

Fiscal and Monetary Policy Interaction under Limited Foresight*

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Abstract

Analyses of the interaction between monetary and fiscal policy often turn crucially on assumptions that are made about outcomes far in the future, sometimes infinitely far. This is a problematic feature of rational-expectations analyses, given the limited basis for assumptions about the distant future. This paper instead considers both short-term effects and long-run consequences of alternative monetary and fiscal policies under an assumption of bounded rationality. In particular, it assumes that explicit forward planning extends only a finite distance into the future, with anticipated situations at that horizon evaluated using a value function learned from past experience. Such an approach makes announcements of future policies relevant, but avoids the debates about equilibrium selection that plague rational-expectations analyses. The combined monetary-fiscal regimes that result in stable long-run dynamics are characterized, and the effectiveness of temporary changes in either type of policy as a source of short-run demand stimulus is analyzed. The effectiveness of a coordinated change in monetary and fiscal policy is shown to be greatest when decision makers' degree of foresight is intermediate in range (average planning horizons on the order of ten years), rather than shorter or longer.

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1 Introduction

The recent economic and financial crisis has led to renewed interest in counter-cyclical fiscal policies.¹ Some recent discussions of the extent to which fiscal policy can be relied upon emphasize the potential benefits of commitments to monetary accommodation of fiscal transfers.² Some have even proposed that a “fiscally dominant” regime (i.e., passive monetary policy with active fiscal policy) would better maintain macro stability in the face of an effective lower bound on nominal interest rates.³

Most of those discussions and analyses have been built on the hypothesis of rational expectation (RE), where the effectiveness of the analyses turns crucially on the assumptions of RE that are made about outcomes far in the future, sometimes infinitely far. Decision makers are assumed to correctly understand the economy well in the distant future, and base their decisions on their expectations regarding the infinite future. However, the assumption that agents can have such foresight is non-plausible and unrealistic. Putting too much weight on the infinite future, the assumption of RE also raises issues such as the “forward guidance puzzle” (e.g., Del Negro, Giannoni, and Patterson, 2015; McKay, Nakamura, and Steinsson, 2016), which views a commitment to future monetary policy being too effective in stimulating output and inflation.

By relaxing this problematic assumption of RE, while keeping other parts as close as possible to the standard New Keynesian model, this paper studies the interplay of fiscal and monetary policy, including policy experiments of fiscal transfers and unconventional monetary policy (forward guidance), in both short-term effects and long-run consequences.⁴ In particular, I emphasize the degree to which foresight influences the effects of fiscal and monetary policy, and their interactions. More generally, this paper investigates the question of how fiscal policy and monetary policy jointly determine inflation and output when decision makers are boundedly rational.

Although the assumption of RE regarding the far future is strong and unrealistic, the belief that decision makers in the short run are still rational is natural. To relax the assumption concerning the long run, but retain the features of RE in the short run, I adopt the approach of finite forward planning recently developed by Woodford (2018).⁵ More specif-

¹For instance, the IMF asks for smarter use of fiscal policy in an environment of high-level public debt and low interest rates. The former managing director of IMF, Christine Lagarde, appeals for the dedicated use of fiscal policy in her April 2019 speech at US Chamber of Commerce: <https://www.imf.org/en/News/Articles/2019/03/29/sp040219-a-delicate-moment-for-the-global-economy>.

²Recent examples include Ascari and Rankin (2013), Buiters (2014), Turner (2017), and Galí (2019).

³Among others, see Jarociński and Maćkowiak (2017).

⁴In terms of fiscal policy, the major focus of this paper is to study transfer-type policy. The analysis of government expenditure, as another type of fiscal stimulus, can be found in the appendix.

⁵Woodford (2018) motivates the approach of finite forward planning from state-of-the-art AI programs

ically, instead of assuming decision makers make infinite contingent plans in each period, this approach assumes limited foresight for decision makers who look only a finite distance into the future, and use a value function learned from past experiences to evaluate situations that may be reached at the end of forward planning. That is, decision makers are still “rational” within their planning horizon but use a *coarse* value function for continuation values to approximate for the future beyond their planning horizon. The finite-planning-horizon model has two key components: the length of the planning horizon, and the value function used to approximate for the future beyond the planning horizon. Intuitively, as the planning horizon of decision makers is longer, or decision makers update their value function faster, the equilibrium under finite horizon planning becomes closer to the case under RE.

Note that if the decision makers can obtain an accurate state-dependent value function, the finite-horizon problem resembles the standard dynamic programming problem as in the analyses of RE. But in practice, when the real world is complex or new information is coming in, acquiring an accurate value function is computationally too costly. Thus, a simplification of the accurate state-dependent value function, that is, the value function learned from past experiences through a rule of constant-gain, would be a useful approximation, and decision makers can abstract from deriving a complex value function through finite forward planning.⁶

In this paper, I build a New Keynesian model with finite forward planning to study the interaction between monetary policy and fiscal policy. More specifically, I characterize the combined monetary-fiscal regimes that result in stable long-run dynamics, and analyze the short-run effectiveness and long-run consequences of temporary changes in either type of policy as a source of short-run demand stimulus. I evaluate the effects of counter-cyclical fiscal stimulus such as debt-financed lump-sum transfer, and unconventional monetary policy such as forward guidance, and their interactions. Importantly, this paper emphasizes how the degree of foresight influences the effects of monetary-fiscal policies and their interactions.

The monetary-policy instrument for the central bank is represented by a rule of nominal interest rate, which is specified by the Taylor rule as in the standard New Keynesian model. Fiscal policy imposed by the government is specified by a lump-sum taxation scheme (e.g., Leeper, 1991; Woodford, 2003; Cochrane, 2019), and thus the evolution of real public debt is endogenous.⁷ The lump-sum taxation scheme has three fiscal policy instruments: one-time lump-sum transfer, the speed of taxation collections with respect to the level of real public debt, and the long-run target of real public debt.

The baseline model is first built upon the representative agent by assuming all decision

to play games.

⁶The learning rule of constant-gain is also a rule in the type of error-correction.

⁷In the appendix, I also discuss the effects of government expenditure with finite forward planning, and emphasize its interaction with monetary policy.

makers (i.e., households and firms) share the same planning horizon. Then, I introduce heterogeneous agents to allow some fraction of the agents in the population to have short foresight while others have far foresight. For simplicity, I assume the distribution of planning horizon following an exponential distribution, in which a single parameter measures the (average) planning horizon of the whole population. Then, the aggregate behavior of the economy is “smooth” in terms of the (average) planning horizon. In this case, the characterization of the equilibrium with the endogenous evolution of real public debt can be summarized by a “hybrid” five-equation linear system. Thus, the case of heterogeneous agents can be easily compared with the standard New Keynesian three-equation system and allows us to derive closed-form analytical solutions for specific policy experiments.⁸ Methodologically, the paper develops a tractable method for the finite-forward-planning approach to analyze the dynamics of aggregate variables in the case of heterogeneous agents with an endogenous path of debt evolution.

This paper suggests that with finite forward planning, fiscal and monetary policy always jointly determine aggregate inflation and output, and Ricardian equivalence always breaks down. By contrast, the literature of the representative-agent model with rational expectations implies a limited impact of fiscal policy in the long run, which dismisses the adoption of fiscal stimulus. In particular, under the parameterization of “Ricardian” fiscal policy as defined in Woodford (2003), that is, when the nominal interest rate responds more than one to one to the inflation rate (“Taylor principle”) and lump-sum tax collections respond strongly enough to the government’s real public debt, the standard New Keynesian literature suggests the output and inflation are purely determined by monetary policy (and that Ricardian equivalence holds for fiscal policy).⁹ But with finite forward planning, fiscal policy always matters even in this policy regime.

A *unique* equilibrium with finite forward planning always exists regardless of the parameterization of monetary or fiscal policy rules. This uniqueness in equilibrium is a key merit of the finite-planning-horizon model, which makes announcements of future policies relevant, but avoids the debates about equilibrium selection that plague rational-expectations analyses. If the limiting values of the equilibrium exist as the planning horizon approaches infinity, the finite-forward-planning approach also provides an equilibrium-selection criterion for all possible solutions under RE. It therefore adds to the discussion of equilibrium selection, such as “minimum-state-variable criterion” or “E-stability criterion,” and clarifies an important

⁸The three-equation system for the standard New Keynesian model includes the IS curve, the Phillips curve, and the rule of monetary policy.

⁹In Leeper (1991), the Ricardian fiscal policy is classified as passive fiscal policy (under the policy regime of active monetary policy with passive fiscal policy).

issue for monetary economics and the fiscal theory of price level (FTPL).¹⁰

Two scenarios of combined monetary-fiscal regimes, depending on the degree of foresight, can ensure long-run stability. In the language of Leeper (1991), they are “active” monetary policy (AM) with “passive” fiscal policy (PF) or “passive” monetary policy (PM) with “active” fiscal policy (AF).¹¹ The fiscal-policy instrument (among the three) – the speed of tax collections with regard to the government’s real public debt level – is what determines the long-run stability of equilibrium together with monetary policy. The boundary condition for fiscal policy between the two scenarios changes little with respect to the length of the planning horizon, whereas the boundary condition of monetary policy relies heavily on the length of the planning horizon. Furthermore, as the length of the planning horizon becomes shorter, the policy space for long-run stability increases under the “AM/PF” policy regime, and decreases under the regime of the “PM/AF.”

If the government and the central bank do not have good knowledge of the actual planning horizon of the population, however, adopting a policy combination to ensure long-run stability under the regime of “AM/PF” pinned down with the hypothesis of an infinite planning horizon is more robust and safer. That is, in contrast to the recent studies proposing a “fiscally dominant” regime (e.g., Jarociński and Maćkowiak, 2017) in facing the effective zero lower bound, the government might appreciate the policy combination of “AM/PF” to better ensure long-run stability from the concerns of limited foresight.¹²

In other combined monetary-fiscal policy regimes, the finite-planning-horizon model with exponential distribution of planning lengths indicates the summation across agents does not converge (in a given period). It suggests the hypothesis of exponential distribution is not appropriate in studying such parameterization. But in general, as long as the maximum planning horizon is finite, or the summation across heterogeneous agents converges, a unique equilibrium path always exists in any policy regime. This distinction is an important one

¹⁰See McCallum (1999) for “minimum-state-variable criterion,” and Evans and Honkapohja (2001) and McCallum (2007) for the discussion of “E-stability.”

¹¹The definition of “active” or “passive” policy depends on whether it is forward-looking or backward-looking in the equilibrium. Intuitively, “active” policy indicates the policy authority is free to set the policy rule depending on past, current, or expected future variables, whereas “passive” policy indicates the authority is constrained by the active authority’s decision in order to balance the government budget constraint.

¹²I show analytically that, in the absence of a fiscal sector and with extremely slow learning in the value function, the Taylor principle is relaxed; that is, as the planning horizon becomes shorter, the requirement on the nominal interest rate in response to the change in inflation to ensure long-run stability is looser. But different from the literature with rational expectations or alternative approaches of modeling bounded rationality, such as incomplete information, cognitive discounting, or “Level-k” thinking (e.g., Angeletos and Lian, 2017; Gabaix, 2018; Farhi and Werning, 2019), when the Taylor principle is violated, the issue of multiple equilibria does not exist in the finite-forward-planning model. As long as the maximum planning horizon is finite, or the summation across heterogeneous agents converge (in any given period), a unique equilibrium path always exists.

with respect to the literature under RE in which the other monetary-fiscal regimes suggest multiple equilibria (by monetary policy and fiscal policy both being “passive”) or no bounded equilibrium solution (by the two policies both being “active”).

Given the policy specification that ensures long-run stability, what are the effects of temporary changes in monetary or fiscal policy as a source of short-run demand stimulus? I investigate the effects of transfer policy, and emphasize its interaction with (conventional) monetary policy in both short-run effects and long-run consequences, as well as its interaction with unconventional monetary-policy “forward guidance.” In particular, I highlight how the degree of foresight affects the interactions between fiscal and monetary policy.

Intuitively, because monetary policy affects the economy through agents’ looking ahead, the effect of a monetary shock or a policy change decreases as decision makers become more short-sighted. By contrast, as the length of the planning horizon becomes shorter, a fiscal stimulus in the type of lump-sum transfer becomes more powerful. The intuition is that, people only incorporate the taxation in a finite future into today’s decision-making.

Before moving to the policy experiments, the key difference between short-run and long-run analysis needs to be clarified, that is, whether decision makers update their value function used as continuation values for the future beyond the planning horizon. In the short run, the value function that decision makers use is assumed to be a given one learned from the steady-state stationary equilibrium, where the steady-state value function is the fixed point of the general (constant-gain) learning process. It is valid and helpful for the study of short-term effects in which decision makers in the economy have stayed in the steady-state stationary equilibrium for a long time and do not have many experiences with those policies in the past. In the analysis of evaluating long-run consequences, however, incorporating the learning process in the value function is necessary and important, which in general dampens the stimulative effects of fiscal transfers found in the short run over time.

First, in terms of short-term effects, given conventional monetary policy specified by the Taylor rule, the three fiscal policy instruments, namely, a large one-time lump-sum transfer financed by public debt, or a slow speed of tax collections after a lump-sum transfer, or an increase in the long-run target of real public debt, can be stimulative for both output and inflation. The stimulative effects of fiscal stimulus on output increase exponentially as decision makers become more short-sighted. For instance, consider a one-time lump-sum transfer fully financed by real public debt and the public debt being kept unchanged thereafter. Quantitatively, with the size of the lump-sum transfer being equal to one-quarter GDP, output (permanently) increases by 0.9%, if the (average) planning horizon is one-quarter as estimated in Gust, Herbst, and López-Salido (2019).¹³ In the absence of an update

¹³Empirically, Gust, Herbst, and López-Salido (2019) apply the model in Woodford (2018), which does not

in the value function, the “fiscal-transfer multiplier” (defined as the discounted aggregate response of output with respect to the size of the initial lump-sum transfer) is large and equals to 0.94.¹⁴

Putting monetary and fiscal policy together, I show that more accommodative monetary policy in general amplifies the stimulative effect of fiscal stimulus, but the impact of monetary policy accommodation on the fiscal effect is *hump shaped* with respect to decision makers’ foresight. It is the greatest when decision makers’ degree of foresight is in the intermediate range (average planning horizons on the order of 10 years), rather than shorter or longer. The intuition is that when the planning horizon is long, the equilibrium is nearly Ricardian-equivalent, and thus fiscal policy is of little effect in stimulating output and inflation. When the planning horizon is short, because monetary policy works through forward-looking behavior, it becomes ineffective and thus matters little for fiscal policy. Thus, how accommodative monetary policy matters most for the effect of fiscal policy only when agents have an intermediate degree of foresight.

Nowadays, in a world with lower nominal interest rates, discussion about unconventional monetary policy, such as forward guidance, has increased. As decision makers plan for shorter distances into the future, the stimulative effect of forward guidance deteriorates, which generates a demand for fiscal stimulus. More importantly, if forward guidance is accompanied by a simultaneous fiscal stimulus through lump-sum transfer, can it achieve anything that a simple summation of the two policies cannot? I analytically show a positive interaction between forward guidance and fiscal lump-sum transfer, and the aggregate effect of the two policies is larger than the simple summation of the two. Furthermore, the positive interaction is maximized also when agents have an intermediate length of planning horizon.

Fiscal stimulus can be effective in the short run, whereas in the long-run, as decision makers update their value function used to approximate for the future beyond their planning horizon, they start to incorporate the effect of those fiscal policies from a more distant future as time passes. Then, the short-term stimulative effect becomes transitory, and dampens over time. If a long-run stationary equilibrium exists after the policy changes, the equilibrium in the long run will finally converge back to the steady-state stationary equilibrium before the

include fiscal sector, to match with US aggregate data such as inflation and output by Bayesian estimation. Their estimation indicates the average length of the planning horizon is about one quarter, and less than 1% of the population plans for more than two years into the future. Their paper also suggests very slow learning in the decision maker’s value function, and show the approach of finite forward planning outperforms other behavioral macro models such as Angeletos and Lian (2017) and Gabaix (2018) in terms of matching with aggregate data.

¹⁴Under the assumption of no update in the value function, simply borrowing the calibrated parameters from the discounted Euler equation in McKay, Nakamura, and Steinsson (2016) implies the (average) planning horizon is around eight years. Then, the positive response of output to the one-time fiscal transfer (with the size of one-quarter GDP) is about 0.3% and the fiscal-transfer multiplier is near 0.31.

policy changes.¹⁵ Nevertheless, as the speed of learning in the value function is slower, the effect of a fiscal stimulus is more persistent.

A natural question emerges regarding how quantitatively important the fiscal transfer is after incorporating the learning process in the value function. For illustration, still consider the one-time lump-sum transfer fully financed by public debt with the monetary policy specified by the Taylor rule. Borrowing the estimates of the (average) planning horizon and learning process from Gust, Herbst, and López-Salido (2019) by matching with US aggregate data, the fiscal-transfer multiplier is about 0.94 in the case of no update in the value function, whereas it becomes 0.27 by calibrating to the US data with a (non-zero) constant-gain learning process.¹⁶

Related Literature. This paper contributes to the increasing interest in introducing behavioral elements into a macroeconomic model. Many of the papers in this fast-developing area provide an explanation for the forward guidance puzzle in various forms of micro-foundation.¹⁷ Christiano, Eichenbaum, and Trabandt (2019) give a brief survey on the new development of DSGE models in this strand. More specifically, Angeletos and Lian (2017) relax the assumption of complete information in the New Keynesian model, and argue the effect of forward guidance is attenuated without common knowledge. Farhi and Werning (2019) obtain a similar limited effect of forward guidance by replacing the hypothesis of rational expectations with “Level-k” thinking, an approach first proposed in García-Schmidt and Woodford (2019). Gabaix (2018) instead builds a behavioral New Keynesian model through cognitive discounting, and discusses the effect of fiscal stimulus by imposing an exogenous evolution of real public debt.¹⁸ Woodford (2018) first develops the framework of finite forward planning grounded on the New Keynesian model, abstracting from the fiscal sector, and provides a natural explanation for the forward guidance puzzle through limited foresight. Instead of limiting attention to the forward guidance puzzle, this paper adds to the discussion of bounded rationality in macroeconomics by focusing on the role of fiscal policy, and studies how monetary and fiscal policy determines inflation and output in both short-run effects and long-run consequences.

Notably, the aggregate behavior of the economy predicted by alternative micro-founded

¹⁵It converges to the steady-state stationary equilibrium with the same inflation and output as before the fiscal stimulus, but could end up with a higher level of real public debt.

¹⁶The estimation in Gust, Herbst, and López-Salido (2019) suggests a small gain in the learning process of value function by matching with US data.

¹⁷Instead of assuming bounded rationality, McKay, Nakamura, and Steinsson (2016) provide an explanation for the forward guidance puzzle through incomplete markets and households’ borrowing constraints by the approach of a Bewley-Aiyagari-Huggett structure.

¹⁸The online appendix of Gabaix (2019) has a brief discussion regarding a mean-reverting public debt formation.

behavioral approaches can be nested as a special case in the finite-planning-horizon model by the assumption of no update in the value function and there being an exponential distribution of the planning horizon among the population. Naturally, when the distribution of the planning horizon changes, or a learning process is in the value function, the finite-forward-planning approach in this paper will have different implications with regard to alternative approaches, especially in the long-run dynamics.

This paper highlights the importance of bounded rationality in analyzing fiscal policy in an expanding literature of the fiscal theory of price level (FTPL). Most of the existing studies are built on the assumption of rational expectation. Early research includes Leeper (1991), Woodford (1996), Cochrane (2001), and seminal references on this topic. Cochrane (2019) and Canzoneri, Cumby, and Diba (2010) survey the existing development of FTPL. For recent works, Eusepi and Preston (2018) propose the scale and composition of the public debt mattering for inflation based on imperfect knowledge and learning. Hagedorn (2018) argues that allowing for heterogeneous agents and incomplete financial markets makes a big difference in contrast to a standard representative-agent model, and show that monetary policy and fiscal policy determine the price level together. Farmer and Zabczyk (2019) challenge the established views about what constitutes a good combination of fiscal and monetary policies by replacing the (infinite-lived) representative-agent model with the overlapping generations (OLG) model. This paper instead introduces bounded-rational agents into the New Keynesian model by replacing the hypothesis of rational expectations with finite forward planning, and shows that fiscal policy together with monetary policy determines inflation and output even in the canonical parameterization of Ricardian fiscal policy. In addition, this paper highlights the role of the degree of foresight in affecting the interaction of monetary and fiscal policy.

Whereas many works in studying the effect of fiscal policy adopt the approach of the OLG model (with rational expectations), the finite-planning-horizon model differs from the OLG model in two major aspects: (i) If the OLG model is calibrated seriously, it corresponds to the case of relatively long planning horizons, for example, sixty years; (ii) conceptually, in the finite-planning-horizon model, agents still care about the infinite future, whereas agents in the OLG model do not. Furthermore, the finite-planning-horizon model has different implications regarding long-run dynamics for a permanent increase in public debt. Given a permanent increase in government debt, agents in the finite-planning-horizon model will finally incorporate the effects of such policy changes, and the output and inflation will converge back to the initial steady-state level after a long enough time, whereas the OLG model predicts a permanent change in output and inflation.¹⁹

¹⁹Ascari and Rankin (2013) show that, in the OLG model with staggered prices, if the monetary rule is

Other recent studies in the literature of heterogeneous-agent New Keynesian model (HANK) allow for non-Ricardian fiscal policy through incomplete markets and borrowing constraints on consumers. Farhi and Werning (2016) provide a survey on the existing literature. This paper contributes to this literature by suggesting that, on top of assuming any physical constraint on agents in the economy, fiscal policy can be non-Ricardian just because of how decision makers form expectations about the future. If a physical constraint like a borrowing constraint is further introduced with bounded rationality, the effect of fiscal stimulus can be even stronger.

This paper adds to the literature discussing the policy interaction between monetary and fiscal policy by introducing bounded rationality, and emphasizes that the policy interaction is maximized when decision makers have an intermediate degree of foresight. Some of the works in this topic overlap with those in FTPL, including Benhabib, Schmitt-Grohé, and Martin Uribe (2001), Eusepi and Preston (2008), Leith and von Thadden (2008), Davig and Leeper (2011), Ascari and Rankin (2013), and so on.

This paper also highlights the role of fiscal stimulus and its interaction with monetary policy in the discussion of stimulative policies through short-term demand. Eggertsson and Woodford (2003, 2004) discuss the optimal monetary and fiscal policy in a liquidity trap, followed by seminal research works on this topic. A recent paper by Sims and Wu (2019) study the tools of unconventional monetary policies including quantitative easing (QE), forward guidance, and negative interest rate policy (NIRP) in a unified DSGE model. Bernanke, Kiley, and Roberts (2019) study the quantitative performance of monetary-policy strategies for a low-rate environment including various forms of the Taylor rule and price-level targeting through the Federal Reserve’s principal simulation model. Although many existing papers focus on the tools of monetary policy, Woodford and Xie (2019), in particular, explore the short-term effects for a variety of alternative monetary-fiscal policy options at the zero lower bound, including strict inflation targeting, debt-financed lump-sum transfer, government expenditure, temporary price-level targeting, and systematic price-level targeting, by assuming finite forward planning with a given value function learned from the steady-state stationary equilibrium. This paper deviates from that paper and contributes to the literature by introducing an endogenous path of debt evolution, and focuses on a broader class of fiscal policy instruments in the transfer type. More importantly, this paper not only studies the interaction of fiscal stimulus and monetary policy in the short run, but also discusses the long-run consequences of such policies by incorporating a learning process in the value function and characterizes the combined monetary-fiscal policy regimes that can ensure long-run stability.

kept unchanged, the long-run output and inflation will have a non-zero response to a permanent increase in public debt.

