gain from undercutting is higher. This standard result carries over to staggered price increase as it applies to both types of period  $t$  and  $t + 1$  incentive constraints. The second comparative statics result with respect to the number of leaders is more subtle. First, if there is more than one follower, the followers' period  $t$  constraint is always implied by the followers  $t + 1$  constraint. In period  $t + 1$ , a deviating follower can increase his market share from  $(1 - s)/(n - k)$  to 1, which is larger than the increase from  $1/(n - k)$  to 1. Moreover, the total market value is higher in period  $t + 1$  as all firms charge  $p_2$ .

As a consequence, there are two possible regimes for the optimal organization of the cartel. Either, the period  $t + 1$  constraints are binding which allows firms to implement a price increase under the same condition as a simultaneous price increase, that is, for  $\delta \geq (n - 1)/n$ . Or, the binding constraints are the period  $t$  constraint of the leaders and the period  $t + 1$  constraint of the followers. Raising the number of leaders  $k$  increases the market share of an individual follower and makes condition (15) easier to sustain. At the same time, the market share of an individual leader decreases which makes condition (16) harder to sustain. However, the absolute effect on the discount factor is stronger for the leaders as the stream of discounted cartel profits starts one period later. This implies that the more price leaders there are, the harder it is to sustain collusion. Or, a staggered price increase is easiest to sustain with exactly one leader.

## 5.2 Intertemporal Demand and Strategic Buyers

So far we have assumed that demand is static and not responding to the cartel's staggered pricing. In this subsection we consider strategic buyers who anticipate the cartel's price increase and are able to bring forward their purchase. We investigate how such intertemporal demand patterns affects the sustainability of price staggering.

To fix ideas, we introduce buyers who can purchase the product one period ahead of consumption. In particular, we consider an overlapping generations model where in each period a cohort of buyers of size one enters the market. A proportion  $\rho$  of these consumers have to buy and consume the product in the same period. A proportion  $1 - \rho$  can either buy immediately and keep the product or wait and buy in the next period. These patient buyers consume the product at the end of the second period and then leave the market. We assume that patient buyers purchase in the first period of their life if and only if it yields a strictly larger pay-off. When expected prices are the same across the two periods, they buy in the period of consumption. Suppose that buyers are differentiated with respect to their willingness to pay  $\theta$  which is distributed according to a log-concave cumulative density  $F(\theta)$ . Demand at a price  $p$  from a given set of patient or impatient buyers of measure 1 is then simply  $D(p) = 1 - F(p)$ .

We consider a staggered price increase from  $p_1 > c$  to  $p_2 \in (p_1, p^m]$ . The leader increases its price in period  $t$ , the other firm follows in  $t + 1$ , and both firms charge  $p_2$  from that period onwards. The initial price increase is unexpected which means that the proportion of patient buyers from period  $t - 1$  are present in period  $t$ . Those consumers plus the impatient buyers entering in period  $t$  buy in this same period. The proportion  $1 - \rho$  patient buyers entering in  $t$ , however, buy in period  $t + 1$  if and only if there is a weakly lower price next period.

First consider the constraints in period  $t + 2$  on the equilibrium path. The industry prices in period  $t + 1$  and  $t + 2$  are the same which implies that in  $t + 2$ , the patient buyer from  $t + 1$  plus the impatient buyers from  $t + 2$  buy. Total demand in this and in any future period is therefore  $D(p_2)$ . This means the binding constraints for the leader and follower are given by (1) and (2).

In period  $t + 1$ , there is a proportion of  $\rho$  impatient buyers willing to buy on the

equilibrium path. The patient buyers of period  $t$  have bought in the previous period while the current patient buyers wait for period  $t + 2$ . This means that the leader's period  $t + 1$  incentive constraint is given by

$$s\rho\pi(p_2) + s\frac{\delta}{1-\delta}\pi(p_2) \geq \rho\pi(p_2).$$

Note that the deviation profits are lower compared to our benchmark model. If the leader undercuts the follower, consumers anticipate lower, competitive prices in period  $t + 2$  and patient buyers delay their purchase. Similarly, the follower's period  $t + 1$  constraint is given by

$$(1-s)\rho\pi(p_2) + (1-s)\frac{\delta}{1-\delta}\pi(p_2) \geq \rho\pi(p_2).$$

In both constraints, the current period gains from deviation are less than in period  $t + 2$  while the future gains from cooperation are the same. Hence, if conditions (1) and (2) are satisfied, then the period  $t + 1$  constraints always hold.

