Cauchy-Weighted Measure for Nonlinear Conditional Dependencies within the High-Dimensional Factor Model Framework

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Abstract

Conditional independence testing is a fundamental and challenging problem with many important applications. This paper proposes a new approach to test conditional independence within the framework of high-dimensional factor models. A Cauchy-weighted measure is introduced to quantify the dependence between the idiosyncratic components and develop a corresponding conditional independence test. This measure, which ranges from 0 to 1, equals 0 if and only if conditional independence is true. It is robust to extreme values and computationally efficient. The proposed test is asymptotically distribution-free under the null hypothesis and capable of detecting nonlinear dependencies under the alternative hypothesis in high-dimensional scenarios. Furthermore, this paper demonstrates that the approach of first directly estimating the factors using pooled data, followed by performing the test, is invalid when factors are unobserved. Instead, this paper proposes to estimate the factors within the factor-augmented regression model framework and shows that the corresponding test remains valid. Extensive simulation studies and real data analysis are conducted to validate the effectiveness of this method, demonstrating its superior performance in high-dimensional scenarios.

KEY WORDS: Conditional independence; Distance correlation; Factor models; Nonlinear dependency.