The rest of the paper proceeds as follows: Section 2 introduces the basic model with finite planning horizon and policy specification; Section 3 incorporates the learning process in decision makers’ value function, and characterizes the long-run stability (or determinacy) condition; Section 4 discusses the short-term effects of fiscal stimulus (i.e., lump-sum transfer policies), and their interaction with monetary policy under the parameterization that ensures long-run stability; in particular, Section 4.2 focuses on the gain from the interaction of lump-sum transfer and forward guidance; Section 5 evaluates the long-run consequences of those fiscal policies considered in Section 4; and Section 6 concludes the paper.

2 A New Keynesian DSGE Model with Finite Planning Horizon

This section lays out the New Keynesian DSGE model with finite forward planning built upon the approach developed in Woodford (2018).²⁰ Households and firms make contingent plans for a finite distance into the future, and use a value function learned from past experiences to evaluate all possible terminal states at the ending period of the planning horizon. The central bank sets a monetary policy following the Taylor rule, and the fiscal authority (the government) specifies a fiscal policy in terms of lump-sum taxation. Intuitively, decision makers are “rational” within their planning horizon as in the standard New Keynesian model with rational expectations (e.g., Woodford, 2003; Galí, 2015), but instead of making an infinite-horizon contingent plan, they use a *coarse* value function from their past experiences to approximate the future beyond their planning horizon. In this aspect, decision makers are bounded rational and the model departs from the rational-expectations assumption.

In this section, I first build up the general framework with finite forward planning, and then characterize the equilibrium with log-linearization around the steady state equilibrium by assuming the value function used by decision makers is a given one learned from the steady-state stationary equilibrium. Then, Section 3 introduces the constant-gain learning process (an error-correction rule) of the value function over time, where the value function learned from the steady state is the fixed point of such a learning rule. For simplicity, I model the value function by considering a perturbation around the steady-state value function.

²⁰Woodford (2018) abstracts from fiscal policy (e.g., government debt or government expenditure) and focuses on the issues of monetary policy such as forward guidance puzzle and Neo-Fisher effect.

2.1 Optimal Finite-horizon Planning for Households

Instead of making an infinite-horizon state-contingent expenditure plan, an infinitely-lived household makes state-contingent plans over a fixed period h by maximizing the expected utility within her planning horizon and approximating the future with a value function for continuation values. More specifically, the objective function for the representative household i with planning horizon- h in period t is to choose a state-contingent expenditure plan $\{C_\tau^i\}$ for any date $t \leq \tau \leq t + h$ to maximize

$$E_t^h[\sum_{\tau=t}^{t+h} \beta^{\tau-t} u(C_\tau^i) + \beta^{h+1} v(B_{t+h+1}^i; s_{t+h})]$$

subject to the budget constraint

$$B_{\tau+1}^i = (1 + i_\tau)[B_\tau^i/\Pi_\tau + Y_\tau - \mathcal{T}_\tau - C_\tau]$$

where the parameter β is the subjective discount factor, and the variable B_τ^i is the financial wealth, i.e., a one-period riskless nominal bond, carried into date τ by household i deflated by the aggregate price index $P_{\tau-1}$, and $v(B_{t+h+1}^i; s_{t+h})$ is the value function that the household uses to evaluate the continuation value for each possible state s_{t+h} at the ending period of planning horizon. By definition, B_τ^i is a real variable that is pre-determined at date $\tau - 1$. The variable $\Pi_\tau = P_\tau/P_{\tau-1}$ is gross inflation, Y_τ is the income of household i at date τ , and \mathcal{T}_τ is the lump-sum tax imposed by the government. The variable i_τ is the interest rate of a one-period riskless nominal bond set by the central bank, and C_τ is the household's consumption on the composite good, where $C_\tau^i = [\int_0^1 (C_\tau^i(f))^{\frac{\theta-1}{\theta}} df]^{\frac{\theta}{\theta-1}}$, and the price of the composite good is P_τ .²¹ Operator $E_t^h[\cdot]$ refers to the expectation of a decision maker with planning horizon- h in period t .

To focus on the household's intertemporal decision regarding consumption and savings, I abstract endogenous labor supply from households' decision-making. More specifically, the labor market contains an organization in which a large number of representatives bargain wage contracts with firms on behalf of households. When a given labor supply is agreed upon with a given wage, each household is required to supply its share of the aggregate labor demanded by the representatives, and thus each household's labor income is equal to its share of the total value Y_τ of composite consumption-goods production. That is, the income of household i is out of her control.

The finite forward planning of household i with horizon $h \geq 1$ yields the following

²¹The variable $C(f)$ is the consumption of the differentiated good f indexed by $f \in [0, 1]$.

first-order conditions for any date $t \leq \tau \leq t + h - 1$

$$u_c(C_\tau^i) = \beta E_t^h[(1 + i_\tau)/\Pi_{t+1} u_c(C_{\tau+1}^i | s_\tau)]$$

and for the ending period of planning horizon $\tau = t + h$ (or the case of $h = 0$),

$$u_c(C_{t+h}^i) = \beta(1 + i_{t+h})v_B(B_{t+h+1}^i; s_{t+h})$$

The decision maker's finite planning problem has two key ingredients: (i) expectation formation and (ii) the value function used for approximating continuation values at the end of planning horizon. If the household's expectation $E_t^h[\cdot]$ is model-consistent and the value function $v(B_{t+h+1}^i; s_{t+h})$ is the accurate model-consistent value for the household's continuation problem, the household's expenditure problem regenerates the standard rational-expectations problem in which decision makers make the optimal infinite-horizon contingent plan.

In terms of expectation formation, instead of assuming model-consistent expectations, households with planning horizon- h make a contingent plan for date t to $t+h$. To incorporate the idea that households only plan for the finite h periods, at each date $t + j$ within their planning horizon ($0 \leq j \leq h$), they are assumed to only plan forward for $h - j$ periods. Households also assume that at any date $t + j$ within their planning horizon, spending and pricing decisions made by other households and firms are made with planning horizon $h - j$. To clarify, in each period t , households choose a contingent plan for the following h periods, but only implement the plan in the current period t . In the following period $t + 1$, they will re-optimize the contingent h -period plan, which is generally not the same plan as the one made in the previous period, and only implement the new plan in the next period $t + 1$.

In other words, by assumption, the expectation formation for agents with planning horizon- h can be written into model-consistent expectation, that is,

$$E_t^h[Z_\tau | s_\tau] = E_t Z_\tau^{t+h-\tau}, \quad t \leq \tau \leq t + h$$

and

$$E_t^h[Z_{\tau+1} | s_\tau] = E_\tau Z_{\tau+1}^{t+h-\tau}, \quad t + 1 \leq \tau \leq t + h$$

for any endogenous variable Z_τ at date τ and any future state s_τ .

Therefore, in the forward-planning exercise, the household's Euler equation with any planning horizon $h \geq 1$ can be translated into model-consistent expectations as

$$u_c(C_\tau^{t+h-\tau}) = \beta E_\tau[(1 + i_\tau^{t+h-\tau})/\Pi_{\tau+1} u_c(C_{\tau+1}^{t+h-\tau-1})] \quad (2.1)$$

for any date $t \leq \tau \leq t + h - 1$; for the ending period of forward planning $\tau = t + h$ (or the case of $h = 0$), it satisfies

$$u_c(C_\tau^0) = \beta(1 + i_\tau)v_B(B_{\tau+1}^0; s_\tau) \quad (2.2)$$

The second key component in the finite-forward-planning problem is the value function used to approximate continuation values at the end of the planning horizon. Although households are sophisticated enough to make plans within their planning horizon, it is computationally too costly for them to correctly deduce the accurate (and complete) state-dependent value function as in the canonical dynamic programming problem. Instead, the value function is *coarse* in terms of state-dependence structure – the value function that households use is assumed to only depend on their real financial position, and households learn the value function by averaging past experiences.

For simplicity, in this section, I assume the value function that agents use to approximate for the future beyond their planning horizon is the one learned from the steady-state stationary equilibrium. This is the situation in which the economy has stayed in the steady-state stationary equilibrium for a long time, and thus households and firms have learned the correct value function for such environment. Then, in Section 3, I introduce the constant-gain learning process in the value function by considering a perturbation around the steady-state value function, where the steady-state value function is the fixed point of this learning process.

The value function learned from the steady-state stationary equilibrium can be derived as²²

$$v(B) = (1 - \beta)^{-1}u(\bar{Y} - \bar{T} + (1 - \beta)B/\bar{\Pi})$$

By log-linearizing the first-order condition (2.1), the finite-forward-planning exercise for households with horizon $h \geq 1$ at any date $t \leq \tau \leq t + h - 1$ satisfies

$$c_\tau^{t+h-\tau} - g_\tau = E_\tau[c_{\tau+1}^{t+h-\tau-1} - g_{\tau+1}] - \sigma[i_\tau^{t+h-\tau} - E_\tau\pi_{\tau+1}^{t+h-\tau-1}]$$

and for the ending period of planning exercise $\tau = t + h$ (or the case of $h = 0$), the first-order condition (2.2) yields

$$c_\tau^0 - g_\tau = -\sigma i_\tau^0 + (1 - \beta)b_{\tau+1}^0$$

where the parameter $\sigma = -u_c(\bar{C})/(u_{cc}(\bar{C})\bar{C}) > 0$ is the intertemporal elasticity of substitution of households, the variable g_τ is a demand or preference shock, and $b_{\tau+1}^0$ is the real

²²The value function learned from the steady-state stationary equilibrium is derived by solving the Bellman equation; that is, $v(B) = \max_C[u(C) + \beta v(B')]$ subject to $B' = \beta^{-1}[B + (\bar{Y} - \bar{T} - \bar{C})\bar{\Pi}]$, where the stationary nominal interest rate $\bar{i} = \beta^{-1}\bar{\Pi} - 1$ has been imposed in the intertemporal budget constraint.

financial position at the end of the planning exercise. All variables in lowercase refer to the percentage deviation from their steady-state values, namely, $c_t = \log(C_t/\bar{C})$, $y_t = \log(Y_t/\bar{Y})$, $\pi_t = \log(\Pi_t/\bar{\Pi})$, $\hat{i}_t = \log(1 + i_t/1 + \bar{i})$. In particular, to allow a non-zero steady-state financial position, I define the deviation in real public debt as $b_t = (B_t - \bar{B})/(\bar{\Pi}\bar{Y})$.

2.2 Optimal Price-setting of Firms with Finite Forward Planning

The optimal pricing problem of firms with finite forward planning is similar to the standard New Keynesian model with sticky price in the style of Calvo (1983) except that, firms plan ahead for only finite periods and use a value function to approximate future profits beyond their planning horizon. The expectation formation with finite forward planning is the same as in the previous section.

More specifically, a continuum of firms exists, and each firm produces a differentiated good indexed by $f \in [0, 1]$ sold in a monopolistically competitive market. As implied by the Dixit-Stiglitz (CES) preference, the demand for good f is given by $Y_t(f) = Y_t(P_t^f/P_t)^{-\theta}$, where Y_t is the aggregate demand for the composite good, P_t^f is the price of good f , and P_t is the price of the composite good. Each firm uses labor as the only input for producing good f with a production function $Y_t(f) = A_t L_t(f)^{1/\phi}$, where A_t represents the level of productivity and $L_t(f)$ is the labor hired by the firm.²³

Following Calvo (1983), each firm can adjust its price freely only with probability $1 - \alpha$ in any given period, regardless of the timing of the last adjustment. In other words, a measure of $1 - \alpha$ of producers in each period can reset their prices, whereas the rest α keep their prices unchanged. Then, the optimal pricing problem with finite planning horizon- h for a firm producing good f in period t is given by

$$\max_{P_t^f} \hat{E}_t^f [\sum_{\tau=t}^{t+h} (\alpha\beta)^{\tau-t} \Lambda_\tau H(P_t^f \bar{\Pi}^{\tau-t}/P_\tau; Z_\tau) + (\alpha\beta)^{h+1} \tilde{v}(P_t^f \bar{\Pi}^h/P_{t+h}; s_{t+h})]$$

where $\tilde{v}(P_t^f \bar{\Pi}^h/P_{t+h}; s_{t+h})$ is the firm's estimate of the value of discounted real profits since date $t + h + 1$ onward (conditional on reaching state s_{t+h} in at date $t + h$), $\Lambda_\tau = \int u_c(C_\tau^i) di$ is the average marginal utilities of households, $H(P_t^f \bar{\Pi}^{\tau-t}/P_\tau; Z_\tau)$ is the real profits of firm f at date τ , Z_τ indicates the vector of real state variables that are out of the firm's control but matter for firm profit (including Y_τ , Λ_τ , A_τ , and exogenous disturbances), and $\hat{E}_t^f[\cdot]$ indicates the expectation used by firm f in the planning exercise in period t . The definition of Λ_τ implies the assumption that shares of the firms are not traded and each household i receives an equal share of firm profits.

Two more key parts remain left to fully establish the firm's pricing problem – the value

²³Households own the firms and receive all the profits of the firms through dividends (with equal shares).

function used by firms to approximate future profits beyond the planning horizon and the labor cost of production. Similar to the assumption in the previous section, the value function used by firm f is the one learned from the steady-state stationary equilibrium and is *coarse* in terms of state-dependence structure, that is, only depending on the firm's relative price. More specifically, the steady-state value function of the firm is given by

$$\tilde{v}(P^f/P) = (1 - \alpha\beta)^{-1} \bar{\Lambda} H(P^f/P; \bar{Z})$$

where the variable $\bar{\Lambda} = u_c(\bar{C})$ is the constant value of Λ_t in the perfect-foresight steady state. In Section 3, a constant-gain learning process to update this value function is introduced for more general analysis.

The labor wage is pinned down by the idea of abstracting the decision-making of labor supply from any individual household but still maintaining the aggregate labor-supply curve as in the canonical New Keynesian model (e.g., Woodford, 2003; Galí, 2015). As mentioned in the household's problem, there are a lot of representatives within the organization of the labor market who bargain wages on behalf of households. For any given wage, a representative determines the number of working hours provided by households, and households must supply that number of hours and receive the same wage. Since a large number of such representatives exist, no one has any monopoly power. Therefore, the representatives will choose the number of hours L_t to maximize the average utility of the households in the economy, which yields

$$v_L(L_t) = \Lambda_t W_t$$

Similar to the derivations in the standard New Keynesian model with Calvo-pricing rigidity, the Phillips curves implied by firms (and households) with planning horizon $h \geq 1$ at any date $t \leq \tau \leq t + h - 1$ within the forward-planning exercise are given by

$$\pi_\tau^h = \kappa(y_\tau^h - y_\tau^*) + \beta E_\tau \pi_{\tau+1}^{h-1}$$

and for the ending period of the planning exercise $\tau = t + h$ (or the case of $h = 0$),

$$\pi_\tau^0 = \kappa(y_\tau^0 - y_\tau^*)$$

where $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \xi$, $\xi = \frac{\phi-1+\eta\phi}{1+(\phi-1)\theta}$, and $\eta = \bar{L} w_{LL} / w_L > 0$ is the Frisch elasticity of labor supply with $w = v_L(L)L$.²⁴ The variable y_t^* refers to exogenous supply shocks such as productivity shocks.²⁵

²⁴The function $w(\cdot)$ is the period-disutility of labor for households.

²⁵The derivation of these expressions and a more detailed description of the labor market can be found in

2.3 Monetary and Fiscal Policy

In this section, I specify monetary and fiscal policy. The central bank sets monetary policy reaction function following the Taylor rule, that is, $\hat{i}_t = i_t^* + \phi_{\pi,t}\pi_t$.²⁶ The coefficient $\phi_{\pi,t}$ can be time-variant to incorporate the policy experiment of forward guidance, whereas in other policy experiments, for simplicity, $\phi_{\pi,t}$ is assumed to be time-invariant. Then, in a forward-planning exercise of decision makers with horizon $h \geq 0$ at time t , the decision makers' expectation regarding the nominal interest rate is given by

$$\hat{i}_\tau^{t+h-\tau} = i_\tau^* + \phi_{\pi,\tau}\pi_\tau^{t+h-\tau}$$

for any date $t \leq \tau \leq t+h$ within the planning exercise. In the case of time-varying coefficient, the coefficient $\phi_{\pi,\tau}$ is the one from the policy announcement in period t .

The fiscal policy is a (net) lump-sum taxation scheme; that is, the rule of tax collections \mathcal{T}_t in period t is given by

$$\mathcal{T}_t = (1 - \Gamma)\bar{\mathcal{T}} + \Gamma\left(\frac{B_t}{\Pi_t} - \frac{\bar{B}}{1+i_t}\right)$$

where the variable $\bar{\mathcal{T}} = \frac{\bar{B}}{\bar{\Pi}} - \frac{\bar{B}}{1+\bar{i}}$ is the lump-sum tax collection associated with the steady-state equilibrium, and \bar{B} is the steady-state level of real public debt. The parameter Γ captures how lump-sum tax collections respond to the level of real public debt as in the FTPL (e.g., Leeper, 1991; Woodford, 2003). In Section 4, policy experiments such as a one-time lump-sum transfer or a change in the long-run debt target are discussed in detail for the analysis of counter-cyclical fiscal stimulus.

With lump-sum taxation (and no government expenditure), the intertemporal budget constraint of the government is given by

$$B_{t+1} = (1 + i_t)[B_t/\Pi_t - \mathcal{T}_t]$$

where \mathcal{T}_t is the net tax collections by the government in period t .²⁷

By log-linearization and substituting the path of tax collections into the government budget constraint, and also noting the steady-date nominal interest rate satisfies $\beta^{-1}\bar{\Pi} =$

Woodford (2018).

²⁶The nominal interest rate is assumed to only respond to inflation and not output gap, in order to have a single parameter $\phi_{\pi,t}$ measuring how accommodative the monetary policy is. This assumption is also the standard case studied in the FTPL. More generally, the policy reaction function of nominal interest rate can be extended to respond to both inflation and output gap and the major conclusions of the paper do not change.

²⁷The government flow budget constraints can also be written in nominal terms as $B_{t+1}P_t = (1 + i_t)[B_tP_{t-1} - \mathcal{T}_tP_t]$.

$\bar{i} + 1$, the evolution of real public debt is given by

$$b_{t+1} = \beta^{-1}(1 - \Gamma)b_t - \beta^{-1}(1 - \Gamma)s_b\pi_t + (1 - \Gamma)s_b\hat{l}_t \quad (2.3)$$

where the variable $b_t = \frac{B_t - \bar{B}}{\Pi Y}$ is the deviation of real public debt from its steady-state value and $s_b = \frac{\bar{B}}{\Pi Y}$ is the relative size of steady-state real public debt relative to output.

2.4 Equilibrium Characterization with Common Planning Horizon

Now, I characterize the full equilibrium by assuming households and firms have the same planning horizon- h . The households are further assumed to start with the same initial financial position, and therefore they make identical decisions in each period. In Section 2.5, the assumption of common planning horizon is extended to heterogeneous planning horizons across decision makers in order to allow some fraction of the whole population to have short foresight and some to have long foresight.

Goods market clearing yields $y_t^h = c_t^h$. Then, given the state variable of pre-determined real asset position $\{b_t\}$ and exogenous disturbances in period t , the equilibrium output, inflation, and nominal interest rate $\{y_t^h, \pi_t^h, \hat{i}_t^h\}$ are pinned down by the solution of the forward-planning problem with horizon- h , and then the asset position $\{b_{t+1}\}$ in the period $t + 1$ is given by the evolution path of real public debt.

The planning problem in period t is characterized by the system of equations as follows: For any date $t \leq \tau \leq t + h - 1$ in the planning exercise with $h \geq 1$,

$$c_\tau^{t+h-\tau} - g_\tau = E_\tau[c_{\tau+1}^{t+h-\tau-1} - g_{\tau+1}] - \sigma[\hat{i}_\tau^{t+h-\tau} - E_\tau\pi_{\tau+1}^{t+h-\tau-1}] \quad (2.4)$$

$$\pi_\tau^{t+h-\tau} = \kappa(y_\tau^{t+h-\tau} - y_\tau^*) + \beta E_\tau\pi_{\tau+1}^{t+h-\tau-1} \quad (2.5)$$

and for the ending period of planning exercise $\tau = t + h$ (or the case of $h = 0$),

$$c_{t+h}^0 - g_{t+h} = -\sigma\hat{i}_{t+h}^0 + (1 - \beta)b_{t+h+1}^0 \quad (2.6)$$

$$\pi_{t+h}^0 = \kappa(y_{t+h}^0 - y_{t+h}^*) \quad (2.7)$$

with the interest rate rule and the evolution of real public debt

$$\hat{i}_\tau^{t+h-\tau} = i_\tau^* + \phi_{\pi,\tau}\pi_\tau^{t+h-\tau} \quad (2.8)$$

$$b_{\tau+1}^{t+h-\tau} = \beta^{-1}(1 - \Gamma)b_\tau^{t+h+1-\tau} - \beta^{-1}(1 - \Gamma)s_b\pi_\tau^{t+h-\tau} + (1 - \Gamma)s_b\hat{l}_\tau^{t+h-\tau} \quad (2.9)$$

where the variable $b_t^{h+1} = b_t$ is the initial asset position in the planning exercise.

The system of equations (2.4)-(2.9) can be solved for the variables $\{y_t^h, \pi_t^h, \hat{i}_t^h\}$ with a unique solution under any parameterization of monetary-fiscal policy rules. Because all the decision makers have the same planning horizon, it follows that the equilibrium inflation and output are $\pi_t = \pi_t^h$ and $y_t = y_t^h$ with nominal interest rate $\hat{i}_t = i_t^* + \phi_{\pi,t}\pi_t$. The real public debt in period $t + 1$ is then given by the debt-evolution equation (2.9).

The characterization of the equilibrium implies a merit of the finite-forward-planning framework – the equilibrium of finite forward planning is always *uniquely* determined. Therefore, it allows us to study the situation in which the standard New Keynesian model with rational expectations indicates multiple equilibria or no bounded equilibria, which is one of the key issues in monetary economics and in the study of fiscal policy. Furthermore, if the limiting value of the equilibrium exists as the planning horizon approaches infinity $h \rightarrow \infty$, it provides an equilibrium-selection criterion for the model with rational expectations if we consider the appropriate equilibrium under RE is the one pinned down by the limiting case of finite forward planning.

2.5 Heterogeneous Agents in Terms of Planning Horizon

The assumption of common planning horizon can be relaxed to accommodate heterogeneous agents. On the one hand, conceptually, in contrast to the homogeneous case with a sharp truncation of the planning horizon for the whole population, the case of heterogeneous agents allows for some fraction of the population to have short foresight and some to have long foresight. On the other hand, technically, with heterogeneous agents, the equilibrium of aggregate variables can be characterized by a similar linear-equation system as in the standard New Keynesian model, which allows easy comparison with the literature and sheds light on the role of bounded rationality in affecting the effects of fiscal and monetary policy. It also yields closed-form analytical solutions.

Suppose in each period that a ω_h fraction of households and a $\tilde{\omega}_h$ fraction of firms have planning horizon h , where $\sum_h \omega_h = 1$ and $\sum_h \tilde{\omega}_h = 1$, respectively. Assume households and firms with planning horizon- h make decisions by assuming all other decision makers share the same planning horizon. Then, the system of equations (2.4)-(2.9) in the previous section still apply for any agent with horizon- h . To simplify the analysis in the heterogeneous case, further assume that, at the beginning of each period, each household has the possibility of being a horizon- h agent with the probability of distribution the same as the population distribution (i.i.d. in each period). It follows that, given the aggregate (average) real public debt b_t known in period t , the group of decision makers with any horizon- h as a whole start

their forward planning with the same level of initial asset b_t . Such an assumption allows us to abstract from tracking the heterogeneous asset accumulation across agents and focus on the behaviors of aggregate variables.²⁸ The aggregate output, inflation, and real asset position can then be defined as

$$y_t = \sum_h \omega_h y_t^h, \quad \pi_t = \sum_h \tilde{\omega}_h \pi_t^h, \quad b_t = \sum_h \omega_h b_t^h$$

with the implicit assumption that the summations converge.