Now consider incentives in period  $t$ . The leader has no incentive to deviate if

$$\delta s\rho\pi(p_2) + s\frac{\delta^2}{1-\delta}\pi(p_2) \geq \pi(p_1). \quad (18)$$

The leader's deviation profits are the same as in the benchmark model. Shaving the price of the follower means that prices are competitive from period  $t + 1$ . This implies that only patient consumers from  $t - 1$  and impatient consumers from  $t$  buy after a deviation. At the same time, future cartel profit are less than in the benchmark model for any  $\rho < 1$  since patient buyers anticipate the price increase and purchase in the current period. Hence, the leader's incentives are harder to sustain.

The follower's incentive constraint in period  $t$  is given by

$$\delta(1-s)\rho\pi(p_2) + (1-s)\frac{\delta^2}{1-\delta}\pi(p_2) \geq \pi(p_2) - (2-\rho)\pi(p_1). \quad (19)$$

Again, the future cartel profits on the LHS are less relative to the benchmark due to patient buyers pre-empting the price increase. However, the follower also benefits as his

current period demand increases by  $(1 - \rho)D(p_1)$ . It thus depends on the relative size of these two effects as to whether strategic buyers weaken or strengthen the follower's incentives. We consider the overall effect in the next proposition.

**Proposition 7** *The presence of strategic buyers and intertemporal demand makes staggered price increases easier to sustain if and only if*

$$\delta^2 \leq \frac{\pi(p_1)}{\pi(p_2)} \leq \frac{1 + \rho}{4}.$$

*Otherwise, they are strictly harder to sustain relative to the benchmark model.*

The effect of strategic buyers on the sustainability of staggered price increases is ambiguous. On the one hand, strategic buyers anticipate higher prices and purchase the products right at the beginning of the cartel's moves. This reduces future demand and profits of the cartel and weakens incentives for both the leader and follower. However, at the same time, the follower benefits from the increased demand from patient buyers. This allows the cartel to relax the follower's constraint when it is binding in the optimal cartel scheme. This occurs when the initial price is sufficiently low relative to the target price. If the initial price is too low though, that is, the step size is large, then the value of the follower's current demand gains is outweighed by the loss in future cartel profits from impatient buyers in  $t + 1$ . As a consequence, strategic buyers increase the potential for collusion if the step size is intermediate and the fraction of those strategic buyers is relatively small.

## 6 Conclusions

The literature on cartels is usually concerned with the sustainability of collusion at some long run target conditions such as the industry monopoly price. There is very little research into the initiation and gradual formation of cartels despite the fact that antitrust case documents suggest that firms face various obstacles on their way to a long term stable collusive scheme. One important hurdle is potential detection by buyers and competition authorities due to collusive patterns in firms' market behaviour. This paper

considers staggered price increases, which are a commonly observed strategy by cartels to avoid the suspicion of price collusion. We discuss the cartel's optimal organization of such price increases and their impact on cartel formation.

Our analysis allows us to explain pricing patterns of discovered cartels that have not been addressed before. For example, we show that a cartel scheduling two consecutive price increases might find it optimal to alternate price leadership. Moreover, when firms face asymmetric cost structures, a staggered price increase might be easier to sustain when the high-cost firm is leading. The results in this paper also contribute towards distinguishing between competitive barometric price leadership and collusive behaviour. A few collusive markers follow straight from our analysis. A single staggered price increase is more likely to be sustainable by a cartel if the step size is neither too small nor too large. When firms implement two consecutive price increases, a cartel uses the same leader twice (alternates leadership) if the initial industry price is low (high). Alternating price leadership typically implies a small price increase followed by a larger price increase. If the smaller, less efficient firm leads a price increase, then it is likely to be collusive. Finally, a cartel finds it easier to sustain a staggered price increase when the number of leaders is small.

