# Price Staggering in Cartels

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**Abstract**

In this paper we investigate the optimal organization of staggered price increases in cartels. Staggered price increases impose a cost during cartel formation as the price leader initially loses sales. We show that for intermediate discount factors, staggered price increases can only be sustained when the increase is neither too low nor too high. When a cartel executes two consecutive price increases, the choice between using the same leader or alternating leadership depends on the initial price level in the industry. We also discuss the allocation of price leadership in the presence of cost asymmetry, product differentiation and consider the effect of strategic buyers on price staggering.

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# 1 Introduction

Over the last two decades antitrust authorities around the world have stepped up their efforts to prosecute hard-core cartels. Records from recent cartel cases show that companies are well aware of the increased threat. Strategies to avoid detection from buyers and antitrust authorities have become part of the cartel's organization. A particularly well documented practice of cartels to avoid the appearance of collusion is price staggering, the orchestration of sequential rather than simultaneous price increases.[1] Consider, for example, the *Electrical and Mechanical Carbon and Graphite Products (EMCG)* cartel prosecuted by the European Commission (EC):

> *For the new prices to take effect, one of the cartel members would circulate its new price list to customers at some time between January and March in the year following the Technical Committee meeting. The other cartel members would follow suit and issue their new price lists over the following weeks or months, thereby trying to create the impression that the companies concerned took their pricing decisions autonomously.*[2]

Similarly, the *Vitamins* cartel used sequential price increases as they "...could be passed off, if challenged, as the result of price leadership in an oligopolistic market."[3] However, staggering a price increase is costly as the price leader risks losing sales before the follower raises the price. This was a clear concern in the *Rubber Chemicals* cartel:

> *Bayer was not sure about the success of the increase because it was so large. It needed a three-month waiting period for the adjustment of prices to avoid the appearance of collusion. Flexsys agreed but was worried about losing significant market share. Bayer assured that there was no reason to worry and that it would follow in three months.*[4]

Flexsys' concerns were later attested as it lost significant volumes and key customers.[5]

[1] For example, staggered pricing is reported in the case documentation of 29 out of the 41 cartel cases of the European Commission during the period 2002-2007.

[2] See EC Case 38.359, 3 December 2003, para 101.

[3] See EC Case 37.512, 21 November 2001, para 203-4.

[4] See EC Case 38.443, 21 December 2005, para 70.

[5] A significant loss of sales also almost caused the *Fine Arts Auction House* cartel to collapse early on. As part of the first price increase agreement, Christie's had already announced its new non-negotiable

In this paper, we develop a theoretical model of staggered pricing in cartels and analyze the optimal organization of collusion in different settings. In our benchmark model, we consider a Bertrand duopoly with homogenous goods when firms compete in prices with an infinite horizon. The firms are interested in raising the industry price from its (exogenously given) current level towards a target price. A staggered price increase involves the cartel specifying a leader who raises the price to the target level in period $t$. In period $t+1$, the other cartel member follows suit and raises its price to the same level. We show that for intermediate discount factors the firms' incentive constraints affect the type of price increases that can be implemented. In particular, a staggered price increase can be sustained if and only if the target price is neither too small nor too large relative to the current industry price. Large price increases give the follower a strong incentive to deviate and raise its price and profits by shaving the target price. By contrast, small price increases are unable to satisfy the leader's incentive constraint to raise its price and forego current period sales.

We then explore a situation where the cartel is scheduling two consecutive staggered price increases. We show that whether a cartel chooses the same leader in each increase or alternating price leadership depends on the initial industry price level. A low current industry price requires a larger first increase and gives the first follower a stronger incentive to deviate. Allocating this firm the follower role in the second move might thus increases the sustainability of the staggered price increases. Vice versa, if the current price is higher, the first leader is more tempted to deviate and is assigned the follower role in the second increase to stabilize the cartel. We also show that two price increases are sufficient to take the cartel from any starting price to the monopoly price.

Towards endogenizing the price leader role in the cartel, we consider staggered price increases when firms face asymmetric cost of production. We show that the cartel increases the sustainability of a price move if it assigns the leading role to the small, high-cost firm. This firm stands to gain less from undercutting as a leader while the low-cost firm is less tempted to increase price and reduce demand when deviating as a

schedule with a minimum seller commission of 2 percent. Sotheby's followed three weeks later. In the meantime, Sotheby's won a very significant jewellery consignment from Alghanim worth nearly US$10 million. Apparently, the CEO of Christie's was furious when he received the news and began to fear that Sotheby's would double-cross him (Mason, 2004, p.166-167).

follower.