For simplicity, assume $\omega_h = \tilde{\omega}_h = (1 - \rho)\rho^h$ fraction of households and firms has planning horizon h with $0 < \rho < 1$. Then, the average planning horizon in the population is given by $E(h) = \rho/(1 - \rho)$, and thus this assumption allows us to have a single parameter ρ to measure how forward-looking the economy is on average. The evolution of aggregate endogenous variables $\{y_t, \pi_t, \hat{i}_t, b_{t+1}\}$ can then be characterized by averaging the system of equations (2.4)-(2.9) across the population, which yields the IS equation, Phillips curve, rule of nominal interest rate, and path of debt evolution given by

$$y_t - g_t = \rho E_t(y_{t+1} - g_{t+1}) - \sigma(\hat{i} - \rho E_t \pi_{t+1}) + (1 - \rho)(1 - \beta)b'_t$$

$$\pi_t = \kappa(y_t - y_t^*) + \beta \rho E_t \pi_{t+1}$$

$$\hat{i}_t = i_t^* + \phi_{\pi,t} \pi_t$$

$$b_{t+1} = \beta^{-1}(1 - \Gamma)b_t - \beta^{-1}(1 - \Gamma)s_b \pi_t + (1 - \Gamma)s_b \hat{i}_t$$

where $b'_t = \psi_{b,t} b_t + \psi_{g,t} g_t + \psi_{y,t} y_t^* + \psi_{i,t} i_t^*$ with

$$\psi_{b,t} = \beta^{-1}(1 - \Gamma) / [1 - (\phi_{\pi,t} - \beta^{-1})s_b(1 - \Gamma) \frac{(1 - \beta)\kappa}{1 + \sigma\phi_{\pi,t}\kappa}]$$

The time variation in the coefficients $\{\psi_{b,t}, \psi_{g,t}, \psi_{y,t}, \psi_{i,t}\}$ only comes from the variation in $\phi_{\pi,t}$, and if the parameter $\phi_{\pi,t}$ in the monetary policy reaction function is time-independent, these coefficients become constant as well over time. Details of the derivation and expressions for these coefficients can be found in Appendix A. In the limiting case of the (average) planning horizon approaching infinity $\rho \rightarrow 1$, the system of equations becomes the standard New Keynesian model with fiscal policy as discussed in Woodford (2003) as long as the summation across heterogeneous agents converges.

Compared with the standard New Keynesian model, the system of equations for the

²⁸The random possibility of being horizon- h is not being incorporated into the decision maker's forward-planning exercise, where the forward planning exercise conditional on being horizon- h is the same as described in Section 2.1.

finite planning horizon differs in two aspects: (i) a discount factor ρ before each expectation operator captures how far ahead decision makers on average plan for the future; (ii) the IS equation has one extra term $(1 - \rho)(1 - \beta)b'_t$, which is composed of the current level of real public debt and exogenous disturbances. Therefore, in general, fiscal policy *joint* with monetary policy determines inflation and output, and Ricardian equivalence always breaks down unless the real public debt is kept constant at all times (that is, $b_t = 0$ for any period t ; defined as inactive fiscal policy). In other words, no “Ricardian fiscal policy” exists as long as the fiscal policy is active.

More specifically, the first difference as shown by the discount factor ρ before each expectation operator indicates limited effects of monetary policy. Intuitively, because monetary policy affects the economy through decision makers’ forward-looking behavior, as agents have less foresight (i.e., smaller value of ρ), the effects of temporary monetary policy changes (or monetary shocks) on output and inflation become weaker. By contrast, the second difference as captured by the extra term, $(1 - \rho)(1 - \beta)\psi_{b,t}b_t$, indicates strong effects of fiscal transfer policy. Because the parameter $\psi_{b,t}$ is independent of ρ , as the degree of (average) foresight ρ becomes smaller, the coefficient $(1 - \rho)(1 - \beta)\psi_{b,t}$ before b_t in the IS equation is larger. In other words, as agents are less forward-looking, a lump-sum fiscal transfer (or taxation) has larger effects on output and inflation.

Importantly, under the assumption of the steady-state value function with no update process and an exponential distribution in the planning horizon, the above system of equations is robust to alternative approaches of modeling bounded-rationality such as incomplete information, cognitive discounting, or “Level-k” thinking (e.g., Angeletos and Lian, 2017; Gabaix, 2018; Farhi and Werning, 2019).²⁹ In other words, the dynamics of aggregate variables predicted by alternative approaches are nested in such a special case of the finite-planning-horizon model. With a different distribution of the planning horizon, or with a learning process in the value function used to approximate the future, the finite-horizon model has different implications with regard to those alternative approaches. For instance, as shown in Section 5, the model with finite planning horizon has different implications of the equilibrium in terms of long-run dynamics as long as a non-zero gain is present in the learning process of the value function.³⁰

²⁹Angeletos and Lian (2017) and Farhi and Werning (2019) focus on the effects of monetary policy. Based on cognitive discounting, Gabaix (2019) derives a similar modification of the equations for the standard New Keynesian model with an exogenous evolution of real public debt. The online appendix of Gabaix (2019) has a brief discussion regarding a mean-reverting public debt formation.

³⁰As another example, with a non-zero gain in the learning process of the value function, Woodford (2018) points out the “neo-Fisherian” fallacy implied by the finite-planning-horizon model, whereas Gabaix (2019) concludes that the economy is “neo-Fisherian” in the long-run but “Keynesian” in the short-run.

3 Long-run Stability of Monetary-fiscal Policy Interaction

Thus far, the analysis has been based on the assumption that the value function used to approximate the future beyond the planning horizon is learned from the steady-state stationary equilibrium, and that no learning occurs for households and firms to update their value function. Before discussing specific policy experiments in Section 4, first understanding under what conditions the specification of monetary and fiscal policy can ensure long-run stability, or determinacy, is essential. Then, incorporating the learning process in decision makers' value function becomes necessary.

Thus, in this section, I introduce the learning process in the value function specified by a rule of constant gain (i.e., a type of error-correction rule), and discuss the long-run stability (determinacy) condition of monetary-fiscal policy regime. I first model the learning process by assuming all the agents sharing the same planning horizon, and characterize the equilibrium under such an assumption. Then, I introduce heterogeneous agents for analytical purposes. Methodologically, I develop a tractable method for the finite-forward-planning approach to analyze the dynamics of aggregate variables in the case of heterogeneous agents with an endogenous path of debt evolution.

3.1 Learning Process in the Value Function

The learning process is assumed to be a rule of constant gain (or an error-correction rule).³¹ Assume all the decision makers share the same planning horizon, and households use $v_t(B)$ as the value function in their forward-planning exercise and $v_t^{est}(B)$ as the estimated value function from period- t decisions. Similarly, I define $\tilde{v}_t(r)$ and $\tilde{v}_t^{est}(r)$ for firms, where the variable r defined as $r = P^f/P$ is the relative price index of the firm's product to aggregate price index.³² Then, the update of beliefs for the value function are assumed to follow

$$v_{t+1}(B) = \gamma v_t^{est}(B) + (1 - \gamma)v_t(B)$$

$$\tilde{v}_{t+1}(r) = \tilde{\gamma} \tilde{v}_t^{est}(r) + (1 - \tilde{\gamma})\tilde{v}_t(r)$$

where the parameter γ and $\tilde{\gamma}$ measures how fast the households and firms update their beliefs.

Now, I consider a local approximation to the dynamics implied in the above system

³¹The technique of modeling the learning process in the value function is consistent with that in Woodford (2018).

³²The variable of P^f is the price of the firm's product as if it can freely set prices.

through a perturbation around the steady-state value function, where the steady-state value function denoted as $\{v^*, \tilde{v}^*\}$ is the fixed point of the above learning process in the situation of no exogenous disturbances and no change in fiscal and monetary policies.

First, consider a local approximation for v_t . Because the household's optimal finite-horizon plan involves the derivative $v'(B)$ of the value function, it is parameterized as

$$\log(v'_t(B)/v^{*'}(\bar{B})) = -\sigma^{-1}[\nu_t + \chi_t b]$$

Denote $C_t^i(B)$ to be the optimal expenditure plan of households under the counter-factual assumption $B_t^i = B$, and then the derivative of the estimated value function will satisfy

$$v_t^{est'}(B) = \hat{E}_t^i[u_C(C_t^i(B))/\Pi_t]$$

which implies

$$\log(v_t^{est'}(B)/v^{*'}(\bar{B})) = -\sigma^{-1}(c_t^h(b) - g_t) - \pi_t^h$$

The log-linear approximation to the optimal household plan $c_t^h(b) = c_t^h(0) + (c_t^h)'b$ allows us to express the right-hand side as a function of b . Note the right-hand side can be approximate as $-\sigma^{-1}(\nu_t^{est} + \chi_t^{est}b)$, and by equating coefficients, it gives

$$\nu_t^{est} = y_t^h - g_t - \chi_t^{est}b_t + \sigma\pi_t^h$$

$$\chi_t^{est} = (c_t^h)' \equiv g_h(\chi_t)$$

Therefore, the belief-updating system can be written as

$$\nu_{t+1} + \chi_{t+1}b = \gamma[\nu^{est} + \chi_t^{est}b] + (1 - \gamma)[\nu_t + \chi_t b] \quad (3.1)$$

From the households' optimal expenditure plan, it follows that

$$g_h(\chi) = \frac{\chi}{\beta^{h+1} + \frac{1-\beta^{h+1}}{1-\beta}\chi}$$

By the equation (3.1), the variable χ_t is converging to $1 - \beta$ for any initial value $\chi_0 > 0$. Without losing generality, I then assume $\chi_t = 1 - \beta$; that is, the convergence has already occurred.

Thus, the learning dynamics of households reduces to

$$\nu_{t+1} = \gamma\nu^{est} + (1 - \gamma)\nu_t \quad (3.2)$$

where

$$\nu_t^{est} = y_t^h - g_t - (1 - \beta)b_t + \sigma\pi_t^h \quad (3.3)$$

Similarly, I define $\tilde{\nu}_t$ and $\tilde{\chi}_t$ for the value function of firms, and it can be shown that $\tilde{\chi}_t$ is converging to 1 from any initial value $\tilde{\chi}_0 > 0$. Without losing generality, $\tilde{\chi}_t$ is thus set to 1. Then, the learning dynamics of firms can be captured by one endogenous variable $\tilde{\nu}_t$, where the dynamics of $\tilde{\nu}_t$ is given by³³

$$\tilde{\nu}_{t+1} = \tilde{\gamma}\tilde{\nu}^{est} + (1 - \tilde{\gamma})\tilde{\nu}_t \quad (3.4)$$

where

$$\tilde{\nu}_t^{est} = (1 - \alpha)^{-1}\pi_t^h \quad (3.5)$$

3.2 Equilibrium Characterization with Common Planning Horizon

Now, I characterize the complete dynamics of (aggregate) endogenous variables $\{y_t, \pi_t, \hat{t}_t\}$ and $\{b_{t+1}, \nu_{t+1}, \tilde{\nu}_{t+1}\}$ for horizon- h decision makers with learning in the value function.³⁴ In this section, I keep the assumption that all the decision makers have the same planning horizon. Monetary policy and fiscal policy are specified as in Section 2.3.

For horizon- h decision makers in period t with known state variables $\{b_t, \nu_t, \tilde{\nu}_t\}$, at any date $t \leq \tau \leq t + h$ within the planning exercise, the predicted equilibrium evolution of the endogenous variables $\{y_\tau^j, \pi_\tau^j, \hat{t}_\tau^j, b_{\tau+1}^j\}_{j=t+h-\tau}$ can be written in two components

$$y_\tau^j = \tilde{y}_\tau^j + \bar{y}_\tau^j, \quad \pi_\tau^j = \tilde{\pi}_\tau^j + \bar{\pi}_\tau^j, \quad \hat{t}_\tau^j = \tilde{t}_\tau^j + \bar{t}_\tau^j, \quad b_{\tau+1}^j = \tilde{b}_{\tau+1}^j + \bar{b}_{\tau+1}^j$$

where the tilde component indicates the predicted value for variables under no learning in the value function; that is, $\nu_t = \tilde{\nu}_t = 0$, in all periods of the planning exercise, but taking all exogenous shocks and policy changes into consideration with beginning asset position \tilde{b}_t^{h+1} (to be clarified later) in the planning. To be clear, although no learning $\nu_t = \tilde{\nu}_t = 0$ is assumed in the planning exercise for the tilde components, the beginning asset position \tilde{b}_t^{h+1} can be a function of ν_t and $\tilde{\nu}_t$. The bar component represents the discrepancy from this prediction as a result of variation in ν_t and $\tilde{\nu}_t$ throughout the forward planning with beginning asset position \bar{b}_t^{h+1} . Note that $\{\tilde{b}_t^{h+1}, \bar{b}_t^{h+1}\}$ satisfies the condition $b_t = \tilde{b}_t^{h+1} + \bar{b}_t^{h+1}$, and as long as this condition is satisfied, how specifically \tilde{b}_t^{h+1} and \bar{b}_t^{h+1} are defined does not affect the calculation of aggregate endogenous variables $\{y_t^h, \pi_t^h, \hat{t}_t^h\}$ in period t . Because all

³³Details of the derivation for firms are similar to those in Woodford (2018).

³⁴Note that the endogenous variables $\{b_{t+1}, \nu_{t+1}, \tilde{\nu}_{t+1}\}$ are pre-determined in period t .

the decision makers share the same planning horizon, it follows that $y_t = y_t^h$, $\pi_t = \pi_t^h$, and $\hat{i}_t = \hat{i}_t^h$. The pre-determined endogenous variables $\{b_{t+1}, \nu_{t+1}, \tilde{\nu}_{t+1}\}$ are then given by the relations (2.3), (3.2)-(3.3), and (3.4)-(3.5).

Intuitively, the bar component captures the trend of aggregate endogenous variables $\{y_t, \pi_t\}$ resulted from the learning in the value function, whereas the tilde component captures the deviation of aggregate variables from this trend. For the deviation component, it follows from Section 2.4 at any date $t \leq \tau \leq t + h - 1$ within the planning horizon and $h \geq 1$,

$$\tilde{y}_\tau^j - g_\tau = E_\tau[\tilde{y}_{\tau+1}^{j-1} - g_{\tau+1}] - \sigma[\tilde{\nu}_\tau^j - E_\tau\tilde{\nu}_{\tau+1}^{j-1}] \quad (3.6)$$

$$\tilde{\pi}_\tau^j = \kappa[\tilde{y}_\tau^j - y_\tau^*] + \beta E_\tau\tilde{\pi}_{\tau+1}^{j-1} \quad (3.7)$$

where $j = t + h - \tau$. For the ending period of forward planning $\tau = t + h$ (or the case of $h = 0$), it satisfies

$$\tilde{y}_{t+h}^0 - g_{t+h} = -\sigma\tilde{\nu}_{t+h}^0 + (1 - \beta)\tilde{b}_{t+h+1}^0 \quad (3.8)$$

$$\tilde{\pi}_{t+h}^0 = \kappa[\tilde{y}_{t+h}^0 - y_{t+h}^*] \quad (3.9)$$

The rule of nominal interest rate and the evolution of real public debt in the planning exercise for the deviation component are given by

$$\tilde{\nu}_\tau^j = i_\tau^* + \phi_\pi\tilde{\pi}_\tau^j \quad (3.10)$$

$$\tilde{b}_{\tau+1}^j = \beta^{-1}(1 - \Gamma)\tilde{b}_\tau^{j+1} - \beta^{-1}(1 - \Gamma)s_b\tilde{\pi}_\tau^j + (1 - \Gamma)s_b\tilde{\nu}_\tau^j \quad (3.11)$$

with the initial value \tilde{b}_t^{h+1} in period t .³⁵

For the trend component, that is, fluctuations solely coming from learning in the value function, it follows for any date $t \leq \tau \leq t + h - 1$ (with $j = t + h - \tau$) and $h \geq 1$,

$$\bar{y}_\tau^j = \bar{y}_{\tau+1}^{j-1} - \sigma[\bar{\nu}_\tau^j - \bar{\pi}_{\tau+1}^{j-1}] \quad (3.12)$$

$$\bar{\pi}_\tau^j = \kappa\bar{y}_\tau^j + \beta\bar{\pi}_{\tau+1}^{j-1} \quad (3.13)$$

and for the ending period of forward planning $\tau = t + h$ (or the case of $h = 0$), the first-order intertemporal relation of households specified by the expression (2.2) (and a similar

³⁵In this section, the parameter $\phi_{\pi,t}$ in the Taylor rule is assumed to be time-invariant, and thus I simply denote it as ϕ_π .

condition for firms) yields³⁶

$$\bar{y}_{t+h}^0 = -\sigma \bar{i}_{t+h}^0 + (1 - \beta) \bar{b}_{t+h+1}^0 + \nu_t \quad (3.14)$$

$$\bar{\pi}_{t+h}^0 = \kappa \bar{y}_{t+h}^0 + (1 - \alpha) \beta \tilde{\nu}_t \quad (3.15)$$

The rule of nominal interest rate, and the evolution of real public debt with the initial value \bar{b}_t^{h+1} , for the trend component are given by

$$\bar{v}_\tau^j = \phi_\pi \bar{\pi}_\tau^j \quad (3.16)$$

$$\bar{b}_{\tau+1}^j = \beta^{-1}(1 - \Gamma) \bar{b}_\tau^{j+1} - \beta^{-1}(1 - \Gamma) s_b \bar{\pi}_\tau^j + (1 - \Gamma) s_b \bar{v}_\tau^j \quad (3.17)$$

Because all the agents have the same planning horizon, it follows that $y_t = y_t^h$, $\pi_t = \pi_t^h$, and $\hat{i}_t = \hat{i}_t^h$. Therefore, the complete system of endogenous variables $\{y_t, \pi_t, \hat{i}_t, b_{t+1}, \nu_{t+1}, \tilde{\nu}_{t+1}\}$ with state variable $\{b_t, \nu_t, \tilde{\nu}_t\}$ in period- t is characterized by the following: (1) the six equations (3.6)-(3.11) capturing the *forward-looking* system of the deviation component, which can be solved recursively to obtain $\{\tilde{y}_t^h, \tilde{\pi}_t^h, \tilde{\hat{i}}_t^h\}$; (2) the six equations (3.12)-(3.17) capturing the *static* system of the trend component, which can be solved to obtain $\{\bar{y}_t^h, \bar{\pi}_t^h, \bar{\hat{i}}_t^h\}$, and thus together with the deviation component, pinning down aggregate variables $\{y_t, \pi_t, \hat{i}_t\}$; (3) ν_t^{est} and $\tilde{\nu}_t^{est}$ can be computed through the equations (3.3) and (3.5) of Section 3.1 as a function of $\{y_t^h, \pi_t^h, b_t\}$ with exogenous disturbances; (4) the evolution of ν_{t+1} and $\tilde{\nu}_{t+1}$ are specified in the equations (3.2) and (3.4); and (5) the evolution of b_{t+1} is described by the evolution of real public debt (2.3).

Notably, as long as $b_t = \tilde{b}_t^{h+1} + \bar{b}_t^{h+1}$, the choice of the beginning asset position $\{\tilde{b}_t^{h+1}, \bar{b}_t^{h+1}\}$ for calculating the deviation and trend components in the planning exercise does not affect the characterization of the aggregate equilibrium process $\{y_t, \pi_t, \hat{i}_t, b_{t+1}, \nu_{t+1}, \tilde{\nu}_{t+1}\}$. This crucial feature allows us to tractably characterize the behaviors of aggregate variables when I introduce heterogeneous agents into the model in the next section. Thus far, without loss of generality, we can simply assume $\bar{b}_t^{h+1} = 0$ and $\tilde{b}_t^{h+1} = b_t$.

The characterization of the equilibrium under the assumption that all decision makers share the same planning horizon suggests that with finite forward planning, the aggregate equilibrium is always uniquely determined after incorporating the learning process in the value function.

³⁶Details of a derivation for the condition of firms can be found in Woodford (2018).

3.3 Heterogeneous Agent with Learning in the Value Function

To be comparable with the standard New Keynesian model with rational expectations, I introduce heterogeneous agents as in Section 2.5 into the finite-planning-horizon model, while incorporating the learning process in decision makers' value function. All the discussions regarding long-run stability (or determinacy) in the following section are based on the case of heterogeneous agents for the ease of closed-form analysis.

Suppose in each period that $\omega_h = \tilde{\omega}_h = (1 - \rho)\rho^h$ fraction of households and firms have planning horizon h , respectively. Similar to the assumption in Section 2.5, at the beginning of each period, each household has the possibility of being a horizon- h agent with the same probability of exponential distribution as the population distribution (i.i.d. in each period). Then, given aggregate (average) real public debt b_t known in period t , the group of decision makers with any horizon- h as a whole start their forward-planning exercise with initial asset b_t .³⁷ Furthermore, each household and firm makes forward plans by assuming all others share the same planning horizon and use the same value function.

From the linear equations (3.14)-(3.15), only the average belief of ν_t and $\tilde{\nu}_t$ matters for the aggregate endogenous variables such as $\{y_t, \pi_t, \hat{i}_t\}$, and thus I refer to these two variables as representing population averages. Then, the estimated value function in period t as specified in equations (3.3)-(3.5) can be re-written as

$$\nu_t^{est} = \sum_h \omega_h [y_t^h - g_t - (1 - \beta)b_t + \sigma \pi_t^h]$$

$$\tilde{\nu}_t^{est} = \sum_h \tilde{\omega}_h (1 - \alpha)^{-1} \pi_t^h$$

The general steps in this section characterizing the equilibrium with heterogeneous agents are as follows: Given the pre-determined aggregate endogenous variables $\{b_t, \nu_t, \tilde{\nu}_t\}$ in period t , I first characterize the trend and deviation components of $\{y_t, \pi_t, \hat{i}_t\}$, and thus the aggregate endogenous variables $\{y_t, \pi_t, \hat{i}_t\}$; then, I describe the evolution path of $\{b_{t+1}, \nu_{t+1}, \tilde{\nu}_{t+1}\}$ from period t to $t+1$ by the aggregate endogenous variables $\{y_t, \pi_t, \hat{i}_t\}$ and $\{b_t, \nu_t, \tilde{\nu}_t\}$. I assume that, throughout the analysis in this section, the aggregation across agents converges, though not necessarily (always) true. In the discussion of the determinacy condition in the following Section 3.4, I double-check the assumption of convergence for each combined monetary-fiscal regime.

With a focus on characterizing aggregate endogenous variables $\{y_t, \pi_t, \hat{i}_t\}$ in the first step, as discussed in Section 3.2, any specific definition of beginning asset positions $\{\bar{b}_t^{h+1}, \tilde{b}_t^{h+1}\}$ for “trend” and “deviation” variables does not affect the calculation of aggregate endogenous

³⁷Although each agent has a random probability of being horizon- h in the next period, they do not take this randomness into consideration for making forward planning.

variables as long as $b_t = \tilde{b}_t^{h+1} + \bar{b}_t^{h+1}$. In other words, the aggregate variables are robust to the way of defining $\{\bar{b}_t^{h+1}, \tilde{b}_t^{h+1}\}$. Therefore, I impose a specific structure on the heterogeneous (beginning) asset position $\{\bar{b}_t^{h+1}, \tilde{b}_t^{h+1}\}$ in period t for any horizon- h in a particular way to allow for easy and tractable aggregation across agents, but still preserving the aggregate variables unchanged.