## Appendix

### Proof of Lemmas 1 and 2

As (1) and (2) are never jointly satisfied with  $\delta < 1/2$ , we only consider  $\delta \geq 1/2$ . Condition (3) is satisfied if

$$s \geq \frac{1 - \delta \pi(p_1)}{\delta \pi(p_2)} \equiv s_L.$$

The RHS is decreasing and convex in  $\delta$ . It takes value  $\pi(p_1)/\pi(p_2)$  at  $\delta = 1/2$  and is equal to 0 at  $\delta = 1$ . Hence, if  $\pi(p_1)/\pi(p_2) \leq 1/2$ , then condition (3) is implied by condition (1). Otherwise, if  $\pi(p_1)/\pi(p_2) > 1/2$ , then conditions (2) and (3) can be jointly satisfied if and only if  $\delta \geq \delta'_L$  where

$$\delta'_L = \{\delta | \delta^2 \pi(p_2) = (1 - \delta) \pi(p_1)\}.$$

At  $\delta'_L$  the RHS of condition (3) is greater than  $1/2$ . It follows that (1) is satisfied for all  $\delta \geq 1/2$ . Condition (4) is satisfied if

$$s \leq 1 - \frac{1 - \delta}{\delta} \frac{\pi(p_2) - \pi(p_1)}{\pi(p_2)} \equiv s_F.$$

The RHS is increasing and concave in  $\delta$ . It takes value  $\pi(p_1)/\pi(p_2)$  at  $\delta = 1/2$  and is equal to 1 at  $\delta = 1$ . Thus, if  $\pi(p_1)/\pi(p_2) > 1/2$ , then condition (4) is implied by condition (2). Otherwise, if  $\pi(p_1)/\pi(p_2) \leq 1/2$ , then conditions (1) and (4) are jointly satisfied if and only if  $\delta \geq \delta'_F$  where

$$\delta'_F = \{\delta | \delta^2 \pi(p_2) = (1 - \delta)(\pi(p_2) - \pi(p_1))\}.$$

At  $\delta'_F$  the RHS of condition (4) is less than  $1/2$ . This implies that (2) is satisfied.

From the above follows that if  $\pi(p_1) > \pi(p_2)/2$ , then the minimum discount factor is given by  $\delta'_L$  and the binding constraints are (2) and (3). Otherwise, the discount factor threshold is  $\delta'_F$  and conditions (1) and (4) are binding. Lemma 1 follows. Furthermore, using total differentiation, we get

$$\frac{d\delta'_L}{d\pi(p_1)} = \frac{1 - \delta}{2\delta\pi(p_2) + \pi(p_1)} > 0$$

and

$$\frac{d\delta'_F}{d\pi(p_1)} = -\frac{1 - \delta}{2\delta\pi(p_2) + \pi(p_1)} < 0.$$

Note that  $\delta'_L = \delta'_F = 1/2$  for  $\pi(p_1) = \pi(p_2)/2$ . This means it holds that  $\delta'_L > 1/2$  if  $\pi(p_1) > \pi(p_2)/2$  and  $\delta'_F > 1/2$  if  $\pi(p_1) < \pi(p_2)/2$ . Lemma 2 follows. QED.

## Proof of Corollary 1

The highest price is obtained when conditions (2) and (3) can be jointly satisfied. This implies that

$$\bar{p} = \{p_1 | \delta^2 \pi(p^m) = (1 - \delta)\pi(p_1)\}.$$

From our analysis in the proof of Lemma 2 follows that  $\bar{p}$  is increasing in  $\delta$  and  $\pi(\bar{p}) = \pi(p^m)/2$  at  $\delta = 1/2$ . The lowest price is achieved when conditions (1) and (4) are jointly

satisfied, that is,

$$\underline{p} = \{p_1 | \delta^2 \pi(p^m) = (1 - \delta)(\pi(p^m) - \pi(p_1))\}.$$

Note that  $\underline{p}$  is decreasing in  $\delta$  and  $\pi(\underline{p}) = \pi(p^m)/2$  at  $\delta = 1/2$ . The corollary follows. QED.