More specifically, consider the forward planning exercise of “trend” variables in period t . For any $h \geq 1$ in period t , the equations (3.12)-(3.13) yield

$$\bar{y}_t^h(\bar{b}_t^{h+1}) = \bar{y}_{t+1}^{h-1}(\bar{b}_{t+1}^h) - \sigma[\bar{i}_t^h(\bar{b}_t^{h+1}) - \bar{\pi}_{t+1}^{h-1}(\bar{b}_{t+1}^h)] \quad (3.18a)$$

$$\bar{\pi}_t^h(\bar{b}_t^{h+1}) = \kappa \bar{y}_t^h(\bar{b}_t^{h+1}) + \beta \bar{\pi}_{t+1}^{h-1}(\bar{b}_{t+1}^h) \quad (3.18b)$$

$$\bar{i}_t^h = \phi_\pi \bar{\pi}_t^h \quad (3.18c)$$

$$\bar{b}_{t+1}^h = \beta^{-1}(1 - \Gamma)\bar{b}_t^{h+1} - \beta^{-1}(1 - \Gamma)s_b \bar{\pi}_t^h + (1 - \Gamma)s_b \bar{i}_t^h \quad (3.18d)$$

For $h = 0$, the equations (3.14)-(3.15) yield

$$\bar{y}_t^0(\bar{b}_t^1) = -\sigma \bar{i}_t^0(\bar{b}_t^1) + (1 - \beta)\bar{b}_{t+1}^0 + \nu_t$$

$$\bar{\pi}_t^0(\bar{b}_t^1) = \kappa \bar{y}_t^0(\bar{b}_t^1) + (1 - \alpha)\beta \tilde{\nu}_t$$

with

$$\bar{i}_t^0 = \phi_\pi \bar{\pi}_t^0$$

$$\bar{b}_{t+1}^0 = \beta^{-1}(1 - \Gamma)\bar{b}_t^1 - \beta^{-1}(1 - \Gamma)s_b \bar{\pi}_t^0 + (1 - \Gamma)s_b \bar{i}_t^0$$

Then, I define $\bar{b}_t^1 = 0$ for agents with horizon $h = 0$. For any $h \geq 1$, the beginning asset position \bar{b}_t^{h+1} of “trend” variables for agents with horizon- h is defined in such a way that their asset position in the planning exercise at date $t + 1$ will equal the beginning asset position of agents with horizon $h - 1$ at date t . That is, \bar{b}_t^{h+1} is defined by backward induction in equations (3.18a)-(3.18d) such that $\bar{b}_{t+1}^h = \bar{b}_t^h$, which is a function of only ν_t and $\tilde{\nu}_t$. To keep the aggregate endogenous variables being unchanged, the beginning asset position \tilde{b}_t^{h+1} of “deviation” variables in period t is defined by $\tilde{b}_t^{h+1} = b_t - \bar{b}_t^{h+1}$.

By aggregating across the whole population for the “trend” variables, it follows

$$\bar{y}_t = \rho \bar{y}_t - \sigma[\bar{i}_t - \rho \bar{\pi}_t] + (1 - \rho)v_t + (1 - \rho)(1 - \beta)\bar{b}_{t+1}^0 \quad (3.19a)$$

$$\bar{\pi}_t = \kappa \bar{y}_t + \beta \rho \bar{\pi}_t + (1 - \rho)(1 - \alpha)\beta \tilde{\nu}_t \quad (3.19b)$$

$$\bar{i}_t = \phi_\pi \bar{\pi}_t \quad (3.19c)$$

where $\bar{b}_{t+1}^0 = \psi_\nu \nu_t + \psi_{\tilde{\nu}} \tilde{\nu}_t$. The derivation and expressions for $\{\psi_\nu, \psi_{\tilde{\nu}}\}$ can be found in Appendix B.

Through the system of (3.19a)-(3.19c), $\{\bar{y}_t, \bar{\pi}_t\}$ can be written in ν_t and $\tilde{\nu}_t$; that is,³⁸

$$B \begin{bmatrix} \bar{y}_t \\ \bar{\pi}_t \end{bmatrix} = \Xi \begin{bmatrix} \nu_t \\ \tilde{\nu}_t \end{bmatrix} \quad (3.20)$$

Recall that the beginning asset position for the deviation component of decision makers with planning horizon- h in period t is given by $\tilde{b}_t^{h+1} = b_t - \bar{b}_t^{h+1}$ for $\forall h \geq 0$. Also note the structural equations of deviation components are the same as those specified in Section 2.4 with no update in the value function. By averaging across the population, the structural equations determining the aggregate plans $\{y_t, \pi_t, b_t\}$ must satisfy

$$y_t - g_t - \bar{y}_t = \rho E_t(y_{t+1} - g_{t+1} - \bar{y}_{t+1}) - \sigma[\hat{i} - \bar{i} - \rho E_t(\pi_{t+1} - \bar{\pi}_{t+1})] \quad (3.21a)$$

$$+(1 - \rho)(1 - \beta)(b_{t+1}^0 - \bar{b}_{t+1}^0)$$

$$\pi_t - \bar{\pi}_t = \kappa(y_t - y_t^* - \bar{y}_t) + \beta \rho E_t(\pi_{t+1} - \bar{\pi}_{t+1}) \quad (3.21b)$$

$$\hat{i}_t - \bar{i}_t = i_t^* + \phi_\pi(\pi_t - \bar{\pi}_t) \quad (3.21c)$$

where $b_{t+1}^0 - \bar{b}_{t+1}^0 = \psi_b b_t + \psi_g g_t + \psi_y y_t^* + \psi_i i_t^*$ is the same as the expression in the case of no update in the value function as in Section 2.4 (as showed in Appendix A).

Thus far, given the pre-determined state variables $\{b_t, \nu_t, \tilde{\nu}_t\}$ in period t , I have characterized the system of equations capturing aggregate variables $\{y_t, \pi_t, \hat{i}_t\}$. Then, I describe the evolution process of $\{b_t, \nu_t, \tilde{\nu}_t\}$ over time. From Section 3.1, the dynamics of the value-function adjustment is captured by

$$\nu_{t+1} = \gamma \nu_t^{est} + (1 - \gamma) \nu_t$$

$$\tilde{\nu}_{t+1} = \tilde{\gamma} \tilde{\nu}_t^{est} + (1 - \tilde{\gamma}) \tilde{\nu}_t$$

$$\nu_t^{est} = y_t - g_t + \sigma \pi_t - (1 - \beta) b_t$$

$$\tilde{\nu}_t^{est} = (1 - \alpha)^{-1} \pi_t$$

³⁸The expressions of B and Ξ are given by $B = \begin{bmatrix} 1 - \rho & \sigma(\phi_\pi - \rho) \\ -\kappa & 1 - \beta\rho \end{bmatrix}$, $\Xi = \begin{bmatrix} (1 - \rho)[1 + (1 - \beta)\psi_\nu] & (1 - \rho)(1 - \beta)\psi_{\tilde{\nu}} \\ 0 & (1 - \rho)(1 - \alpha)\beta \end{bmatrix}$.

which can be re-written as

$$\begin{bmatrix} \nu_{t+1} \\ \tilde{\nu}_{t+1} \end{bmatrix} = \Omega \Phi \begin{bmatrix} y_t - g_t \\ \pi_t \end{bmatrix} + \Omega \begin{bmatrix} -(1 - \beta) \\ 0 \end{bmatrix} b_t + (I - \Omega) \begin{bmatrix} \nu_t \\ \tilde{\nu}_t \end{bmatrix} \quad (3.23)$$

where Ω is a 2×2 diagonal matrix with diagonal elements $(\gamma, \tilde{\gamma})$ and $\Phi = \begin{bmatrix} 1 & \sigma \\ 0 & (1 - \alpha)^{-1} \end{bmatrix}$. Together with the evolution path of aggregate real public debt captured by equation (2.3), the equilibrium under heterogeneous agents has been fully characterized.

Now, I re-write the whole system of equations of the equilibrium in a more compact form, and use $\{\bar{y}_t, \bar{\pi}_t\}$ to substitute $\{\nu_t, \tilde{\nu}_t\}$ for capturing the “trend” evolution of the aggregate variables. By substituting (3.20) into (3.23), the “trend” variables follows that

$$\bar{x}_{t+1} = Fx_t + G\bar{x}_t + Hb_t \quad (3.24)$$

where $x_t = \begin{bmatrix} y_t - g_t & \pi_t \end{bmatrix}^T$, $\bar{x}_t = \begin{bmatrix} \bar{y}_t & \bar{\pi}_t \end{bmatrix}^T$, $F = B^{-1}\Xi\Omega\Phi$, $G = B^{-1}\Xi(I - \Omega)\Xi^{-1}B$ and $H = B^{-1}\Xi\Omega \begin{bmatrix} -(1 - \beta) & 0 \end{bmatrix}^T$.

The system (3.21a)-(3.21c) of the “deviation” variables can be written as

$$E_t(x_{t+1} - \bar{x}_{t+1}) = A(x_t - \bar{x}_t) + Cb_t + Ku_t \quad (3.25)$$

where u_t captures exogenous disturbances, and the definition of A , C , and K can be found in Appendix A.

Also note the evolution path of real public debt (2.3) follows

$$b_{t+1} = \beta^1(1 - \Gamma)b_t + D \cdot x_t \quad (3.26)$$

where $D = \begin{bmatrix} 0 & (\phi_\pi - \beta^{-1})(1 - \Gamma)s_b \end{bmatrix}$.

Therefore, the whole system is fully characterized by the evolution of trend components (3.24), deviation components (3.25), and the evolution of real public debt (3.26). It can be summarized as

$$E_t \begin{bmatrix} x_{t+1} \\ \bar{x}_{t+1} \\ b_{t+1} \end{bmatrix} = \Upsilon \begin{bmatrix} x_t \\ \bar{x}_t \\ b_t \end{bmatrix} + \begin{bmatrix} Ku_t \\ 0 \\ 0 \end{bmatrix} \quad (3.27)$$

with three pre-determined variables $\{\bar{y}_t, \bar{\pi}_t, b_t\}$ and two non-predetermined variables $\{y_t, \pi_t\}$. The expression of Υ can be found in Appendix C.

3.4 Determinacy Condition for Monetary Policy with Inactive Fiscal Policy

Consider the first case in which fiscal policy is inactive; that is, the government bond is always in fixed supply $B_t = \bar{B}$ (which corresponds to $\Gamma = 1$). In such a case, it implies $b_t = \bar{b} = 0$ all the time. The only policy instrument is monetary policy, which determines the equilibrium path.

The system of equilibrium (3.27) can then be written into

$$E_t \begin{bmatrix} x_{t+1} \\ \bar{x}_{t+1} \end{bmatrix} = \begin{bmatrix} A + F & -A + G \\ F & G \end{bmatrix} \begin{bmatrix} x_t \\ \bar{x}_t \end{bmatrix} + \begin{bmatrix} Nu_t \\ 0 \end{bmatrix}$$

When the learning process is slow, i.e., $\gamma, \tilde{\gamma} \rightarrow 0$, the “trend” variables \bar{x}_t become constant (with $\Omega = 0$, $F = 0$ and $G = I$). Then, the entire system is determinant if and only if the equilibrium path of aggregate variables x_t is determinant. Following Blanchard and Kahn (1980), the equilibrium path of x_t is determinant if and only if the two eigenvalues of A are outside the unit circle, which is equivalent to³⁹

$$\phi_\pi > -\frac{1}{\kappa\sigma} [\beta\rho^2 - \rho(1 + \beta + \kappa\sigma) + 1] \equiv l(\rho) \quad (3.28)$$

Because $0 < \rho < 1$, the right-hand-side $l(\rho)$ is strictly decreasing in ρ . As the (average) length of planning horizon approaches infinity $\rho \rightarrow 1$, it follows $l(\rho) \rightarrow 1$, which features the boundary condition of the Taylor principle as in the standard New Keynesian model with rational expectations. As ρ becomes smaller, that is, the population is (on average) less forward-looking, the requirement on monetary policy to ensure long-run stability is relaxed. Even in the case of $\phi_\pi = 0$, monetary policy can still ensure stability in the long-run equilibrium as long as agents’ foresight is short enough, that is, ρ is small enough (and the learning in the value function is really slow).

Furthermore, different from the literature of RE with multiple equilibria when the Taylor principle is violated $\phi_\pi < 1$, for the case of $\phi_\pi < l(\rho)$ with finite forward planning, the summation across heterogeneous agents does not converge in any given period. It indicates the assumption of exponential distribution is not appropriate to study the policy regime under such parameterization. In fact, the aggregation across heterogeneous agents converges if and only if the condition in (3.28) is satisfied.⁴⁰ In general, as long as the maximum planning horizon in the population is finite, or the summation across agents converges, a

³⁹Proofs for the determinacy condition can be found in Appendix D.

⁴⁰Details can be found in Appendix D.

unique equilibrium path with finite forward planning always exists.

3.5 Stability (Determinacy) Condition with Monetary-fiscal Policy Interaction

More generally, the evolution of real public debt can be endogenous as specified in equation (3.26). Leeper (1991) categorizes monetary and fiscal policy into two groups – “active” or “passive” policy depending on whether the policy is forward-looking or backward-looking in the equilibrium. Intuitively, “active” policy indicates the policy authority is free to set the policy rule depending on past, current, or expected future variables, whereas “passive” policy indicates the authority is constrained by the active authority’s decision in order to balance the government budget constraint. In this section, I focus on the stability (or determinacy) condition in the “active/passive” language under the environment of heterogeneous agents with finite forward planning.

From the system of equations (3.27), because it has two non-predetermined variables, the equilibrium is determinate if and only if two eigenvalues of Υ are outside the unit circle.

In the limiting case of $\rho \rightarrow 1$, it implies $\Xi = 0$, and the system of trend components (3.20) requires that $\bar{y}_t = 0$, $\bar{\pi}_t = 0$ as long as $\phi_\pi \neq 1$. In such a case, the system of equations characterizing aggregate endogenous variables $\{y_t, \pi_t, b_t\}$ becomes the standard New Keynesian model with rational expectations as discussed in Woodford (2003); that is,

$$E_t(x_{t+1}) = Ax_t + Ku_t$$

$$b_{t+1} = \beta^{-1}(1 - \Gamma)b_t - \beta^{-1}(1 - \Gamma)s_b\pi_t + (1 - \Gamma)s_b\hat{i}_t$$

where the policy rule of nominal interest rate is $\hat{i}_t = i_t^* + \phi_\pi\pi_t$. A unique (locally) bounded solution exists if and only if (i) $|\beta^{-1}(1 - \Gamma)| < 1$ and $\phi_\pi > 1$, or (ii) $|\beta^{-1}(1 - \Gamma)| > 1$ and $\phi_\pi < 1$.⁴¹ In the language of Leeper (1991), condition (i) features active monetary policy (AM) and passive fiscal policy (PF), whereas the condition (ii) features passive monetary policy (PM) and active fiscal policy (AF).⁴²

The limiting case of the finite-planning-horizon model, however, differs from the discussion in Leeper (1991) when the standard model in the literature under RE indicates no bounded equilibrium solution (i.e., $|\beta^{-1}(1 - \Gamma)| > 1$ and $\phi_\pi > 1$) or multiple equilibria (i.e., $|\beta^{-1}(1 - \Gamma)| < 1$ and $\phi_\pi < 1$). In these two scenarios, similar to the discussion

⁴¹It is verified that, under either condition (1) or condition (2) holds, the sum of $\Sigma\omega_h y^h$ and $\Sigma\tilde{\omega}_h \pi^h$ in a given period converges. More details can be found in Appendix E.

⁴²In addition, for the fiscal policy under condition (i), Woodford (2003) also names it as “Ricardian” fiscal policy.

with inactive fiscal policy, the summation across heterogeneous agents does not converge, which indicates the assumption of exponential distribution is not appropriate for studying the policy regime under such parameterization. Generally, as long as the maximum planning horizon is finite, or the summation across agents converges, one unique equilibrium path in the finite-planning-horizon model always exists even in the region where the canonical model with RE indicates multiple equilibria.

Now, I focus on the more general situation of $0 < \rho < 1$; that is, decision makers are not forward-looking into the infinite future. For simplicity, consider the case in which the learning in the value function is slow, i.e., $\gamma, \tilde{\gamma} \rightarrow 0$. From equation (3.24), the evolution of \bar{x}_t no longer depends on x_t and becomes a constant (with $\Omega = 0$, $F = 0$, $H = 0$, and $G = I$). Without loss of generality, I assume $\bar{x}_t = 0$ all the time. The complete system reduces to

$$E_t \begin{bmatrix} x_{t+1} \\ b_{t+1} \end{bmatrix} = \Upsilon_s \begin{bmatrix} x_t \\ b_t \end{bmatrix} + \begin{bmatrix} Ku_t \\ 0 \end{bmatrix}$$

where Υ_s is given by

$$\Upsilon_s = \begin{bmatrix} A & C \\ D & \beta^{-1}(1 - \Gamma) \end{bmatrix}$$

The equilibrium is determinant if and only if two eigenvalues of Υ_s are outside the unit circle. Although there is no further refined analytical solution to the determinacy condition, the boundary conditions of the determinacy condition can be mostly captured by $\phi_\pi = l(\rho)$ and $|\beta^{-1}(1 - \Gamma)| = 1$.⁴³ The determinacy condition is illustrated through numerical exercises as indicated in Figure 1.

For quantitative analysis, the values of parameters for calibration are borrowed from Eggertsson and Woodford (2004) with a quarter model as summarized in Table 1. For a quarter model, the subject discount factor is set to $\beta = 0.99$, implying a 4% (annual) natural rate, the intertemporal elasticity of substitution for households is set at $\sigma = 0.5$, and the response of inflation to the output gap in the Phillips curve is set at $\kappa = 0.02$. The level of steady-state real public debt is calibrated to be 60% of annual GDP, that is, $s_b = 2.4$.

By the calibration of parameters summarized in Table 1, the shaded areas in Figure 1 show the regime of parameterization for $\{\phi_\pi, \Gamma\}$ in which the equilibrium reaches long-run stability (or determinacy). The four subfigures vary in different average planning horizons. The dotted lines in each subfigure (captured by $\phi_\pi = l(\rho)$ and $|\beta^{-1}(1 - \Gamma)| = 1$) largely overlap with the solid lines and represent the thresholds of ϕ_π and Γ , respectively. Notably,

⁴³Note that $l(\rho) = -\frac{1}{\kappa\sigma}[\beta\rho^2 - \rho(1 + \beta + \kappa\sigma) + 1]$.

Table 1: Calibrated Values of Parameters

	Parameter	Value	Notes
Subjective discount factor	β	0.99	4% annual real interest rate (a quarter model)
Intertemporal elasticity of substitution	σ	0.5	Standard value in literature
Probability of being not able to reset prices (Calvo)	α	0.66	An average length of price contract of 3 quarters
Response of inflation to output gap in Phillips curve	κ	0.02	Following Eggertsson and Woodford (2004)
Real public debt to output rate (steady state)	s_b	2.4	Implying real public debt as 60% of GDP

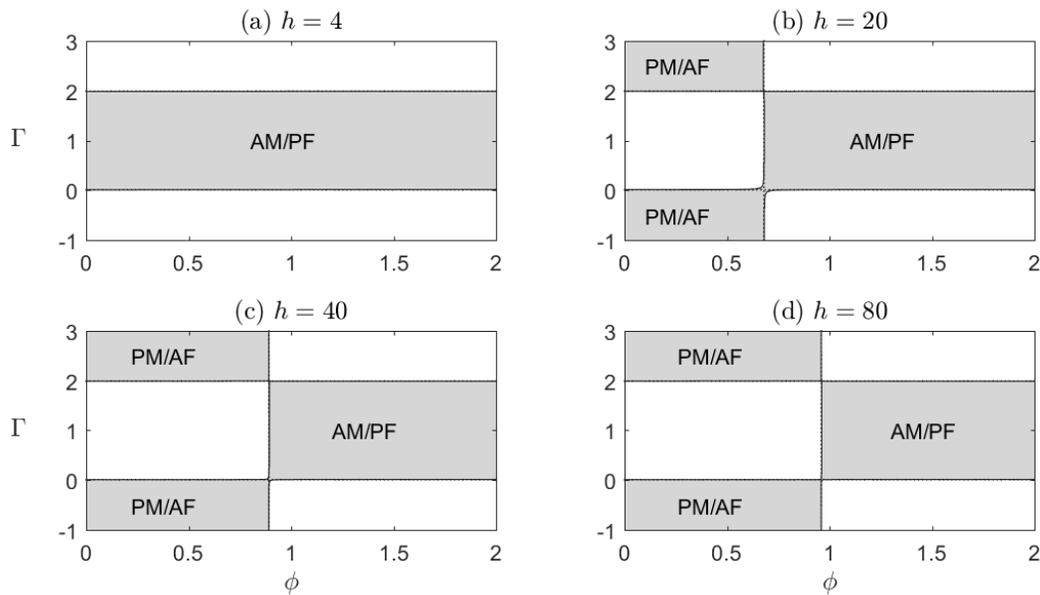


Figure 1: Determinacy condition for $\{\phi, \Gamma\}$ with respect to different planning horizons

Notes: $\gamma = \tilde{\gamma} = 0$, $s_b = 2.4$; h in quarters.

the shaded areas within the dotted lines also ensure the convergence of aggregation across heterogeneous agents in a given period.⁴⁴ For those blank areas in Figure 1, it can be numerically verified that under such parameterization, the aggregation across agents does not converge, which rejects the validity of assuming exponential distribution in the planning horizon for analyzing such scenarios.

From Figure 1, the more short-sighted decision makers are, the larger the policy space of the “AM/PF” regime with long-run stability in a reasonable range of parameterization (i.e., $0 \leq \Gamma \leq 1$ and $\phi_\pi > 0$) becomes, while the policy space for “PM/AF” becomes smaller. “A” in the figure represents “active” policy, and “P” represents “passive” policy. As shown in the bottom-right subfigure (d), when the average planning horizon is around 20 years, or $h = 80$ quarters, the boundary conditions are close to those in the standard New Keynesian

⁴⁴Details can be found in Appendix E.

model with rational expectations. When the planning horizon is short enough, as shown in the upper-right subfigure (b), that is, five years or $h = 20$ quarters, the requirement for long-run stability on ϕ_π in “AM/PF” is significantly smaller than 1.

Importantly, if the government does not have good knowledge of how forward-looking the population is, an active monetary policy with passive fiscal policy satisfying $\phi_\pi > 1$ and $|\beta^{-1}(1 - \Gamma)| < 1$ is robust to the length of the planning horizon; that is, the government might appreciate the policy regime of “AM/PF” to better ensure long-run stability. By contrast, some recent studies (e.g., Jarociński and Maćkowiak, 2017) propose that a “fiscally dominant” regime (“PM/AF”) would better maintain macro stability in the face of an effective lower bound on nominal interest rates.

Although Figure 1 shows the case under the assumption of a slow learning process in the value function, the impact of the length of the planning horizon still applies in the case with a quicker adjustment in the learning process. But a quicker adjustment in updating the value function mitigates the impact of a shorter planning horizon, and leads the boundary conditions of determinacy closer to the standard model with rational expectations.