## Proof of Lemma 3 and Proposition 2

The leader's constraint in period  $t$  is given by

$$s \geq \frac{1 - \delta}{\delta^2} \frac{\pi(p_0)}{\pi(p^m)} \equiv s_L^S.$$

This condition is satisfied for any  $s \geq 1 - \delta$  if and only if  $\delta^2 \geq \pi(p_0)/\pi(p^m)$ . If  $\pi(p_0) \leq \pi(p^m)/4$ , then this holds for any  $\delta \geq 1/2$ . Since conditions (1) and (2) can only be jointly satisfied if  $\delta \geq 1/2$ , the first part of the lemma follows.

To show the second part of the lemma, note that (6) is hardest to satisfy for  $\pi(p_0) = 0$  which yields

$$\frac{\delta^2}{1 - \delta} (1 - s) \pi(p^m) \geq (1 - \delta) \pi(p_1).$$

Condition (3) puts an upper bound on the value of  $\pi(p_1)$ . Plugging this value into the above condition gives

$$\frac{\delta}{1 - \delta} \geq \frac{s}{1 - s}$$

which holds for any value that satisfies (2).

To prove the proposition, consider  $p_0 > p''$ . There are two cases to consider. If  $p_1 \leq p'$ , then the binding constraints for the cartel is (5) and either (2) or (4). In the former case, the cartel can be sustained if and only if  $s_L^S \leq \delta$  or  $\delta \geq \delta_{S1}$  with

$$\delta_{S1} = \{\delta | \delta^3 \pi(p^m) = (1 - \delta) \pi(p_0)\}.$$

In the latter case, the minimum discount threshold is given by

$$\delta_{S2} = \{\delta | \delta(1 - \delta) \pi(p_1) + \delta(2\delta - 1) \pi(p^m) = (1 - \delta) \pi(p_0)\}.$$

It holds that the binding discount factor is  $\max\{\delta_{S1}, \delta_{S2}\}$ . By total differentiation

$$\frac{d\delta_{S2}}{d\pi(p_1)} = -\frac{(1-\delta)\delta^2}{(2-\delta)\pi(p_0) + \delta(\pi(p^m) - \pi(p_1))} < 0.$$

At  $\pi(p_1) = \pi(p^m)/2$  we have

$$\delta_{S2} = \left\{ \delta \left| \frac{1}{2} \delta (3\delta - 1) \pi(p^m) = (1 - \delta) \pi(p_0) \right. \right\}.$$

The LHS of this definition is larger than the LHS in the definition of  $\delta_{S1}$  whereas the RHS are identical. Since  $\delta_{S1}$  is independent of  $\pi(p_1)$ , it follows that the lowest collusive discount factor is  $\delta_{S1}$ .

If  $p_1 > p'$ , then the binding constraints for the cartel is (2) and either (3) or (5). The resulting minimum threshold is thus given by  $\max\{\delta_{S1}, \delta'_L\}$ . Note that  $\delta'_L$  takes value  $1/2$  at  $\pi(p_1) = \pi(p^m)/2$  and increases in  $\pi(p_1)$  whereas  $\delta_{S1}$  is independent of  $\pi(p_1)$  and strictly larger than  $1/2$  for  $\pi(p_0) > \pi(p^m)/4$ . It follows that the overall lowest collusive discount factor is given by  $\delta_{S1}$ . Increasing  $\pi(p_1)$  raises cartel profits without affecting sustainability up to the point where condition (3) holds with equality. The proposition follows. QED.