4 Short-term Effects of Stimulative Fiscal Policy and Interaction with Monetary Policy

In this section, under the parameterization of the policy regime (“AM/PF”) that can ensure long-run stability as discussed in the previous section, I study the short-term effects of stimulative fiscal policy and its interaction with monetary policy. More specifically, I analyze three fiscal policy instruments in a unified framework: a one-time lump-sum transfer from government to the private sector (e.g., new financial claims on the government), the speed of tax collections by the government (with respect to the level of its real public debt), and a change in the long-run target of real public debt. In addition, I also briefly discuss the effects of unconventional monetary policy, namely forward guidance, and its interaction with fiscal stimulus, and whether fiscal policy can achieve a stimulative effect that a pure commitment on future monetary policy cannot.⁴⁵

Before the detailed description of policy experiments, in analyzing the short-term effects of this section, the value function decision makers use is assumed to be a given one learned from the steady-state stationary equilibrium as specified in Section 2. This assumption is valid and helpful to study the short-term effects of counter-cyclical fiscal policy and unconventional monetary policy, in the sense that decision makers in the economy have stayed in

⁴⁵Woodford (2018) shows that, in a model without a fiscal sector, the shorter the length of decision makers’ planning horizon is, the less effective the forward guidance policy is.

the steady-state stationary equilibrium for a long time and do not have much experience with such policies in the past. The following Section 5 relaxes this assumption by incorporating the learning process in the value function as in Section 3, and focuses on the long-run dynamics of these policies.

Now, consider the following policy experiment: Prior to period $t = 0$, the economy has stayed at the steady state for a long time with a fiscal rule as specified in Section 2.3; in the period $t = 0$, the government makes a one-time lump-sum transfer T_0 (in real value), and the government sets a *new* long-run target for the real public debt denoted by B^* , which is characterized by $B^* - \bar{B} = \lambda(1+i_0)T_0$, as well as the rule of tax collections \mathcal{T}_t . In each period from $t = 0$ onward, *net* lump-sum taxes are collected in each period, which is specified by

$$\mathcal{T}_0 = (1 - \Gamma)\bar{\mathcal{T}} + \Gamma\left(\frac{B_0}{\bar{\Pi}_0} - \frac{B^*}{1 + i_0} + T_0\right) - T_0, \quad t = 0$$

and

$$\mathcal{T}_t = (1 - \Gamma)\bar{\mathcal{T}} + \Gamma\left(\frac{B_t}{\bar{\Pi}_t} - \frac{B^*}{1 + i_t}\right), \quad \forall t \geq 1$$

where \bar{B} , $\bar{\Pi}$, \bar{i} , $\bar{\mathcal{T}}$ is the real public debt, inflation, nominal interest rate, and lump-sum taxation associated with the steady state before the one-time transfer occurs. While the first term in the rule of tax collection from $t = 0$ indicates the initial steady-state taxation, the second term indicates the tax collections needed to make B_{t+1} directly equal to the long-run debt target, with weight $(1 - \Gamma)$ on the first amount and weight Γ on the second ($0 \leq \Gamma \leq 1$). The parameter Γ captures not only how strongly lump-sum tax collections respond to the level of real public debt, but also captures how fast the real public debt will converge to the new long-run target. In the case of no lump-sum transfer $T_0 = 0$ and no change in long-run debt target $B^* = \bar{B}$, the fiscal rule becomes the same one as in Section 2.3.

By log-linearization and substituting the path of tax collections into the government budget constraint, the evolution of real public debt for any period $t \geq 1$ is given by

$$b_{t+1} = \beta^{-1}(1 - \Gamma)b_t - \beta^{-1}(1 - \Gamma)s_b\pi_t + (1 - \Gamma)s_b\hat{t}_t + \Gamma b^*$$

and for the period $t = 0$,

$$b_1 = \beta^{-1}(1 - \Gamma)b_0 - \beta^{-1}(1 - \Gamma)s_b\pi_0 + (1 - \Gamma)s_b\hat{t}_0 + (1 - \Gamma)t^* + \Gamma b^*$$

where the variable $b_t = \frac{B_t - \bar{B}}{\bar{\Pi}Y}$ is the deviation of real public debt from its initial steady-state value before the one-time transfer occurs (relative to output), $b^* = \frac{B^* - \bar{B}}{\bar{\Pi}Y}$ is the deviation of the long-run real debt target from the initial steady-state value of real public debt, $t^* = \frac{T_0}{\bar{\Pi}Y}$

is the size of a one-time lump-sum transfer relative to output, and $s_b = \frac{\bar{B}}{\Pi Y}$ is the relative size of initial steady-state real public debt to output. By the definition of B^* , it follows that $b^* = \lambda t^*$. To sum up, three fiscal policy instruments exist $\{t^*, \lambda, \Gamma\}$.

Several special situations help clarify the role of the policy instruments $\{\Gamma, \lambda\}$. The parameter Γ measures how fast the real public debt converges to the long-run debt target. Intuitively, as Γ increases from 0 to 1, converging to the debt target takes less time. In the case of $\Gamma = 0$, the debt target is irrelevant, and taxation in each period is \bar{T} . Then, the path of real public debt purely depends on the path of interest rates and inflation, and there is no expectation that tax collections will ensure the real public debt under control. When $\Gamma = 1$, the public debt b_1 in period $t = 1$ directly increases from the initial steady-state value 0 to the new long-run target value b^* , and remains unchanged thereafter.

To see the role of λ , consider the case of $\Gamma = 1$, and in this scenario, the meaning of λ is straightforward – indicating the proportion of the lump-sum transfer that is financed by (long-run) real public debt. As λ becomes larger, it follows that a bigger proportion of the lump-sum transfer is financed by the long-run level of public debt, and the *net* transfer to households is larger. For instance, in the case of $\lambda = 1$ (and $\Gamma = 1$), the lump-sum transfer is fully financed by real public debt, and the net transfer is t^* , whereas if $\lambda = 0$ (and $\Gamma = 1$), the lump-sum transfer is simultaneously offset by increased taxes in the same period, and thus no effect can rise.

4.1 Lump-sum Transfer Financed by Debt with Monetary Policy under Taylor Rule

In this section, I study the effects of counter-cyclical fiscal policies as specified in the previous section with monetary policy under the (time-invariant) Taylor rule. To be comparable with the standard New Keynesian model, I conduct the analysis by considering the case of heterogeneous agents as specified in Section 2.5. A similar analysis can also be done under the assumption of homogeneous agents as in Section 2.4, and the conclusions do not change. For simplicity, I assume no real disturbances occur.

Under the assumption of heterogeneous agents as in Section 2.5, the equilibrium relations in any period $t \geq 1$ are given by

$$y_t = \rho E_t y_{t+1} - \sigma(\hat{i} - \rho E_t \pi_{t+1}) + (1 - \rho)(1 - \beta)[\psi_b b_t + \psi_{b^*} b^*] \quad (4.1)$$

$$\pi_t = \kappa y_t + \beta \rho E_t \pi_{t+1} \quad (4.2)$$

with nominal interest rate and the evolution of real public debt satisfying

$$\hat{i}_t = \phi_\pi \pi_t \quad (4.3)$$

$$b_{t+1} = \beta^{-1}(1 - \Gamma)b_t - \beta^{-1}(1 - \Gamma)s_b \pi_t + (1 - \Gamma)s_b \hat{i}_t + \Gamma b^* \quad (4.4)$$

where the parameter ψ_b and ψ_{b^*} can be easily solved as in Appendix A, given by

$$\psi_b = \beta^{-1}(1 - \Gamma) / [1 - (\phi_\pi - \beta^{-1})s_b(1 - \Gamma) \frac{(1 - \beta)\kappa}{1 + \sigma\phi_\pi\kappa}]$$

$$\psi_{b^*} = \Gamma / [1 - (\phi_\pi - \beta^{-1})s_b(1 - \Gamma) \frac{(1 - \beta)\kappa}{1 + \sigma\phi_\pi\kappa}]$$

In the period $t = 0$, the equations of the IS and Phillips curve (4.1)-(4.2), and the evolution of real public debt (4.4) become

$$y_0 = \rho E_0 y_1 - \sigma(\hat{i} - \rho E_0 \pi_1) + (1 - \rho)(1 - \beta)[\psi_b b_0 + \psi_{t^*} t^* + \psi_{b^*} b^*]$$

$$\pi_0 = \kappa y_0$$

$$b_1 = \beta^{-1}(1 - \Gamma)b_0 - \beta^{-1}(1 - \Gamma)s_b \pi_0 + (1 - \Gamma)s_b \hat{i}_0 + (1 - \Gamma)t^* + \Gamma b^*$$

where the initial asset position satisfies $b_0 = 0$, and the parameter ψ_{t^*} is given by

$$\psi_{t^*} = (1 - \Gamma) / [1 - (\phi_\pi - \beta^{-1})s_b(1 - \Gamma) \frac{(1 - \beta)\kappa}{1 + \sigma\phi_\pi\kappa}]$$

To illustrate the effects of the three fiscal policy instruments $\{t^*, \lambda, \Gamma\}$ separately and note that $b^* = \lambda t^*$, I consider two policy experiments with closed-form solutions: (i) the case of $\Gamma = 1$, that is, the real public debt directly increases to the new long-run target level and stays unchanged thereafter; (ii) $s_b = 0$ and $\lambda = 0$, that is, both the steady-state level of real public debt and the long-run target of debt are zero.⁴⁶ The first case helps clarify the role of $\{t^*, \lambda\}$, and the second policy experiment helps to study $\{t^*, \Gamma\}$.

⁴⁶Woodford and Xie (2019) study the first case of $\Gamma = 1$, the situation of a permanent increase in real public debt, by imposing a monetary rule of strict inflation targeting, and focus on the effects under ZLB.

4.1.1 Lump-sum Transfer Financed by an Immediate Permanent Increase in Real Public Debt

When $\Gamma = 1$, the real public debt $b_t = b^* = \lambda t^*$ for $\forall t \geq 1$, and $\psi_b = \psi_{t^*} = 0$ with $\psi_{b^*} = 1$. Then, the solution of the equilibrium for any period $t \geq 0$ is time-invariant, given by

$$y_t = \frac{(1 - \beta\rho)(1 - \rho)(1 - \beta)}{(1 - \beta\rho)(1 - \rho) + \kappa\sigma(\phi_\pi - \rho)} \lambda t^* \quad (4.6)$$

$$\pi_t = \frac{\kappa(1 - \rho)(1 - \beta)}{(1 - \beta\rho)(1 - \rho) + \kappa\sigma(\phi_\pi - \rho)} \lambda t^* \quad (4.7)$$

By limiting the attention to the case of “AM/PF” , which implies $\phi_\pi > l(\rho)$, and also noting that $0 < \rho < 1$, the output and inflation are obviously positive in response to the lump-sum transfer, that is, $y_t, \pi_t > 0$. As the size of lump-sum transfer t^* becomes larger, the response of output and inflation increases. Meanwhile, the larger is the proportion of the transfer λ that is financed by (long-term) debt, the larger the response of output and inflation is.

First, consider how the degree of foresight influences the effect of fiscal transfers with a given parameterization of the monetary policy. Given $\phi_\pi > 1$, the output y_t is strictly decreasing with respect to the length of the (average) planning horizon measured by ρ .⁴⁷ In other words, as decision makers in the economy plan less distance into the future, the stimulative effect of lump-sum transfer on output becomes larger.

By the expressions (4.6) and (4.7) and the calibration of parameters from Table 1, Figure 2 shows the output and inflation in response to a one-time debt-financed lump-sum transfer in period $t = 0$. The (average) planning horizon in the figure is defined as $h = \rho/(1 - \rho)$ with unit in quarters. The standard New Keynesian model corresponds to the case of infinite horizon $h \rightarrow \infty$. The two lines in Figure 2 indicates two different sizes of lump-sum transfer. The shape of the two lines suggests that as the length of the planning horizon decreases, a noticeable (persistent) stimulative effect of fiscal lump-sum transfer occurs in output and inflation. In particular, the response in inflation reaches the highest when the planning horizon of decision makers is around four years, or 16 quarters, whereas the response in output increases exponentially with less foresight.

Quantitatively, with the size of the lump-sum transfer being equal to one-quarter GDP as shown in the solid line, the output increases by 0.9% if the (average) planning horizon is one quarter, as estimated in Gust, Herbst, and López-Salido (2019). In this case, the “fiscal-transfer multiplier” (defined as the discounted aggregate response of output with respect to

⁴⁷Note that as $\rho \rightarrow 1$, $-\frac{1}{\kappa\sigma}[\beta\rho^2 - \rho(1 + \beta + \kappa\sigma) + 1] \rightarrow 1$, and thus it requires $\phi_\pi > 1$ for the monotonicity of y_t with respect to ρ .

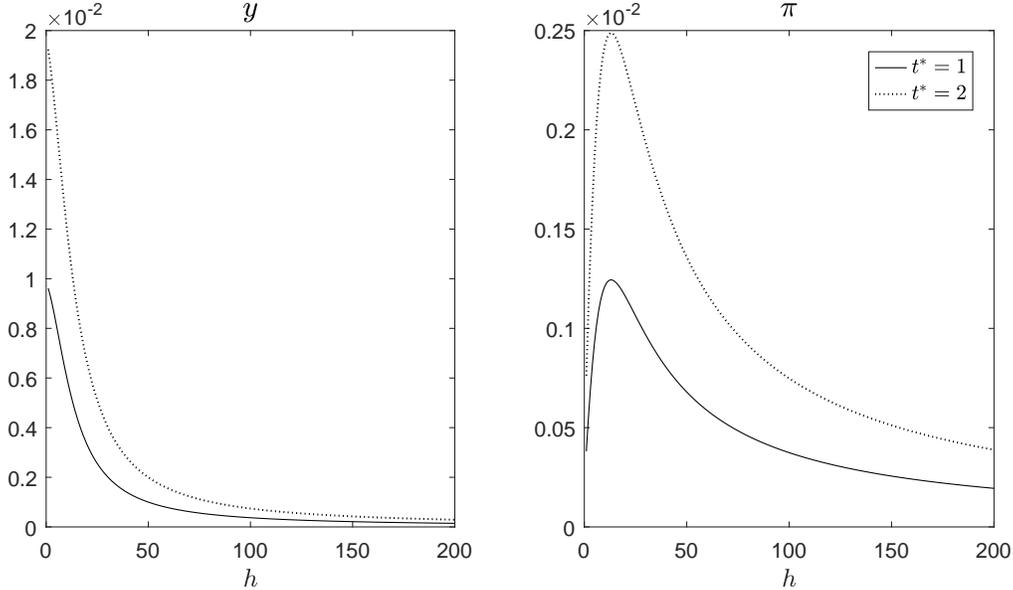


Figure 2: Output and inflation in period 0 as to the planning horizon with a one-time lump-sum transfer financed by a permanent increase in real public debt

Notes: $\phi_\pi = 1.5$, $\Gamma = 1$, $\lambda = 1$; h in quarters; no real disturbances.

the size of the initial lump-sum transfer) is about 0.94. If I simply borrow the calibrated parameters from the discounted Euler equation in McKay, Nakamura, and Steinsson (2016), it implies the (average) planning horizon is around eight years. Then, the positive response of output to the one-time fiscal transfer (with the size of one-quarter GDP) is about 0.3% and the fiscal-transfer multiplier is near 0.31.

From the expressions (4.6) and (4.7), the proportion of lump-sum transfer financed by debt, which is measured by λ , has an effect similar to that of the size of lump-sum transfer t^* .

The response in output and inflation is seemingly permanent, because of the assumption of no update in the value function. But as decision makers update their value function by incorporating the effects of such counter-cyclical fiscal policies, the response in output and inflation becomes transitory.

To shed light on how monetary policy affects the effects of fiscal stimulus, expressions (4.6) and (4.7) indicate the following two propositions:

Proposition 1. *Given the equilibrium is determinant $\phi_\pi > l(\rho)$, and a one-time lump-sum transfer financed by an immediate permanent increase in real public debt $\Gamma = 1$, and $0 < \rho < 1$, it follows that the responses of output and inflation to such a fiscal transfer are strictly decreasing with less accommodative monetary policy, that is, $\frac{\partial y}{\partial \phi_\pi} < 0$ and $\frac{\partial \pi}{\partial \phi_\pi} < 0$.*

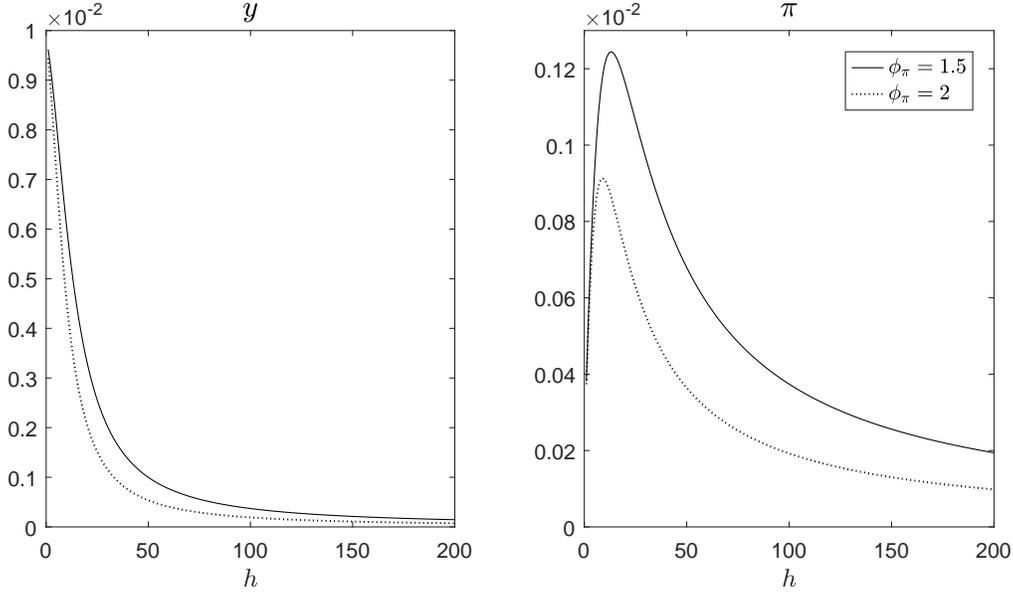


Figure 3: Output and inflation in period 0 as to the planning horizon with a one-time lump-sum transfer under accommodative monetary policy

Notes: $t = 1$, $\Gamma = 1$, $\lambda = 1$; h in quarters; no real disturbances.

Proposition 2. *Given $l(\rho) < \phi_\pi < \frac{1}{\kappa\sigma} + \frac{2}{1+\beta}$, and a one-time lump-sum transfer financed by an immediate permanent increase in real public debt $\Gamma = 1$, and $0 < \rho < 1$, it follows that the impact of monetary policy accommodation on the effect of fiscal transfer in stimulating output is hump-shaped with respect to the degree of foresight; that is, a unique $\bar{\rho} \in (0, 1)$ exists such that $-\frac{\partial^2 y}{\partial \phi_\pi \partial \rho} > 0$ if $0 < \rho < \bar{\rho}$, and $-\frac{\partial^2 y}{\partial \phi_\pi \partial \rho} < 0$ if $\bar{\rho} < \rho < 1$. In the case of $\phi_\pi \geq \frac{1}{\kappa\sigma} + \frac{2}{1+\beta}$, it follows $-\frac{\partial^2 y}{\partial \phi_\pi \partial \rho} < 0$ for any $\rho \in (0, 1)$.*

Proofs of Proposition 1 and 2 can be found in Appendix F. Proposition 1 indicates the effect of fiscal transfer is amplified with more accommodative monetary policy. Going one step further, Proposition 2 shows that as long as the monetary policy does not respond too strongly to inflation, the impact of monetary policy accommodation on fiscal transfer in stimulating output is hump-shaped with respect to the length of the planning horizon.⁴⁸ For a reasonable calibration as shown in Table 1, it requires $\phi_\pi < \frac{1}{\kappa\sigma} + \frac{2}{1+\beta} \approx 101$, which is in the region of our major interest. The intuition of the hump-shaped relationship is that, when the planning horizon is long, the equilibrium is nearly Ricardian-equivalent, and thus fiscal policy is of little effect in stimulating output and inflation. When the planning horizon is short, because monetary policy works through forward looking, it becomes ineffective in this situation, and thus it matters little for fiscal policy.⁴⁹

⁴⁸Similar pattern also follows for the response of inflation, but with a different threshold.

⁴⁹The intuition for why no hump-shaped relationship exists for the case of $\phi_\pi > \frac{1}{\kappa\sigma} + \frac{2}{1+\beta}$ is that in such

Quantitatively, with the same size of one-time lump-sum transfer ($t^* = 1$), Figure 3 shows the output and inflation as to the length of the (average) planning horizon under alternative specifications of the Taylor rule. The solid line indicates a more accommodative monetary policy with $\phi_\pi = 1.5$, and the dotted line shows the case of $\phi_\pi = 2$. Obviously, as the monetary policy becomes more accommodative, the effect of lump-sum transfer is larger. But comparing the gap between the two lines shows that monetary policy matters most for the fiscal policy (in terms of stimulating output) only when decision makers have an intermediate degree of foresight, that is, they plan for about the next 10 years (or $h = 40$ quarters). The specification of monetary policy has little impact on the effects of fiscal policy in the case of short and long planning horizons.

In addition, to illustrate the roles of limited foresight for households and firms, respectively, assume households follow the distribution of $\omega_h = (1 - \rho)\rho^h$, whereas firms follow $\tilde{\omega}_h = (1 - \tilde{\rho})\tilde{\rho}^h$ with $\rho, \tilde{\rho} \in (0, 1)$. Then, the solution to the policy experiment of lump-sum transfer with $\Gamma = 1$ for any period $t \geq 0$ is still time-invariant.⁵⁰ For any given foresight of households $\rho < 1$, if firms become fully rational $\tilde{\rho} \rightarrow 1$, the responses of output and inflation $\{y, \pi\}$ are still positive and follow a pattern similar to the case of assuming the same distribution across households and firms. That is, even if firms are fully rational, Ricardian equivalence still breaks down and fiscal stimulus can be powerful. By contrast, for any given foresight of firms $\tilde{\rho} < 1$, if households become fully rational, $\rho \rightarrow 1$, the responses of output and inflation to the fiscal stimulus become zero. Therefore, the limited foresight of households compared with that of firms plays a more crucial role in the effect of fiscal stimulus.

4.1.2 Lump-sum Transfer Financed by a Temporary Increase in Real Public Debt

Due to the breakdown of Ricardian equivalence, the timing of financing the lump-sum transfer becomes important for output and inflation. To illustrate the effects of the speed of tax collections Γ , consider the case of zero steady-state real public debt $s_b = 0$ with no change in the long-run target of public debt $\lambda = 0$. Then, the evolution of real public debt is exogenously given by $b_1 = (1 - \Gamma)t^*$ and $b_{t+1} = [\beta^{-1}(1 - \Gamma)]^t b_1$ for $\forall t \geq 1$.⁵¹ Given the path of real public debt, the system of equations (4.1)-(4.3) capturing the evolution of equilibrium

a situation, the monetary policy is responding so strongly to the inflation that even agents have extremely short foresight ($\rho \rightarrow 0$), the monetary policy still has a significant impact on output and inflation.

⁵⁰The solution is given by $y_t = \frac{(1-\beta\tilde{\rho})(1-\rho)(1-\beta)}{(1-\beta\tilde{\rho})(1-\rho)+\kappa\sigma(\phi_\pi-\rho)}\lambda t^*$ and $\pi_t = \frac{\kappa(1-\rho)(1-\beta)}{(1-\beta\tilde{\rho})(1-\rho)+\kappa\sigma(\phi_\pi-\rho)}\lambda t^*$.

⁵¹For the real public debt to be non-explosive, I impose the assumption that $|\beta^{-1}(1 - \Gamma)| \leq 1$.

from any period $t \geq 1$ can be re-written as

$$y_t = \rho E_t y_{t+1} - \sigma(\hat{i}_t - \rho E_t \pi_{t+1}) + (1 - \rho)(1 - \beta)\beta^{-1}(1 - \Gamma)b_t$$

$$\pi_t = \kappa y_t + \beta \rho E_t \pi_{t+1}$$

with the rule of nominal interest rate $\hat{i}_t = \phi_\pi \pi_t$.

Conjecturing a solution of the form $y_t = \gamma_y b_t$ and $\pi_t = \gamma_\pi b_t$ for $t \geq 1$, I then substitute $\{y_t, \pi_t\}$ into the above system of equations to get the unique solution in this form, given by⁵²

$$\gamma_y = \frac{[1 - \rho(1 - \Gamma)](1 - \rho)(1 - \beta)\beta^{-1}(1 - \Gamma)}{[1 - \rho\beta^{-1}(1 - \Gamma)][1 - \rho(1 - \Gamma)] + \kappa\sigma[\phi_\pi - \rho\beta^{-1}(1 - \Gamma)]}$$

$$\gamma_\pi = \frac{\kappa(1 - \rho)(1 - \beta)\beta^{-1}(1 - \Gamma)}{[1 - \rho\beta^{-1}(1 - \Gamma)][1 - \rho(1 - \Gamma)] + \kappa\sigma[\phi_\pi - \rho\beta^{-1}(1 - \Gamma)]}$$

Given the path of endogenous variables $\{y_t, \pi_t\}$ for any period $t \geq 1$, in the period $t = 0$, it can easily be solved for $\{y_0, \pi_0\}$.⁵³

By the expression of y_t and π_t , given the speed of tax collections Γ , the effect of one-time lump-sum transfer since period $t = 1$ is transitory and decreases over time. As long as the speed of tax collection Γ is not extremely small, i.e., $\Gamma \geq \max\{1 - \beta, \bar{\Gamma}\}$, where $\bar{\Gamma} = \frac{[\kappa\sigma(\phi_\pi\beta^{-1})]^{1/2} - 1 + \rho}{\rho}$, γ_y and γ_π is strictly decreasing with respect to Γ .⁵⁴ Details of the proof can be found in Appendix G. Thus, the stimulative effect of lump-sum transfer decreases as the government collects taxes more quickly.

Taking the calibration from Table 1, Figure 4 illustrates the role of Γ with respect to the planning horizon h by the response of output and inflation in period $t = 0$. A similar pattern follows in any given period since $t = 0$. The parameter $\Gamma = 0.1$ indicates the half-life for the real public debt to converge back to zero is about 7 quarters, $\Gamma = 0.2$ indicates the half-life is around 3 quarters, and $\Gamma = 0.4$ indicates the half-life is 1 quarter. Without a change in the long-run debt target, a lump-sum transfer together with a slow speed of tax collections to repay the lump-sum transfer improves the short-term effect of fiscal stimulus.

To illustrate the impact of monetary policy accommodation on fiscal stimulus, given

⁵²Given the coefficient in the Taylor rule is larger than one ($\phi_\pi > 1$), it follows that $\gamma_y, \gamma_\pi > 0$ for any $0 < \rho < 1$.

⁵³The output and inflation response in period 0 are given by $y_0 = \frac{(1-\Gamma)t^*}{1+\sigma\phi_\pi\kappa}[\rho(\gamma_y + \sigma\gamma_\pi) + (1 - \rho)(1 - \beta)]$ and $\pi_0 = \frac{\kappa(1-\Gamma)t^*}{1+\sigma\phi_\pi\kappa}[\rho(\gamma_y + \sigma\gamma_\pi) + (1 - \rho)(1 - \beta)]$.

⁵⁴If the coefficient in the Taylor rule is not too small, i.e., $\phi_\pi > \beta^{-1}$, then γ_y and γ_π is strictly decreasing with respect to Γ as long as $\Gamma \geq \max\{1 - \beta, \bar{\Gamma}\}$. If the coefficient in Taylor rule satisfies $\phi_\pi < \beta^{-1}$, the condition for the monotonicity of $\{\gamma_y, \gamma_\pi\}$ with respect to Γ is relaxed; that is, it only requires $\Gamma > 1 - \beta$. Details can be found in Appendix G. But note that ϕ_π can not be too small; otherwise, $\Sigma\omega_h y_t^h$ and $\Sigma\tilde{\omega}_h \pi_t^h$ do not converge. More details about the determinacy condition are discussed in Section 3.

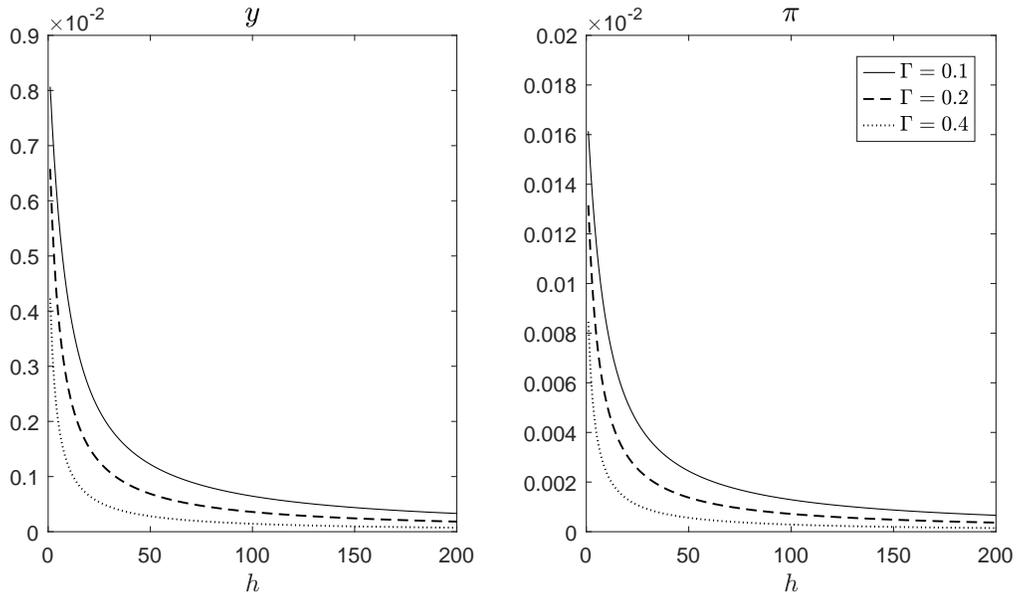


Figure 4: Output and inflation in period 0 with one-time lump-sum transfer financed by a temporary increase in real public debt under different speeds of taxation

Notes: $t = 1$, $\phi_\pi = 1.5$, $\lambda = 0$; h in quarters; no real disturbances.

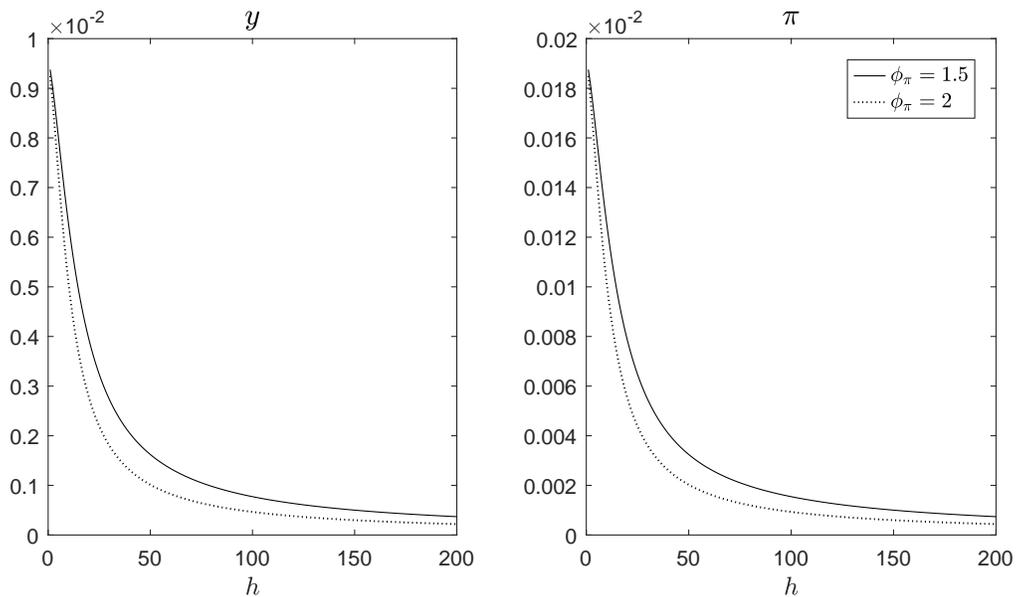


Figure 5: Output and inflation in period 0 with a one-time lump-sum transfer financed by a temporary increase in real public debt under accommodative monetary policy

Notes: $t = 1$, $\Gamma = 0.025$, $\lambda = 0$; h in quarters; no real disturbances.

a low speed of tax collections $\Gamma = 0.025$ (i.e., the half-life of real public debt is around 45 quarters), Figure 5 shows the impact of alternative specifications of monetary policy for the effect of fiscal transfer in period $t = 0$. Similar to the discussion in the previous section, how accommodative monetary policy is matters most for the effect of fiscal policy when decision makers have an intermediate degree of foresight.

4.2 Forward Guidance and Interaction with Lump-sum Transfer

Now, I turn to the discussion of unconventional monetary policy, namely, “forward guidance,” and discuss how fiscal stimulus can add to the unconventional monetary policy. First, with inactive fiscal policy (i.e., the level of real public debt remains unchanged over time), the effect of forward guidance is rebated when decision makers are more short-sighted. The intuition is that, since forward guidance stimulates the output and inflation through foresight, as households and firms are less forward-looking, the stimulative effect of this unconventional monetary policy is more limited. Therefore, the model of finite forward planning provides a natural explanation for the “forward guidance puzzle” (e.g., Del Negro, Giannoni, and Patterson, 2015). Woodford (2018) has a detailed discussion on the effect of forward guidance under finite planning horizon (which abstracts from fiscal sector). Appendix H gives quantitative examples of the forward guidance policy with respect to different lengths of planning horizon.

Quantitatively, borrowing from Gust, Herbst, and López-Salido (2019), the (average) planning horizon in the US is estimated to be one quarter. With the assumption of a steady-state inflation at $\bar{\Pi} = 2\%$ and the calibration from Table 1, a commitment of staying at the effective zero lower bound by the central bank for $T = 10$ quarters has a limited effect in stimulating output and inflation; that is, the response of output and inflation in the period of the policy announcement for forward guidance is 1.0% and 0.04%, respectively. As shown in Appendix H, the central bank’s commitment to stay longer at the effective zero lower bound is of little effect due to the short enough planning horizon. Therefore, it leaves a demand for fiscal stimulus when the (average) planning horizon is short.

More importantly, if a fiscal stimulus through lump-sum transfer is imposed simultaneously with forward guidance, can it achieve anything more than a simple summation of the two? For illustration, consider the policy experiment of the forward guidance as proposed in García-Schmidt and Woodford (2015) and Woodford (2018) together with a one-time lump-sum transfer fully financed by real public debt and the real public debt being unchanged thereafter.

Specifically, suppose prior to period $t = 0$, the economy stays at the steady-state equi-

librium. The central bank announces in period $t = 0$ that from this date to some future date $t = T$, monetary policy will follow the rule of $\phi_\pi = 0$ with $i_t^* = i^* < 0$, and at date $t = T$, the monetary rule will revert back to the “normal” policy reaction function $\hat{i}_t = \phi_\pi \pi_t$. This policy experiment mimics the situation in which the central bank sets the nominal interest rate at the effective zero lower bound for a fixed time, and the negative i^* represents that the nominal interest rate at the effective zero lower bound is smaller than the steady-state inflation rate $\bar{\Pi}$ (target rate). Further suppose a (lump-sum) fiscal stimulus simultaneously happens in period $t = 0$ as discussed in Section 4.1.1; that is, a one-time lump-sum transfer is introduced that is fully financed by debt in period $t = 0$, and then the real public debt is kept constant thereafter (implying $\Gamma = \lambda = 1$). For simplicity, I assume no real disturbances occur.

The fiscal policy ensures the real public debt is kept at $b_t = b^* = t^*$ since period $t = 0$. Then, the equilibrium starting from period $t = T$ is the one described in Section 4.1.1. From the expression (4.6)-(4.7), the endogenous output and inflation for any period $t \geq T$ is given by

$$y_t = \frac{(1 - \beta\rho)(1 - \rho)(1 - \beta)}{(1 - \beta\rho)(1 - \rho) + \kappa\sigma(\phi_\pi - \rho)} t^*$$

$$\pi_t = \frac{\kappa(1 - \rho)(1 - \beta)}{(1 - \beta\rho)(1 - \rho) + \kappa\sigma(\phi_\pi - \rho)} t^*$$

For any period $0 \leq t < T$, from the expression of (4.1)-(4.2) in Section 4.1.1, the system of equations capturing the evolution of the equilibrium is given by

$$x_t = \rho M E_t x_{t+1} + N u^* + N s^*$$

where $x_t = \begin{bmatrix} y_t & \pi_t \end{bmatrix}^T$, $u^* = \begin{bmatrix} -\sigma i^* & 0 \end{bmatrix}^T$, and $s^* = \begin{bmatrix} (1 - \rho)(1 - \beta)b^* & 0 \end{bmatrix}^T$.⁵⁵

It yields a unique solution for all $0 \leq t < T$, that is,

$$x_t = x_T + [I + \rho M + \dots + (\rho M)^{T-t-1}] N u^* \tag{4.8}$$

$$+ [(\rho M)^{T-t} - I] x_T + [I + \rho M + \dots + (\rho M)^{T-t-1}] N s^*$$

Notably, the first term in expression (4.8) solely comes from the fiscal transfer, and the second term solely comes from the policy of forward guidance. The third and fourth terms come from the interaction of lump-sum transfer and forward guidance. The summation of the third and fourth term is proved to be always positive. Proofs can be found in Appendix

⁵⁵The matrices M and N are defined as $M = \frac{1}{1 + \kappa\sigma\phi_\pi} \begin{bmatrix} 1 & -\sigma\phi_\pi \\ \kappa & 1 \end{bmatrix} \begin{bmatrix} 1 & \sigma \\ 0 & \beta \end{bmatrix}$ and $N = \frac{1}{1 + \kappa\sigma\phi_\pi} \begin{bmatrix} 1 & -\sigma\phi_\pi \\ \kappa & 1 \end{bmatrix}$ with $\phi_\pi = 0$.

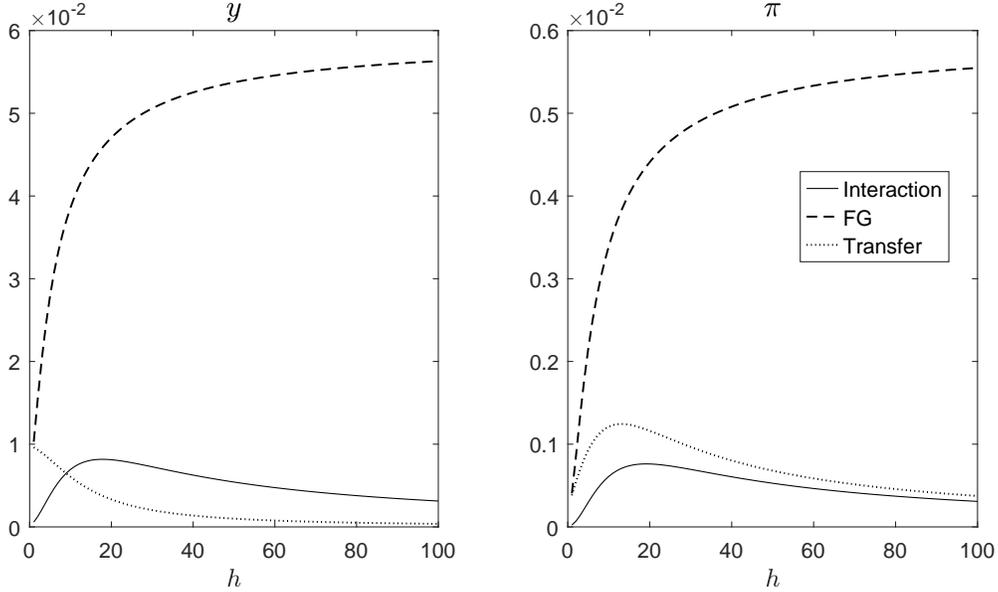


Figure 6: Output and inflation in period 0 regarding the planning horizon with forward guidance and one-time lump-sum transfer financed by a permanent increase in real public debt

Notes: $t = 1$, $\phi_\pi = 1.5$, $\lambda = 1$; h in quarters; no real disturbances.

I. That is, the unconventional monetary policy together with a one-time debt-financed lump-sum transfer has a larger stimulative effect than a simple summation of the two.

Furthermore, the positive gain from the interaction of the two policies, that is, the summation of the third and fourth term in the expression (4.8), is not linear in terms of the planning horizon. By the calibration from Section 4.1.1, Figure 6 shows the effect of interaction between lump-sum transfer and forward guidance in the period $t = 0$ with respect to the (average) length of planning horizon. The solid line represents the interaction between lump-sum transfer and forward guidance, the dashed line represents the effect solely coming from forward guidance, and the dotted line represents the effect solely coming from lump-sum transfer.

From Figure 6, when the (average) length of the planning horizon is around five years (i.e., $h = 20$ quarters), the positive stimulative effect from policy interaction is the highest in the solid line. In either the case of too short a planning horizon or a really long horizon, the effect of the interaction is small. The intuition is similar to the discussion in Section 4.1.1, as the planning horizon increases, monetary policy becomes more effective, while fiscal policy becomes less powerful. The amplification from monetary policy on fiscal stimulus initially plays a dominating role in the situation of a short horizon, and then the effect of policy interaction decreases due to the fiscal policy being more Ricardian-equivalent. Thus, the gain from policy interaction shows a hump shape with respect to the degree of foresight.

5 Long-run Dynamics of Fiscal Transfer Policy

In this section, I review the long-run consequences of those fiscal stimulus policies considered in the previous Section 4 (under the policy regime of “AM/PF” with long-run stability) by incorporating a learning process in decision makers’ value function as modeled in Section 3. Intuitively, as the decision makers adjust their value function more quickly, the behaviors of the equilibrium under the regime of “AM/PF” will be more Ricardian-equivalent. In the data, however, as suggested by Gust, Herbst, and López-Salido (2019), the learning process in the value function in the US is slow as measured by the gain in the learning process $\gamma = \tilde{\gamma} = 0.13$. That is, although the short-term effects of those policy experiments on output and inflation analyzed in Section 4 are rebated in the long run due to the learning in the value function, it is still quite persistent and quantitatively important.

Consider the fiscal stimulus of lump-sum transfer as specified in Section 4 with no real disturbances, and the monetary policy is specified by the Taylor rule. From the equation (3.27) in Section 3.3, which incorporates the constant-gain learning process in the value function, the system of equations characterizing the aggregate equilibrium since period $t = 1$ can be written as

$$E_t \begin{bmatrix} x_{t+1} \\ \bar{x}_{t+1} \\ b_{t+1} \end{bmatrix} = \Upsilon \begin{bmatrix} x_t \\ \bar{x}_t \\ b_t \end{bmatrix} + \begin{bmatrix} B^* \\ 0 \\ \Gamma b^* \end{bmatrix}$$

and in period $t = 0$, it gives⁵⁶

$$E_0 \begin{bmatrix} x_1 \\ \bar{x}_1 \\ b_1 \end{bmatrix} = \Upsilon \begin{bmatrix} x_0 \\ \bar{x}_0 \\ b_0 \end{bmatrix} + \begin{bmatrix} B^* \\ 0 \\ \Gamma b^* \end{bmatrix} + \begin{bmatrix} T^* \\ 0 \\ (1 - \Gamma)t^* \end{bmatrix} \quad (5.1)$$

Before moving to the quantitative analysis, the feature of a long-run stationary equilibrium proceeds with the following proposition:

Proposition 3. *Given the Taylor rule and lump-sum taxation scheme with fiscal stimulus, and gains in updating value function being positive $\gamma, \tilde{\gamma} > 0$, if a stationary equilibrium exists in the long-run, output gap and inflation in such an equilibrium have to satisfy $y = \pi = 0$, and the nominal interest rate satisfies $\bar{i} = 0$.*

Proofs of Proposition 3 can be found in Appendix J. Proposition 3 indicates that if

⁵⁶The matrix B^* and T^* are given by $B^* = \begin{bmatrix} \rho^{-1} & -\sigma(\beta\rho)^{-1} \\ 0 & (\beta\rho)^{-1} \end{bmatrix} \begin{bmatrix} -(1-\rho)(1-\beta)\psi_{b^*}b^* \\ 0 \end{bmatrix}$ and $T^* = \begin{bmatrix} \rho^{-1} & -\sigma(\beta\rho)^{-1} \\ 0 & (\beta\rho)^{-1} \end{bmatrix} \begin{bmatrix} -(1-\rho)(1-\beta)\psi_{t^*}t^* \\ 0 \end{bmatrix}$.

the equilibrium in the long run is stationary, the output gap and inflation have to converge back to the initial steady-state level before policy changes. The intuition is that after a long enough time with learning, agents will finally get the correct value function by incorporating the policy changes, and the long-run real rate will go back to the one before policy changes.

In terms of quantitative analysis, besides those parameters calibrated in Section 4.1.1, the average length of the planning horizon measured by ρ is set to be $\rho = 0.5$ (e.g., Gust, Herbst, and López-Salido, 2019). The coefficient before inflation in the Taylor rule is set to be $\phi_\pi = 1.5$. When no update occurs in the value function $\gamma = \tilde{\gamma} = 0$, the above system of equations becomes the same one as in Section 4.1 for analyzing the short-term effects. Thus, the analyses in Section 4 are nested in the discussion with a learning process in the value function.

Figure 7 shows the policy of one-time debt-financed lump-sum transfer, and the real public debt is kept unchanged thereafter (i.e., the policy experiment considered in Section 4.1.1). The solid line in Figure 7 indicates the case of no update in the value function $\gamma = \tilde{\gamma} = 0$, the dashed line indicates the case of a small gain in learning of the value function $\gamma = \tilde{\gamma} = 0.13$ as estimated in Gust, Herbst, and López-Salido (2019) by matching to US data, and the dotted line indicates a large gain in learning of the value function.⁵⁷

In the case of no update in the value function, the output and inflation permanently increase due to both the one-time lump-sum transfer and the permanent increase in real public debt. As the size of the lump-sum transfer equals one-quarter GDP, the permanent increases in output and inflation are about 0.9% and 0.04%, respectively. The fiscal-transfer multiplier is around 0.94, which is close to one.

When there is non-zero gain of learning in the value function as shown in the dashed line and solid line, the responses of output and inflation in the initial period are similar as in the case of no update in the value function. But the effect of fiscal policy on aggregate output and inflation converges back to zero in the long run, as shown in subfigures (a) and (b). The reason is that the decision makers gradually update their value function, and the value function finally converges back to the one learned from the steady-state stationary equilibrium after a long enough time.⁵⁸

When the policy regime is set to be within the canonical policy regime of “AM/PF”, the standard New Keynesian model (e.g., Woodford, 2003) suggests fiscal policy should have no impact in determining output and inflation. By contrast, Figure 7 indicates fiscal policy under finite forward planning, together with monetary policy, always *jointly* determines the

⁵⁷The gain of leaning in value function is set to be the same across households and firms, i.e., $\gamma = \tilde{\gamma}$.

⁵⁸The oscillating feature of the aggregate variables as shown in Figure 7 comes from the backward looking behavior through the learning in the value function.