## Proof of Lemma 4

For the first part of the lemma, check that (7) holds if and only if

$$s \leq 1 - \frac{1 - \delta}{\delta^2} \frac{\pi(p_0) - \pi(p_1)}{\pi(p^m)}.$$

The RHS is larger than  $s_F$  from condition (4) if

$$\frac{1 - \delta}{\delta^2} \frac{\pi(p^m) - \pi(p_0)}{\pi(p^m)} > 0$$

which holds for any  $\pi(p_0) \leq \pi(p^m)/2$  and  $\delta \geq 1/2$ . For the second part, note that condition (8) holds if

$$s \geq \frac{1 - \delta}{\delta^2} \frac{\pi(p_1) - \pi(p_0)}{\pi(p^m)} \equiv s_F^A.$$

Note that

$$\frac{ds_F^A}{d\pi(p_1)} = \frac{1-\delta}{\delta^2\pi(p^m)} > \frac{1-\delta}{\delta\pi(p^m)} = \frac{ds_L}{d\pi(p_1)} > 0.$$

Moreover, there exist values  $(s, \pi(p_1))$  such that (2) and (3) jointly hold if  $s_L \leq \delta$  or

$$\pi(p_1) \leq \frac{\delta^2}{1-\delta}\pi(p^m).$$

There exist values  $(s, \pi(p_1))$  such that (2) and (8) jointly hold if  $s_F^A \leq \delta$  or

$$\pi(p_1) \leq \pi(p_0) - \frac{\delta^3}{1-\delta}\pi(p^m).$$

Hence, condition (8) is implied by (2) and (3) if

$$\pi(p_0) - \frac{\delta^3}{1-\delta}\pi(p^m) \geq \frac{\delta^2}{1-\delta}\pi(p^m)$$

or  $\pi(p_0) \geq \delta^2\pi(p^m)$ . The second part of the lemma follows. QED.

### Proof of Proposition 3

Point (i): note that the cartel is not sustainable if and only if there exists no pair  $(\pi(p_1), s)$  such that (1), (4) and (8) hold jointly. There always exist values  $s$  such that conditions (1) and (4) are jointly satisfied if

$$\pi(p_1) \geq \pi(\underline{p}) = \frac{1-\delta-\delta^2}{1-\delta}\pi(p^m).$$

Since

$$\frac{ds_F^A}{d\pi(p_1)} = \frac{1-\delta}{\delta^2\pi(p^m)} > \frac{1-\delta}{\delta\pi(p^m)} = \frac{ds_F}{d\pi(p_1)} > 0$$

there exist values  $s$  such that the conditions (4) and (8) are jointly satisfied if

$$\pi(p_1) \leq \frac{\pi(p_0)}{1-\delta} + \frac{2\delta-1}{(1-\delta)^2}\delta\pi(p^m).$$

Both conditions cannot hold simultaneously if

$$\frac{\pi(p_0)}{1-\delta} + \frac{2\delta-1}{(1-\delta)^2} \delta \pi(p^m) < \frac{1-\delta-\delta^2}{1-\delta} \pi(p^m)$$

or

$$\frac{\pi(p_0)}{\pi(p^m)} \leq 1 - \frac{\delta^2(2-\delta)}{1-\delta}. \quad (\text{app-1})$$

Let  $\delta_A$  be the discount factor that satisfies this constraint with equality. Totally differentiating yields

$$\frac{d\delta_A}{d\pi(p_0)} = -\frac{1}{2\delta-1 + \frac{1}{(1-\delta)^2}} < 0$$

for all  $\delta \geq 1/2$ . Check that at  $\pi(p_0) = \pi(p^m)/4$  we get  $\delta_A = 1/2$ . Hence, if  $\pi(p_0) \geq \pi(p^m)/4$ , the cartel's price increases can be sustained for any  $\delta \geq 1/2$ . Otherwise, the price increases are sustainable if  $\delta \geq \delta_A > 1/2$ .

Points (ii) and (iii): The maximum sustainable price  $p_1$  is bounded from above by (3) and (8) while the maximum market share  $s$  is limited by (2) and (4), that is,  $s \leq \min\{s_F, \delta\}$ . Since the slope of  $s_F^A$  is steeper than the slope of  $s_F$ , three cases are possible. The first one is where (2) and (3) are jointly satisfied. In this case the maximum sustainable price has to satisfy

$$\pi(p_1) = \frac{\delta^2}{1-\delta} \pi(p^m) \equiv \pi_1 = \pi(\bar{p}).$$

The second case is where (2) and (8) hold and the maximum sustainable price satisfies

$$\pi(p_1) = \pi(p_0) + \frac{\delta^3}{1-\delta} \pi(p^m) \equiv \pi_2.$$

Finally, when (4) and (8) hold strictly,  $s_F^A = s_F$ , and the maximum price satisfies

$$\pi(p_1) = \frac{\pi(p_0)}{1-\delta} + \frac{\delta(2\delta-1)}{(1-\delta)^2} \pi(p^m) \equiv \pi_3.$$

The maximum sustainable price is  $p_A^*$  such that  $\pi(p_A^*) = \min\{\pi_1, \pi_2, \pi_3\}$ . It is easy to

check that  $\pi_3 \leq \pi_2$  if and only if

$$\frac{\pi(p_0)}{\pi(p^m)} \leq \delta^2 - \frac{2\delta - 1}{1 - \delta} = 1 - \frac{\delta^2(2 - \delta)}{1 - \delta} + \delta(2\delta - 1) \quad (\text{app-2})$$

and  $\pi_1 \leq \pi_2$  if and only if

$$\frac{\pi(p_0)}{\pi(p^m)} \geq \delta^2. \quad (\text{app-3})$$

Since the RHS of (app-2) is smaller than the RHS of (app-3), it follows immediately that if (app-2) holds, then  $\pi(p_A^*) = \pi_3 \leq \pi_2 < \pi_1$  and conditions (4) and (8) are satisfied with equality. Moreover, since the RHS of (app-2) is larger than the RHS of (app-1), there always exist values of  $p_0$  such that the cartel is sustainable and  $\pi(p_A^*) = \pi_3$ . It is easy to check that the maximum price is increasing in  $p_0$ . At the lowest level of  $p_0$  such that the cartel is still sustainable, that is, when (app-1) holds with equality, the maximum price is defined by

$$\pi(p_A^*) = \pi_3 = \frac{1 - \delta - \delta^2}{1 - \delta} \pi(p^m) < \frac{\pi(p^m)}{2}$$

for all  $\delta > 1/2$ . The corresponding market share at  $\pi(p_1) = \pi_3$  is

$$s_F^A = \frac{(1 - \delta)\pi(p_0) + (2\delta - 1)\pi(p^m)}{(1 - \delta)\delta\pi(p^m)}.$$

This value is increasing in  $\pi(p_0)$  and it takes value  $s_F^A = 1 - \delta$  when (app-1) holds with equality and value  $s_F^A = \delta$  when (app-2) holds with equality.

For intermediate values of  $\pi(p_0)$  such that both (app-2) and (app-3) are not satisfied, we have  $\pi(p_A^*) = \pi_2$  and conditions (2) and (8) hold with equality. The maximum price is again increasing in  $p_0$  and the market share is  $s = \delta$ . Finally, if (app-3) holds, we get  $\pi(p_A^*) = \pi_1 = \pi(\bar{p})$  and (2) and (3) are jointly satisfied. This maximum price is independent of  $p_0$  and the market share of the second leader is  $s = \delta$ . Points (ii) and (iii) of the proposition follow. QED.

## Proof of Lemma 5 and 6

Conditions (9-1) and (11) hold if and only if

$$s \geq \max\left\{1 - \delta + \delta \frac{\pi(c_2, c_1)}{\pi(p_2, c_1)}, \frac{\pi(c_2, c_1)}{\delta \pi(p_2, c_1)}\right\}$$

where  $\lambda = \pi(c_2, c_1)/\pi(p_2, c_1)$  while condition (12) requires

$$s \leq \frac{2\delta - 1}{\delta}.$$

From this we get that (11) and (12) can be jointly satisfied if and only if

$$\frac{\pi(c_2, c_1)}{\pi(p_2, c_1)} \leq 2\delta - 1$$

while (9-1) and (12) can be jointly satisfied if and only if

$$\frac{\pi(c_2, c_1)}{\pi(p_2, c_1)} \leq \frac{\delta + \delta^2 - 1}{\delta^2}.$$

The result in Lemma 5 follows. For the second lemma in Section 4 note that condition (14) is equivalent to

$$s \leq \frac{2\delta - 1}{\delta} \left(1 - \frac{\pi(c_2, c_1)}{\pi(p_2, c_1)}\right).$$

This condition can be jointly satisfied with (9-2), which simplifies to  $s \geq 1 - \delta$  if and only if the condition given in the lemma holds. QED.