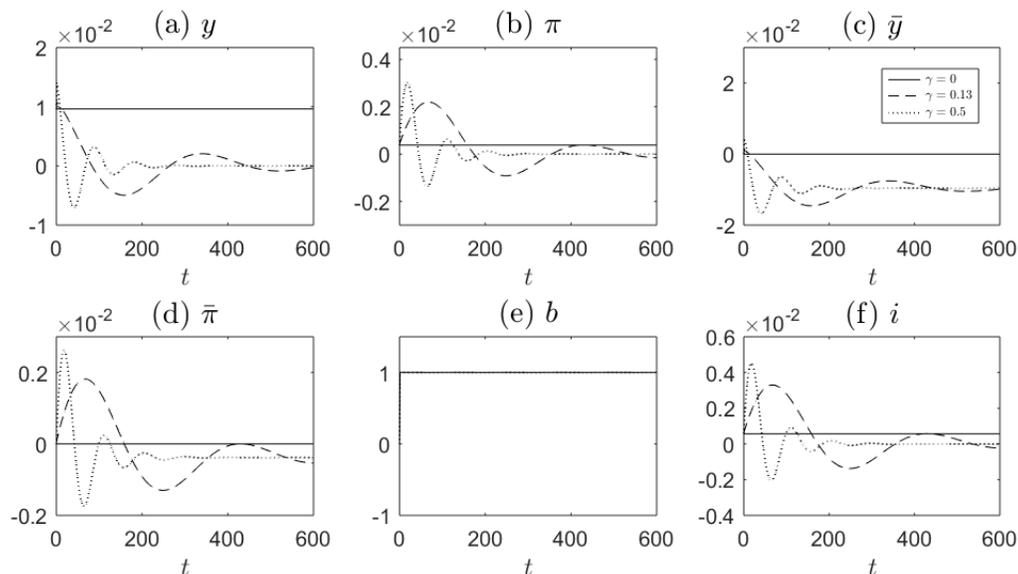


Figure 7: Long-run dynamics with learning in the value function under a one-time debt-financed lump-sum transfer

Notes: $\Gamma = 1$, $\lambda = 1$, $t = 1$, $\phi = 1.5$, $\rho = 0.5$; t in quarters.

output and inflation.

The slower the speed of updating in the value function is, the more persistent the effect of fiscal stimulus becomes, and suggests a larger fiscal-transfer multiplier. In the case of a large gain in the learning process of value function, as shown in the dotted line in Figure 7, the effect of lump-sum transfer disappears after nearly 75 years, or $t = 300$ quarters. In the dashed line calibrated to US data, indicating a small gain in the learning process of the value function, the effect of fiscal stimulus takes more than one hundred years to fade away. The fiscal-transfer multiplier is about 0.27 for the case of a small gain of learning in the value function (dashed line), and about 0.06 for the case of a large gain (dotted line).

While Figure 7 shows the policy experiment with a permanent increase in the real public debt, Figure 8 shows the policy of a one-time lump-sum transfer *temporarily* (fully) financed by debt, and then lump-sum taxes are collected in each period to repay the initial transfer such that the long-run level of public debt does not change (i.e., the policy experiment discussed in Section 4.1.2 with a non-zero steady-state level of real public debt $s_b = 2.4$). In Figure 8, monetary policy is set to be the same under the Taylor rule ($\phi_\pi = 1.5$), and the speed of tax collections is set to be relatively small, namely, $\Gamma = 0.025$. Different from Figure 7, due to the policy specification that the long-run target of real public debt does not change, the path of real public debt as shown in subfigure (e) of Figure 8 converges to its long-run target zero. The subfigures (a) and (b) show that the output and inflation also

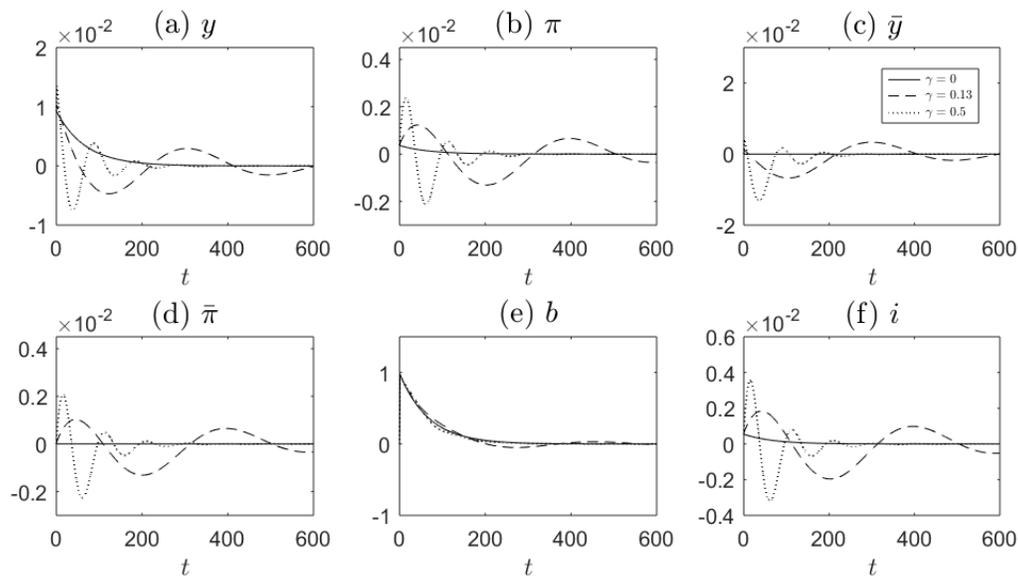


Figure 8: Long-run dynamics with learning in the value function under a one-time debt-financed lump-sum transfer and no change in the long-run debt target

Notes: $\Gamma = 0.025$, $\lambda = 0$, $t = 1$, $\phi = 1.5$, $\rho = 0.5$, $s_b = 2.4$; t in quarters.

converge back to zero in the long run. By comparing the three lines, we see that, as the speed of learning in the value function is slower, the effect of fiscal stimulus becomes more persistent.

6 Conclusions

This paper studies the effect of fiscal policy, specified by a lump-sum taxation scheme, in affecting output and inflation, and its interaction with monetary policy. In contrast to the standard New Keynesian model with rational expectations, fiscal policy and monetary policy always *jointly* determine the output and inflation under finite forward planning. Ricardian equivalence always breaks down, and “Ricardian” fiscal policy no longer exists. With an endogenous evolution of real public debt, as the length of the planning horizon becomes shorter, the policy space for long-run stability under active monetary policy with passive fiscal policy regime (“AM/PF”) increases, and the policy space of “PM/AF” decreases.⁵⁹ The boundary condition for fiscal policy between the two scenarios almost does not change with respect to the degree of foresight, whereas the boundary condition of monetary policy rests heavily on the length of the planning horizon. More importantly, if the government

⁵⁹As decision makers become less forward-looking, the “Taylor principle” for monetary policy becomes more relaxed.

and the central bank do not know the actual planning horizon of the population, however, adopting a policy combination that satisfies the canonical “AM/PF” regime is more robust to ensure long-run stability.

Under the regime of “AM/PF” with long-run stability, this paper then evaluates the effect of fiscal transfers as a source of demand stimulus in both the short run and long run with an emphasis on its interaction with monetary policy. In general, as the length of the planning horizon becomes longer, the effect of monetary policy in stimulating output and inflation decreases because monetary policy works through forward-looking behavior. On the contrary, fiscal stimulus becomes much more powerful as decision makers become more short-sighted. The reason is that, agents take more near-future taxation into today’s decision-making, but not include those taxation in the far future.

Notably, more accommodative monetary policy improves the stimulative effects of fiscal stimulus. But the impact of monetary policy accommodation on the effect of fiscal policy is hump-shaped with respect to the length of the planning horizon.

In addition, the finite-planning-horizon model provides a natural explanation for the “forward guidance puzzle.” The limited effect of monetary policy in stimulating output and inflation generates a demand for fiscal stimulus. This paper suggests an unconventional monetary policy of forward guidance combined with a simultaneous fiscal stimulus such as debt-financed lump-sum transfer can reach an aggregate effect larger than a simple summation of the two. The effect of a positive interaction between the two policies is maximized also when decision makers have an intermediate degree of foresight.

In terms of long-run consequences of those fiscal stimulus through lump-sum transfers, it is initially powerful in stimulating output and inflation. As agents update their value function to incorporate the effects of such stimulus, the stimulative effect dampens over time. As the learning process in decision makers’ value function is slower, the responses of output and inflation become more persistent. Nevertheless, the quantitative analysis shows that the effect of fiscal stimulus is quite persistent even in the case of a relatively large gain in updating the value function.

In a world of widespread high-level debt and lower nominal interest rates in many countries, fiscal policy and government debt have become a more important issue to study. This paper inspires several directions for future research. For instance, Blanchard (2019) suggests a lower natural rate of interest in the long run. Exploring the implications of this observation with finite forward planning would be interesting in future work.

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Appendix

A The System of Equations for Heterogeneous Agents with No Update in the Value Function

Given the assumptions imposed in Section 2.5, suppose in each period $\omega_j = \tilde{\omega}_j = (1 - \rho)\rho^h$ fraction of households and firms have planning horizon- h with $0 < \rho < 1$. For any $h \geq 1$ in period t , the equations of the IS and Phillips curve yield

$$y_t^h - g_t = E_t[y_t^{h-1} - g_{t+1}] - \sigma[\hat{i}_t^h - E_t\pi_{t+1}^{h-1}] \quad (\text{A.1})$$

$$\pi_t^h = \kappa[y_t^h - y_t^*] + \beta E_t\pi_{t+1}^{h-1} \quad (\text{A.2})$$

and for $h = 0$,

$$y_t^0 - g_t = -\sigma\hat{i}_t^0 + (1 - \beta)b_{t+1}^0 \quad (\text{A.3})$$

$$\pi_t^0 = \kappa[y_t^0 - y_t^*] \quad (\text{A.4})$$

where the variable b_{t+1}^0 is the real asset position at the end of the planning horizon for households with horizon $h = 0$. The expression of the variable b_{t+1}^0 will be derived later.

By averaging these equations (A.1)-(A.2) across agents, it follows that

$$y_t - g_t = \rho E_t(y_{t+1} - g_{t+1}) - \sigma(\hat{i}_t - \rho E_t\pi_{t+1}) + (1 - \rho)(1 - \beta)b'_t$$

$$\pi_t = \kappa(y_t - y_t^*) + \beta\rho E_t\pi_{t+1}$$

with the rule of nominal interest rate and the evolution of real public debt

$$\hat{i}_t = i_t^* + \phi_{\pi,t}\pi_t$$

$$b_{t+1} = \beta^{-1}(1 - \Gamma)b_t - \beta^{-1}(1 - \Gamma)s_b\pi_t + (1 - \Gamma)s_b\hat{i}_t$$

where $b'_t = b_{t+1}^0$.

Now, I derive the expression of the variable b_{t+1}^0 as follows. Given the aggregate (average) real public debt b_t in period t and the assumptions imposed in Section 2.5, the decision makers with horizon $h = 0$ start their planing exercise with initial asset b_t in each period t . Their forward planning problem is then characterized by equations (A.3)-(A.4) with the rule of nominal interest rate and the evolution of real public debt

$$\hat{i}_t^0 = i_t^* + \phi_{\pi,t}\pi_t^0$$

$$b_{t+1}^0 = \beta^{-1}(1 - \Gamma)b_t - \beta^{-1}(1 - \Gamma)s_b\pi_t^0 + (1 - \Gamma)s_b\hat{i}_t^0$$

Thus, from the four linear system of equations, the variable b_{t+1}^0 is a function of b_t and exogenous disturbances, i.e.,

$$b'_t \equiv b_{t+1}^0 = \psi_{b,t}b_t + \psi_{g,t}g_t + \psi_{y,t}y_t^* + \psi_{i,t}i_t^*$$

where the parameter $\psi_{b,t}$ is given by

$$\psi_{b,t} = \beta^{-1}(1 - \Gamma)/[1 - (\phi_{\pi,t} - \beta^{-1})s_b(1 - \Gamma)\frac{(1 - \beta)\kappa}{1 + \sigma\phi_{\pi,t}\kappa}]$$

The expression of parameters $\{\psi_{g,t}, \psi_{y,t}, \psi_{i,t}\}$ can be easily solved, but only the parameter $\psi_{b,t}$ is of special interest.

The system of equations characterizing the aggregate equilibrium can be re-written as

$$E_t \begin{bmatrix} x_{t+1} \\ b_{t+1} \end{bmatrix} = \begin{bmatrix} A & C \\ D & \beta^{-1}(1 - \Gamma) \end{bmatrix} \begin{bmatrix} x_t \\ b_t \end{bmatrix} + \begin{bmatrix} K \\ 0 \end{bmatrix} \begin{bmatrix} g_t \\ y_t^* \\ i_t^* \end{bmatrix}$$

where $x_t = [y_t - g_t \quad \pi_t]^T$, K is a 2×3 matrix of less interest, and the matrices A , C , and D are given by

$$A = \begin{bmatrix} \rho^{-1} & -\sigma(\beta\rho)^{-1} \\ 0 & (\beta\rho)^{-1} \end{bmatrix} \begin{bmatrix} 1 & \sigma\phi_{\pi,t} \\ -\kappa & 1 \end{bmatrix} = \begin{bmatrix} \rho^{-1} + \kappa\sigma(\beta\rho)^{-1} & \sigma\rho^{-1}(\phi_{\pi} - \beta^{-1}) \\ -\kappa(\beta\rho)^{-1} & (\beta\rho)^{-1} \end{bmatrix}$$

$$C = \begin{bmatrix} \rho^{-1} & -\sigma(\beta\rho)^{-1} \\ 0 & (\beta\rho)^{-1} \end{bmatrix} \begin{bmatrix} -(1 - \rho)(1 - \beta)\psi_b \\ 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & (\phi_{\pi,t} - \beta^{-1})(1 - \Gamma)s_b \end{bmatrix}$$

B Aggregation across the Population for Characterizing Aggregate “Trend” Variables

From the system of equations (3.18a)-(3.18d) in Section 3.3, by aggregating across the whole population for the “trend” variables, it follows

$$\bar{y}_t = \rho\bar{y}_t - \sigma[\bar{l}_t - \rho\bar{\pi}_t] + (1 - \rho)v_t + (1 - \rho)(1 - \beta)\bar{b}_{t+1}^0 \quad (\text{B.1a})$$

$$\bar{\pi}_t = \kappa \bar{y}_t + \beta \rho \bar{\pi}_t + (1 - \rho)(1 - \alpha) \beta \tilde{\nu}_t \quad (\text{B.1b})$$

$$\bar{i}_t = \phi_\pi \bar{\pi}_t \quad (\text{B.1c})$$

where the variable \bar{b}_{t+1}^0 is the ending period asset position in the planning exercise of households with horizon $h = 0$. To derive \bar{b}_{t+1}^0 , for decision makers with $h = 0$, the planning exercise for “trend” components is given by

$$\bar{y}_t^0 = -\sigma \bar{i}_t^0 + (1 - \beta) \bar{b}_{t+1}^0 + \nu_t$$

$$\bar{\pi}_t^0 = \kappa \bar{y}_t^0 + (1 - \alpha) \beta \tilde{\nu}_t$$

$$\bar{i}_t^0 = \phi_\pi \bar{\pi}_t^0$$

$$\bar{b}_{t+1}^0 = \beta^{-1}(1 - \Gamma) \bar{b}_t^1 - \beta^{-1}(1 - \Gamma) s_b \bar{\pi}_t^0 + (1 - \Gamma) s_b \bar{i}_t^0$$

where $\bar{b}_t^1 = 0$. The above four linear equations yields $\bar{b}_{t+1}^0 = \psi_\nu \nu_t + \psi_{\tilde{\nu}} \tilde{\nu}_t$ with ψ_ν and $\psi_{\tilde{\nu}}$ given by

$$\psi_\nu = (1 - \Gamma) s_b [\phi_\pi - \beta^{-1}] \kappa / [1 + \sigma \phi_\pi \kappa - (\phi_\pi - \beta^{-1})(1 - \Gamma) s_b (1 - \beta) \kappa]$$

$$\psi_{\tilde{\nu}} = (1 - \Gamma) s_b [\phi_\pi - \beta^{-1}] (1 - \alpha) \beta / [1 + \sigma \phi_\pi \kappa - (\phi_\pi - \beta^{-1})(1 - \Gamma) s_b (1 - \beta) \kappa]$$

C The System of Equations for the Equilibrium under Heterogeneous Agents with Learning in the Value Function

From Section 3.3, the system of equations for the whole equilibrium, i.e., equations (3.24), (3.25), and (3.26), can be summarized as

$$\begin{aligned} E_t \begin{bmatrix} x_{t+1} \\ \bar{x}_{t+1} \\ b_{t+1} \end{bmatrix} &= \begin{bmatrix} I & -I & 0 \\ 0 & I & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} A & -A & C \\ F & G & H \\ D & 0 & \beta^{-1}(1 - \Gamma) \end{bmatrix} \begin{bmatrix} x_t \\ \bar{x}_t \\ b_t \end{bmatrix} + \begin{bmatrix} I & -I & 0 \\ 0 & I & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} K u_t \\ 0 \\ 0 \end{bmatrix} \\ &= \Upsilon \begin{bmatrix} x_t \\ \bar{x}_t \\ b_t \end{bmatrix} + \begin{bmatrix} K u_t \\ 0 \\ 0 \end{bmatrix} \end{aligned}$$

where Υ is given by

$$\Upsilon = \begin{bmatrix} A + F & -A + G & C + H \\ F & G & H \\ D & 0 & \beta^{-1}(1 - \Gamma) \end{bmatrix}$$

D Proof for the Determinacy Condition and Convergence Condition with Taylor Rule and Inactive Fiscal Policy

Note that the expression of A is given in Section 3.3 as

$$A = \begin{bmatrix} \rho^{-1} & -\sigma(\beta\rho)^{-1} \\ 0 & (\beta\rho)^{-1} \end{bmatrix} \begin{bmatrix} 1 & \sigma\phi_\pi \\ -\kappa & 1 \end{bmatrix}$$

The eigenvalues of A satisfy the following second-order polynomial

$$f(\lambda) \equiv (\rho\lambda)^2 - [\beta^{-1} + 1 + \kappa\sigma\beta^{-1}](\rho\lambda) + (\beta^{-1} + \kappa\sigma\phi\beta^{-1}) = 0$$

Then, $f(\lambda)$ has two eigenvalues outside unit circle if and only if $f(1) > 0$, which is equivalent to $\phi > -\frac{1}{\kappa\sigma}[\beta\rho^2 - \rho(1 + \beta + \kappa\sigma) + 1] \equiv l(\rho)$. **Q.E.D.**

It can also be proved that the necessary and sufficient condition for the convergence of $\Sigma\omega_h y^h$ and $\Sigma\tilde{\omega}_h \pi^h$ in a given period is that two eigenvalues of A are outside the unit circle.

Proof: For simplicity, assume no exogenous disturbances. Note that, in a given period t , for $\forall h \geq 1$, it follows from Section 3.2 that

$$x^h = (\rho A)^{-1} x^{h-1}$$

where $x^h = \begin{bmatrix} y^h & \pi^h \end{bmatrix}^T$.

In order for $\Sigma\omega_h y^h$ and $\Sigma\tilde{\omega}_h \pi^h$ to converge, it is equivalent to the condition that the growth rate in x^h is smaller than ρ^{-1} for large enough h . It is then equivalent to the condition that there are two eigenvalues of A outside the unit circle. **Q.E.D.**

E Convergence Condition of $\Sigma\omega_h y^h$ and $\Sigma\tilde{\omega}_h \pi^h$ with an Endogenous Fiscal Rule

Following the assumption in Section 3.3, in period t , the group of households with planning horizon- h start with the initial asset position b_t in their planning exercise. By the definition of “trend” variables in Section 3.3, the aggregation across agents for the “trend” variables always converges. Then, it remains to only focus on the “deviation” components. Since the “deviation” variables are those in the case of no update in the value function throughout the planning exercise, the convergence condition of the aggregate endogenous variables is then equivalent to that in Section 2.5 in which there is no update in the value function.

Therefore, I focus on the convergence condition in Section 2.5. For simplicity, assume no real disturbances, and the planning problem for large enough h in period t yields (from Section 2.4)

$$\begin{aligned} \begin{bmatrix} x^h \\ b^{h+1} \end{bmatrix} &= \begin{bmatrix} \rho A & 0 \\ D & \beta^{-1}(1 - \Gamma) \end{bmatrix}^{-1} \begin{bmatrix} x^{h-1} \\ b^h \end{bmatrix} \\ &= \Upsilon^* \begin{bmatrix} \rho^{-1} x^{h-1} \\ b^h \end{bmatrix} \end{aligned} \tag{E.1}$$

where $x^h = [y^h \ \pi^h]^T$ and $\Upsilon^* = \begin{bmatrix} A^{-1} & 0 \\ [\beta^{-1}(1 - \Gamma)]^{-1} D A^{-1} & [\beta^{-1}(1 - \Gamma)]^{-1} \end{bmatrix}$.

In order for $\Sigma\omega_h y^h$ and $\Sigma\tilde{\omega}_h \pi^h$ to converge, it is equivalent to the condition that the growth rate in x^h is smaller than ρ^{-1} for large enough h . It is then equivalent to the condition that either (i) two eigenvalues of A are outside the unit circle, i.e., $\phi_\pi > l(\rho)$, with $|\beta^{-1}(1 - \Gamma)| < 1$, or (ii) only one eigenvalue of A is outside the unit circle, i.e., $\phi_\pi < l(\rho)$, with $|\beta^{-1}(1 - \Gamma)| > 1$.⁶⁰ These two conditions are the shaded areas within the dotted lines in Figure 1. It can be numerically verified that, in other cases as in the blank area of Figure 1, the aggregation across agents does not converge.

⁶⁰A more rigorous method to study the convergence condition across agents is to assume $y^h = \gamma_y^h b^{h+1}$ and $\pi^h = \gamma_\pi^h b^{h+1}$ for any $h \geq 0$. It can be easily showed that $\gamma_y^0 = \frac{(1-\beta)\beta^{-1}(1-\Gamma)}{1+\sigma\phi_\pi\kappa-(1-\beta)s_b(\phi_\pi-\beta^{-1})(1-\Gamma)\kappa}$ and $\gamma_\pi^0 = \kappa\gamma_y^0$. Then, the forward-planning problem for any agent with horizon $h \geq 1$ implies that $\begin{bmatrix} \gamma_y^h \\ \gamma_\pi^h \end{bmatrix} = [\beta^{-1}(1 - \Gamma) + (\phi_\pi - \beta^{-1})s_b(1 - \Gamma)\gamma_\pi^h](\rho A)^{-1} \cdot \begin{bmatrix} \gamma_y^{h-1} \\ \gamma_\pi^{h-1} \end{bmatrix}$, where $(\rho A)^{-1} = \frac{1}{1+\kappa\sigma\phi_\pi} \begin{bmatrix} 1 & -\sigma\phi_\pi \\ \kappa & 1 \end{bmatrix} \begin{bmatrix} 1 & \sigma \\ 0 & \beta \end{bmatrix}$. It can be numerically verified that the convergence condition for $\Sigma\omega_h \gamma_y^h$ and $\Sigma\tilde{\omega}_h \gamma_\pi^h$ is the same as either (i) $\phi_\pi > l(\rho)$ and $|\beta^{-1}(1 - \Gamma)| < 1$, or (ii) $\phi_\pi < l(\rho)$ and $|\beta^{-1}(1 - \Gamma)| > 1$.

F Proof for the Monotonicity of the Responses of Output and Inflation with respect to ϕ_π

Given the expressions of (4.6)-(4.7), since $\phi_\pi > l(\rho)$ and $\rho \in (0, 1)$, it is obvious that the responses of output y and inflation π are strictly decreasing with respect to ϕ_π . Thus, Proposition 1 holds.