## Proof of Proposition 6

Note that the period  $t + 1$  constraint for leaders is given by

$$s \geq (1 - \delta)k,$$

and for followers it is

$$s \leq 1 - (n - k)(1 - \delta).$$

These condition can only be satisfied jointly if  $\delta \geq (n - 1)/n$ . The RHS of the leader's constraint decreases in  $\delta$  whereas the RHS of the follower increases. At  $\delta = (n - 1)/n$ , both RHS are equal to  $k/n$ .

Condition (16) holds if

$$s \geq \frac{1 - \delta}{\delta} k \frac{\pi(p_1)}{\pi(p_2)}.$$

The RHS is decreasing and convex in  $\delta$ . At  $\delta = (n - 1)/n$ , it takes value

$$\frac{k}{n - 1} \frac{\pi(p_1)}{\pi(p_2)} \geq \frac{k}{n} \quad \text{or} \quad \frac{\pi(p_1)}{\pi(p_2)} \geq \frac{n - 1}{n}. \quad (\text{app-5})$$

If this condition does not hold, then (16) is always satisfied if the leader's period  $t + 1$  holds. Otherwise, the condition is potentially binding in the optimal cartel arrangement. For  $k = n - 1$ , condition (17) holds if

$$s \leq 1 - \frac{1 - \delta}{\delta} \frac{\pi(p_2) - \pi(p_1)}{\pi(p_2)}.$$

At  $\delta = (n - 1)/n$  the RHS is smaller than  $k/n = (n - 1)/n$  if

$$\frac{\pi(p_1)}{\pi(p_2)} \leq \frac{1}{n}. \quad (\text{app-6})$$

If this condition does not hold, then (17) is always satisfied if the follower's period  $t + 1$  holds. For  $k < n - 1$ , condition (17) holds if

$$s \leq 1 - \frac{1 - \delta}{\delta} \frac{(n - k - 1)\pi(p_1)}{\pi(p_2)}.$$

The RHS is increasing and concave in  $\delta$ . At  $\delta = (n - 1)/n$  it takes value

$$1 - \frac{n - k - 1}{n - 1} \frac{\pi(p_1)}{\pi(p_2)} > \frac{k}{n}$$

which always holds. Hence, (17) is satisfied if the follower's period  $t + 1$  holds and  $\delta \geq 1/2$ .

We thus have three distinct parameter constellations. First, suppose (app-5) holds. In this case the binding constraints are (16) and the follower's period  $t + 1$  constraint.

The lowest discount factor to support collusion is then defined by

$$\bar{\delta} = \left\{ \delta \left| \frac{\delta}{1-\delta} \frac{1-(n-k)(1-\delta)}{k} \pi(p_2) - \pi(p_1) = 0 \right. \right\}$$

which yields, for  $\delta \geq (n-1)/n$ ,

$$\frac{d\bar{\delta}}{dn} = \frac{\delta(1-\delta)^2}{1-(n-k)(1-\delta)^2} \geq 0, \quad \frac{d\bar{\delta}}{dk} = \frac{\delta(1-\delta)}{k} \frac{1-n+\delta n}{1-(n-k)(1-\delta)^2} \geq 0.$$

Next suppose (app-6) and  $k = n - 1$  hold. The binding constraints are now (17) and the leader's period  $t + 1$  constraint. The lowest discount factor to support collusion is defined by

$$\bar{\delta} = \left\{ \delta \left| \left[ \frac{\delta}{1-\delta} - \delta(n-1) \right] \pi(p_2) - \pi(p_2) + \pi(p_1) = 0 \right. \right\}$$

which yields

$$\frac{d\bar{\delta}}{dn} = \frac{\delta(1-\delta)^2}{2-n+(2-\delta)\delta(n-1)} \geq 0$$

since the denominator increases in  $\delta$  and takes value of  $(n-1)/n + 1/n^2 > 0$  at  $\delta = (n-1)/n$ . Finally, for all other parameter values, the period  $t$  constraints are satisfied when the period  $t+1$  constraints hold. This implies that the price increase is sustainable if  $\delta \geq (n-1)/n$ . The proposition follows. QED.

## Proof of Proposition 7

Condition (18) holds if

$$s \geq \frac{1-\delta}{\delta(\delta+\rho(1-\delta))} \frac{\pi(p_1)}{\pi(p_2)}.$$

The RHS is convex and decreasing in  $\delta$  which implies that there is at most one intersection with  $s = \delta$  and  $s = 1 - \delta$  for  $\delta \geq 1/2$ . In particular, if the RHS at  $\delta = 1/2$  satisfies

$$\frac{2\pi(p_1)}{(1+\rho)\pi(p_2)} \geq 1/2 \quad \text{or} \quad \frac{\pi(p_1)}{\pi(p_2)} \geq \frac{1+\rho}{4}, \quad (\text{app-7})$$

then condition (18) is more restrictive than (1). Similarly, condition (19) holds if

$$s \leq 1 - \frac{1 - \delta}{\delta} \frac{\pi(p_2) - (2 - \rho)\pi(p_1)}{(\delta + \rho(1 - \delta))\pi(p_2)}.$$

As RHS is concave and increasing in  $\delta$ , we know that (19) is more restrictive than (2) if and only if the RHS at  $\delta = 1/2$  is less than  $1/2$  or

$$\frac{\pi(p_1)}{\pi(p_2)} \leq \frac{3 - \rho}{4(2 - \rho)}. \quad (\text{app-8})$$

This yields three cases for the parameter values. First, suppose (app-8) is not satisfied. This implies that (app-7) holds and the binding constraints in the optimal cartel arrangement are (18) and (2). The lowest discount factor to support collusion is then defined by

$$\bar{\delta} = \{\delta | \delta^2 \rho \pi(p_2) + \frac{\delta^3}{1 - \delta} \pi(p_2) - \pi(p_1) = 0\}.$$

Total differentiation then yields

$$\frac{d\bar{\delta}}{d\rho} = - \frac{\delta^2 \pi(p_2)}{\delta \pi(p_2) [(3 - 2\delta)\delta + 2\rho(1 - \delta)^2] / (1 - \delta)^2} < 0.$$

Second suppose both conditions (app-7) and (app-8) hold. In this case, the binding constraints are (5) and (6) and the lowest discount factor is given by

$$\bar{\delta} = \{\delta | \delta^2 \rho \pi(p_2) + \frac{\delta^3}{1 - \delta} \pi(p_2) - \pi(p_1) + (1 - \rho)\pi(p_1) = 0\}.$$

and we get

$$\frac{d\bar{\delta}}{d\rho} = - \frac{\delta \pi(p_2) - \pi(p_1)}{\pi(p_2) [1/(1 - \delta)^2 - 1 + \rho]} < 0$$

for all parameter values that satisfy (app-8). Finally, consider the case where (app-7) is not satisfied which implies that (app-8) holds. The binding constraints are (19) and (1). The lowest discount factor to support collusion is then defined by

$$\bar{\delta} = \{\delta | (2 - \rho)\pi(p_1) + \delta^2 \rho \pi(p_2) + \frac{\delta^3}{1 - \delta} \pi(p_2) - \pi(p_2) = 0\}$$

which yields

$$\frac{d\bar{\delta}}{d\rho} = -\frac{\delta^2\pi(p_2) - \pi(p_1)}{\delta\pi(p_2)[(3 - 2\delta)\delta + 2\rho(1 - \delta)^2]/(1 - \delta)^2} \geq 0$$

if and only if  $\pi(p_1)/\pi(p_2) \geq \delta^2$ . The proposition follows. QED.

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