Now, let's consider the second-order derivatives, and focus on the response of output. It follows that

$$\begin{aligned} \frac{\partial y}{\partial \phi_\pi} &= -\frac{(1-\beta\rho)(1-\rho)(1-\beta)\lambda t^*}{[(1-\beta\rho)(1-\rho) + \kappa\sigma(\phi_\pi - \rho)]^{-2}} < 0 \\ -\frac{\partial^2 y}{\partial \phi_\pi \partial \rho} &= \frac{\lambda t^* \kappa \sigma (1-\beta)}{[(1-\beta\rho)(1-\rho) + \kappa\sigma(\phi_\pi - \rho)]^{-2}} \\ &\quad \left\{ -(1+\beta-2\beta\rho) + 2\frac{(1-\beta\rho)(1-\rho)(1+\beta+\kappa\sigma-2\beta\rho)}{(1-\beta\rho)(1-\rho) + \kappa\sigma(\phi_\pi - \rho)} \right\} \end{aligned}$$

Denote $f(\rho) = 2(1-\beta\rho)(1-\rho)(1+\beta+\kappa\sigma-2\beta\rho)$ and $g(\rho) = [(1-\beta\rho)(1-\rho) + \kappa\sigma(\phi_\pi - \rho)](1+\beta-2\beta\rho)$. Since $\phi_\pi > l(\rho)$, $\beta \in (0, 1)$, and $\rho \in (0, 1)$, both $f(\rho)$ and $g(\rho)$ are strictly decreasing with respect to ρ . Note that $f(1) < g(1)$. Then, due to the monotonicity and continuity of $f(\cdot)$ and $g(\cdot)$, there exists a unique $\bar{\rho} \in (0, 1)$ such that $f(\rho) = g(\rho)$ if and only if $f(0) > g(0)$, which is equivalent to $\phi_\pi < \frac{1}{\kappa\sigma} + \frac{2}{1+\beta}$. Otherwise, if $\phi_\pi \geq \frac{1}{\kappa\sigma} + \frac{2}{1+\beta}$, it follows that $f(\rho) < g(\rho)$ for any $\rho \in (0, 1)$.

Note that $-\frac{\partial^2 y}{\partial \phi_\pi \partial \rho} > 0$ is equivalent to $f(\rho) > g(\rho)$, and vice versa. Therefore, given $\phi_\pi < \frac{1}{\kappa\sigma} + \frac{2}{1+\beta}$, a unique $\bar{\rho} \in (0, 1)$ exists such that $-\frac{\partial^2 y}{\partial \phi_\pi \partial \rho} > 0$ if $\rho \in (0, \bar{\rho})$, and $-\frac{\partial^2 y}{\partial \phi_\pi \partial \rho} < 0$ if $\rho \in (\bar{\rho}, 1)$. Otherwise, if $\phi_\pi \geq \frac{1}{\kappa\sigma} + \frac{2}{1+\beta}$, it follows that $-\frac{\partial^2 y}{\partial \phi_\pi \partial \rho} < 0$ for any $\rho \in (0, 1)$. **Q.E.D.**

G Proof for the Monotonicity of the Responses of Output and Inflation with respect to Γ

Note that $\beta^{-1}(1-\Gamma) \leq 1$ and $0 \leq \Gamma \leq 1$. The expressions of γ_y and γ_π as specified in Section 4.1.2 are given by

$$\begin{aligned} \gamma_y &= \frac{[1-\rho(1-\Gamma)](1-\rho)(1-\beta)\beta^{-1}(1-\Gamma)}{[1-\rho\beta^{-1}(1-\Gamma)][1-\rho(1-\Gamma)] + \kappa\sigma[\phi_\pi - \rho\beta^{-1}(1-\Gamma)]} \geq 0 \\ \gamma_\pi &= \frac{\kappa(1-\rho)(1-\beta)\beta^{-1}(1-\Gamma)}{[1-\rho\beta^{-1}(1-\Gamma)][1-\rho(1-\Gamma)] + \kappa\sigma[\phi_\pi - \rho\beta^{-1}(1-\Gamma)]} \geq 0 \end{aligned}$$

First, the expression of γ_y can be re-written as

$$\begin{aligned}\gamma_y &= \frac{(1-\rho)(1-\beta)\beta^{-1}(1-\Gamma)}{[1-\rho\beta^{-1}(1-\Gamma)] + \frac{\kappa\sigma[\phi_\pi - \rho\beta^{-1}(1-\Gamma)]}{1-\rho(1-\Gamma)}} \\ &= \frac{(1-\rho)(1-\beta)\beta^{-1}(1-\Gamma)}{1-\beta^{-1} + \beta^{-1}\kappa\sigma + \beta^{-1}\{1-\rho(1-\Gamma) + \frac{\kappa\sigma(\phi_\pi\beta-1)}{1-\rho(1-\Gamma)}\}}\end{aligned}$$

If $\phi_\pi\beta < 1$ and note that $1-\rho(1-\Gamma) > 0$, then the term $1-\rho(1-\Gamma) + \frac{\kappa\sigma(\phi_\pi\beta-1)}{1-\rho(1-\Gamma)}$ is strictly increasing with respect to Γ . Since the term $1-\Gamma > 0$ in the numerator of γ_y is strictly decreasing with respect to Γ , γ_y is then strictly decreasing with respect to Γ . If $\phi_\pi\beta > 1$ instead, the term $1-\rho(1-\Gamma) + \frac{\kappa\sigma(\phi_\pi\beta-1)}{1-\rho(1-\Gamma)}$ is strictly increasing with respect to Γ if and only if $1-\rho(1-\Gamma) \geq [\kappa\sigma(\phi_\pi\beta-1)]^{1/2}$, i.e., $\Gamma \geq \frac{[\kappa\sigma(\phi_\pi\beta-1)]^{1/2}-1+\rho}{\rho} \equiv \bar{\Gamma}$. Under such a condition, γ_y is then strictly decreasing with respect to Γ .

For γ_π , since the numerator of γ_π is strictly decreasing with respect to Γ and the denominator is strictly increasing with respect to Γ , it is obvious that γ_π is strictly decreasing with respect to Γ . **Q.E.D.**

H Forward Guidance

Consider the policy experiment of forward guidance as proposed in García-Schmidt and Woodford (2015) and Woodford (2018), and the real public debt is kept at its steady-state level \bar{B} at all times. More specifically, suppose prior to period $t = 0$, the economy stays at the steady-state equilibrium. It is announced in period $t = 0$ that, from this date to some future date $t = T$, monetary policy will follow the rule of $\phi_\pi = 0$ with $i_t^* = i^* < 0$, and at date $t = T$, the monetary rule reverts back to the “normal” policy reaction function $\hat{i}_t = \phi_\pi\pi_t$. This policy experiment mimics the situation in which the central bank sets the nominal interest rate at the effective zero lower bound for a fixed period of time, and negative i^* indicates that the nominal interest rate at the effective zero lower bound is smaller than the steady-state inflation rate $\bar{\Pi}$ (target rate).

Starting from period $t = T$ onward, the endogenous output and inflation for any period $t \geq T$ is given by $y_t = \pi_t = 0$. Then, the system of equations capturing the evolution of the equilibrium for any period $0 \leq t < T$ is given by

$$x_t = \rho M E_t x_{t+1} + N u^*$$

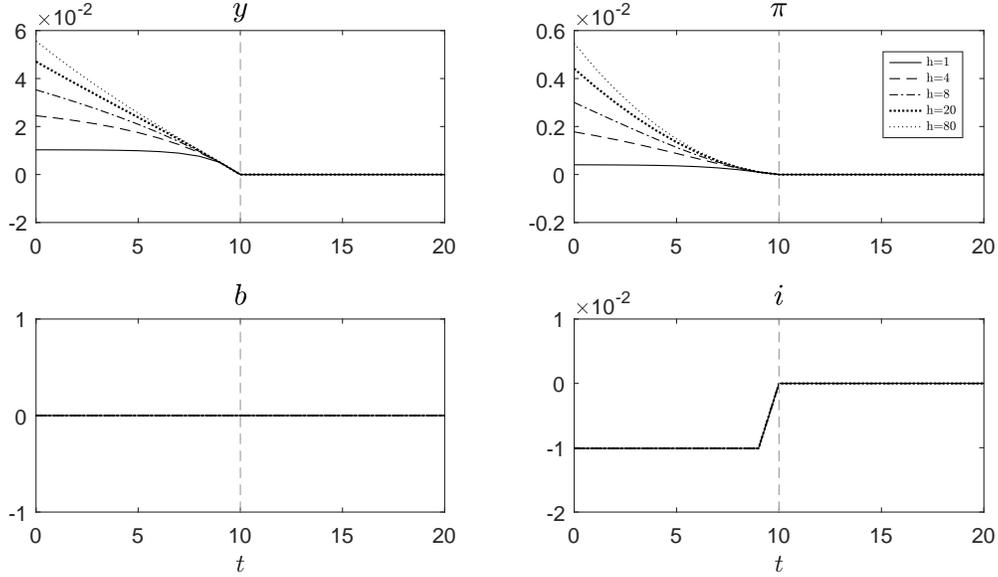


Figure 9: Output and inflation dynamics with different planning horizons under forward guidance

Notes: $T = 10$; h in quarters; no real disturbances.

where $x_t = \begin{bmatrix} y_t & \pi_t \end{bmatrix}^T$, $u^* = \begin{bmatrix} -\sigma i^* & 0 \end{bmatrix}^T$, and $M = \frac{1}{1+\kappa\sigma\phi_\pi} \begin{bmatrix} 1 & -\sigma\phi_\pi \\ \kappa & 1 \end{bmatrix} \begin{bmatrix} 1 & \sigma \\ 0 & \beta \end{bmatrix}$ and $N = \frac{1}{1+\kappa\sigma\phi_\pi} \begin{bmatrix} 1 & -\sigma\phi_\pi \\ \kappa & 1 \end{bmatrix}$ with $\phi_\pi = 0$.

It yields a unique solution for all $0 \leq t < T$, i.e.,

$$x_t = [I + \rho M + \dots + (\rho M)^{T-t-1}] N u^* \quad (\text{H.1})$$

By the calibration in Table 1 with the assumption of a steady-state inflation at $\bar{\pi} = 2\%$, Figure 9 shows the time-series dynamics of output and inflation in response to an interest-rate peg lasting for ten quarters. The lines in the figure indicate the average planning horizon of the decision makers from short to long. Figure 9 shows that, given the interest-rate peg for a fixed length of periods $T = 10$, as households and firms are less forward-looking, the stimulative effect of such a commitment to an interest-rate peg policy, namely forward guidance, becomes less effective. To see how the length of commitment of the forward guidance matters, Figure 10 shows the responses of output and inflation in period $t = 0$ with respect to different planning horizons under alternative lengths of the commitment to an interest-rate peg, i.e., $T = 10$ and $T = 14$, respectively. From Figure 10, when the (average) planing horizon is extremely short, i.e., one quarter as estimated by Gust, Herbst, and López-Salido (2019), the central bank's commitment of staying longer at the effective

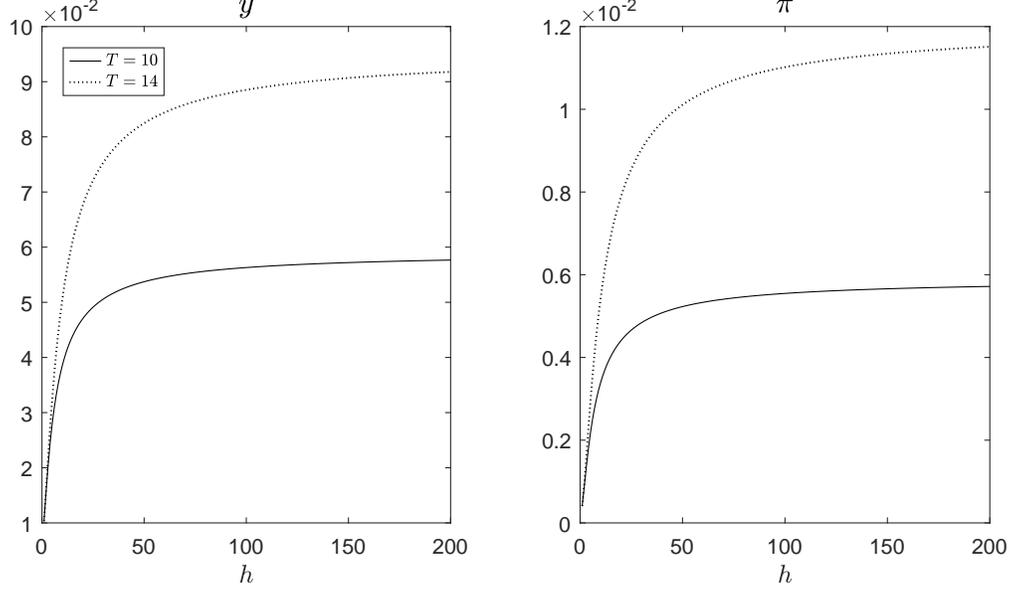


Figure 10: Output and inflation in period 0 as to the planning horizon under forward guidance with different length of commitments

Notes: h in quarters; no real disturbances.

lower bound is of little effect.

I Proof of the Positive Interaction between Forward Guidance and Fiscal Transfer

In this section, I prove the interaction between forward guidance and fiscal transfer, i.e., the summation of the third and fourth terms in the equation (4.8), to be always positive through the method of forward induction. Note that since period $t = T$, the monetary policy reverts back to the normal Taylor rule, and thus it follows $x_T = \rho M^* \cdot x_T + N^* s^*$, where

$$M^* = \frac{1}{1+\kappa\sigma\phi_\pi} \begin{bmatrix} 1 & -\sigma\phi_\pi \\ \kappa & 1 \end{bmatrix} \begin{bmatrix} 1 & \sigma \\ 0 & \beta \end{bmatrix} \text{ and } N^* = \frac{1}{1+\kappa\sigma\phi_\pi} \begin{bmatrix} 1 & -\sigma\phi_\pi \\ \kappa & 1 \end{bmatrix} \text{ with } \phi_\pi > 0.$$

First consider the case of $T = 1$, or the period of $t = T - 1$ for any $T \geq 2$. The interaction term satisfies

$$[\rho M - I]x_T + Ns^* \gg 0$$

$$\iff \rho Mx_T + Ns^* \gg x_T$$

$$\iff \rho[M - M^*]x_T + (N - N^*)s^* \gg 0$$

Since $M - M^* \gg 0$, $N - N^* \gg 0$, $x_T \gg 0$, and $s^* \gg 0$, it follows that the last condition in the above derivation holds.

Then, for any $T \geq 2$, I show that, in any given period $0 \leq t < T - 1$, the interaction term by a commitment for total T periods is larger than that of the commitment for total $T - 1$ periods. Note that we have

$$\begin{aligned}
& [(\rho M)^{T-t} - I]x_T + [I + \rho M + \dots + (\rho M)^{T-t-1}]Ns^* \gg [(\rho M)^{T-t-1} - I]x_T + [I + \rho M + \dots + (\rho M)^{T-t-2}]Ns^* \\
& \iff (\rho M)^{T-t}x_T + (\rho M)^{T-t-1}Ns^* \gg (\rho M)^{T-t-1}x_T \\
& \iff (\rho M)^{T-t-1}[\rho Mx_T + Ns^* - x_T] \gg 0 \\
& \iff (\rho M)^{T-t-1}[\rho(M - M^*)x_T + (N - N^*)s^*] \gg 0
\end{aligned}$$

Since $M - M^* \gg 0$, $M \gg 0$, $N - N^* \gg 0$, $x_T \gg 0$, and $s^* \gg 0$, the last condition in the above derivation holds. Therefore, in any given period $0 \leq t < T - 1$, the interaction term with a commitment for total T periods is larger than that of the commitment for total $T - 1$ periods. Also note that, in period $t = T - 1$ for any $T \geq 2$ and in the case of $T = 1$, the interaction is also positive, and thus the interaction between forward guidance and fiscal transfer is always positive.⁶¹ **Q.E.D.**

J Proof for the Property of a Long-run Stationary Equilibrium

Before proving Proposition 3, let us first consider simpler cases. If the shock or the policy change is temporary, which makes the real public debt converging back to the original steady-state level before the shock happens or policy changes. It is obvious that the equilibrium in the long-run will converge back to the original steady-state stationary equilibrium if the long-run stationary equilibrium exists. Then, I only need to focus on the situations in which the real public debt converges to a new steady-state level in the long run. For simplicity, I first prove Proposition 3 with the fiscal policy that there is a permanent increase in real public debt and the real public debt remains unchanged thereafter ($\Gamma = 1$). Then, I will show that Proposition 3 holds generally.

The monetary policy and fiscal policy are specified as in Section 5. Since there is only one unique (locally) bounded long-run equilibrium under the parameterization of determinacy if a long-run stationary equilibrium exists, I refer to the variables for such a stationary equilibrium by abstracting from time index t .

⁶¹A more rigorous mathematical proof can be done by expanding the matrices in the derivation.

Given $\Gamma = 1$, the equation 3.21a capturing “deviation” variables now becomes

$$y - \bar{y} = \rho(y - \bar{y}) - \sigma[\hat{i} - \bar{i} - \rho(\pi - \bar{\pi})] + (1 - \rho)(1 - \beta)b^*$$

and by equation 3.19a for “trend” variables, it gives

$$\bar{y} = \rho\bar{y} - \sigma(\bar{i} - \rho\bar{\pi}) + (1 - \rho)\nu$$

Given that there is a positive gain of learning in the value function $\gamma, \tilde{\gamma} > 0$, the dynamics of the value-function adjustment yields

$$\nu = y + \sigma\pi - (1 - \beta)b^*$$

By substituting the latter two equations into the first one, it follows that

$$\hat{i} = \pi$$

which indicates that the Fisher equation must hold in the long-run in this environment if the long-run stationary equilibrium exists. Furthermore, because the monetary policy is specified by the Taylor rule, $\hat{i} = \phi_\pi\pi$, it is obvious that $\pi = 0$ as long as $\phi_\pi \neq 1$. Therefore, Proposition 3 holds when $\Gamma = 1$.

Now, I show that Proposition 3 holds generally. By the expressions in Section 3.3, the equations capturing “deviation” variables for the policy specification in Section 5 are given by

$$y - \bar{y} = \rho(y - \bar{y}) - \sigma[\hat{i} - \bar{i} - \rho(\pi - \bar{\pi})] + (1 - \rho)(1 - \beta)(\psi_b b + \psi_{b^*} b^*)$$

$$\pi - \bar{\pi} = \kappa(y - \bar{y}) + \beta\rho(\pi - \bar{\pi})$$

and the equations capturing “trend” variables are given by

$$\bar{y} = \rho\bar{y} - \sigma(\bar{i} - \rho\bar{\pi}) + (1 - \rho)\nu + (1 - \rho)(1 - \beta)(\psi_\nu\nu + \psi_{\tilde{\nu}}\tilde{\nu})$$

$$\bar{\pi} = \kappa\bar{y} + \beta\rho\bar{\pi} + (1 - \rho)(1 - \alpha)\beta\tilde{\nu}$$

By substituting the latter two equations of “trend” variables $\{\bar{y}, \bar{\pi}\}$ into the first two equations, I can get two equations capturing the aggregate output y and inflation π that are fully composed by the aggregate variables $\{y, \pi, b, \hat{i}, \nu, \tilde{\nu}\}$.

Given that there is a positive gain of learning in the value function $\gamma, \tilde{\gamma} > 0$, the

dynamics of the value-function adjustment yields

$$\nu = y + \sigma\pi - (1 - \beta)b^*$$

$$\tilde{\nu} = (1 - \alpha)^{-1}\pi$$

Together with the debt evolution $b = \beta^{-1}(1 - \Gamma)b - \beta^{-1}(1 - \Gamma)s_b\pi + (1 - \Gamma)s_b\hat{i} + \Gamma b^*$, the rule of nominal interest rate $\hat{i} = \phi_\pi\pi$, there is a system of six equations characterizing the aggregate variables $\{y, \pi, b, \hat{i}, \nu, \tilde{\nu}\}$, and there is a unique solution for this system.

From the expressions of $\{\psi_b, \psi_{b^*}, \psi_\nu, \psi_{\tilde{\nu}}\}$, it can be easily verified that the solution to this system is given by

$$y = \pi = \hat{i} = 0, b = \frac{\Gamma b^*}{1 - \beta^{-1}(1 - \Gamma)}$$

$$\nu = -\frac{(1 - \beta)\Gamma b^*}{1 - \beta^{-1}(1 - \Gamma)}, \tilde{\nu} = 0$$

where I have employed the relationship that $\frac{\psi_{b^*}}{\psi_{b-1-(1-\beta)\psi_\nu}} = -\frac{\Gamma}{1-\beta^{-1}(1-\Gamma)}$. **Q.E.D.**

K The Fiscal Multiplier with Government Expenditure under Finite Forward Planning

In this section, I consider the short-term effects of a government expenditure with finite forward planning, and study how the fiscal multiplier changes with respect to the degree of foresight in the situation of no binding ZLB. For the cases under ZLB, Woodford and Xie (2019) have showed that, as long as the government expenditure is not sufficiently large, the fiscal multiplier under ZLB is in general decreasing when decision makers are less forward-looking. But, in the case without binding ZLB, I will show that this relationship is opposite, and the effect of fiscal stimulus through government expenditure follows a pattern similar to the analysis of transfer policies.

For simplicity, I assume decision makers use the value function learned from the steady-state stationary equilibrium to approximate continuation values beyond their planning horizon, and there is an exponential distribution of planning horizon among the population. Also, the monetary policy is specified by the Taylor rule, and the government expenditure is fully financed through lump-sum taxation immediately in the same period when the expenditure is imposed.

Similar to Woodford (2011) with rational expectations, the aggregate output and infla-

tion with finite forward planning can be characterized by

$$y_t - g_t = \rho E_t(y_{t+1} - y_t) - \sigma(\hat{i} - \rho E_t \pi_{t+1})$$

$$\pi_t = \kappa(y_t - \Gamma_g g_t) + \beta \rho E_t \pi_{t+1}$$

where $\Gamma_g = \eta_u/(\eta_u + \eta_w) < 1$ is the fiscal multiplier under flexible-price equilibrium, and $g_t = (G_t - \bar{G})/\bar{Y} > 0$ is the log-deviation of the government expenditure relative to the steady-state level of output, and the nominal interest rate $\hat{i} = \phi_\pi \pi_t$.⁶²

Consider a deterministic path of the government expenditure $g_t = g_0 \eta^t$ with $0 \leq \eta < 1$. Then, the solution of aggregate output and inflation is given by

$$y_t = \gamma_y g_t, \quad \pi_t = \gamma_\pi g_t$$

where

$$\gamma_y = \frac{1 - \rho\eta + \frac{\sigma\kappa(\phi_\pi - \rho\eta)}{1 - \beta\rho\eta} \Gamma_g}{1 - \rho\eta + \frac{\sigma\kappa(\phi_\pi - \rho\eta)}{1 - \beta\rho\eta}}$$

$$\gamma_\pi = \frac{\kappa(\gamma_y - \Gamma_g)}{1 - \beta\rho\eta}$$

It can be showed that, given $\phi_\pi > 1$, the fiscal multiplier γ_y is strictly decreasing with respect to the (average) degree of foresight ρ . In other words, as decision makers are less forward-looking, the effect of the government expenditure without ZLB increases. Similar to the analysis of transfer policies, the impact of monetary policy accommodation on the fiscal multiplier (from government expenditure) is also hump-shaped with respect to the degree of foresight.

⁶²Note that $\eta_u = -\bar{Y}u''/u' > 0$ is the negative elasticity of u' and $\eta_w = \bar{Y}\tilde{w}''/\tilde{w}'$ is the elasticity of \tilde{w} with respect to increases in Y . Similar to the notation in Woodford (2018), the period utility of household i is defined as $u(C_t^i) - w(H_t^i)$, where C_t^i is the quantity consumed in period t and H_t^i is hours of labor supplied in period t . As usual, $u(\cdot)$ is an increasing, strictly concave function, and $w(\cdot)$ is an increasing, convex function. $\tilde{w}(Y) = w(f^{-1}(Y))$ is the dis-utility to the household of supplying a quantity of output Y , and f is the production technology.