Throughout most of the paper we focus on staggered price increase as the optimal pricing strategy in response to the cartel's perceived risk of being investigated and fined by the competition authority. In section 5 we explicitly introduce an antitrust detection function and show under which conditions staggered pricing is easier to sustain than simultaneous price increases. We also extend the benchmark model by considering the effect of product differentiation, the possibility of multiple leaders and followers and strategic buyers in a staggered price increase. While more firms and more leaders in a staggered price increase make collusion harder to sustain, we find that the effects of product differentiation and the presence of strategic buyers are ambiguous.

The literature on cartels is usually concerned with sustaining collusion at a long-running target price.[6] Very little attention has been given to the gradual process of reaching that target level which seems to be a major impediment to cartel formation in practice. Harrington (2005) characterizes the optimal cartel price path to its steady-state level in the presence of an exogenous buyer detection process and (accumulating) damage awards. The steady-state cartel price is strictly below the monopoly level when penalties include (price dependent) damage awards. When penalties are price independent, the cartel reaches the monopoly price in the long run.[7] Harrington (2004) extends the analysis of this framework by considering lower discount factors in which the cartel stability constraint is binding on the optimal price path. In this case, numerical analysis shows that the optimal cartel price might be first increasing and then decreasing. In both papers cartel members charge the same price and raise their prices simultaneously. We consider sequential price increases when buyers and antitrust authorities are suspicious of simultaneous price movements of cartel members.

Our paper also relates to the long-standing discussion of price leadership in industries. Going back to the early work by Stigler (1947) and Markham (1951), three (overlapping) forms of price leadership are typically distinguished. Dominant firm price leadership occurs when one large producer sets its price and a smaller competitor or a competitive

[6] See Ivaldi, Jullien, Seabright, and Tirole (2007), Choi and Gerlach (2014), Levenstein and Suslow (2014) and Green, Marshall, and Marx (2014) for recent surveys.

[7] The reason for this result is that in the steady state, marginal price changes have a negligible effect on detection and only affect penalties through the damage awards. If penalties are not sensitive to the cartel's price, the monopoly price is achieved at no cost.

fringe follows as price takers. Deneckere and Kovenock (1992) show that when two capacity-constrained firms compete in prices, the larger firm emerges as the leader in equilibrium. The reason is that the small firm stands to lose more from being undercut and sets a lower price as leader. By contrast, the large firm can provide a price umbrella which allows the small firm to undercut and sell its entire capacity.[8] Competitive, barometric price leadership refers to situations where changes in prices reflect market conditions and asymmetrically informed firms.[9] Finally, collusive price leadership occurs when the process of price changes is intended to coordinate prices at the collusive level.

Rotemberg and Saloner (1990) consider a repeated game model of a duopoly with differentiated products and asymmetric information. They show that there exists a stationary equilibrium in which the informed firm acts as price leader and the uninformed firm price matches. Although the duopolists achieve a supra-competitive outcome, they are unable to implement the (informationally unconstrained) first-best outcome. More recently, Ishibashi (2008) and Mouraviev and Rey (2011) use a framework that endogenizes the timing of firms' strategy choice in a repeated game. In each period there is an extended stage game with action commitment in which the firm who wants to lead can commit to its price choice. The waiting firm can observe the choice of the leader and select its price before demand is realized. Mouraviev and Rey (2011) show that price leadership can drastically increase the sustainability of price collusion. A deviation of the leader can be immediately punished and the follower can be assigned a higher market share to prevent undercutting. Ishibashi (2008) demonstrates that the firm with the larger capacity emerges as price leader in order to demonstrate its commitment not to deviate. Finally, Marshall, Marx, and Raiff (2008) develop a theoretical model of price announcements in a duopoly and use data from the Vitamins cartel to determine the existence of explicit collusion based on the pattern of price announcements. Our paper differs from this literature in two aspects. We do not require explicit action commitment as it arises endogenously in our model. More importantly, we consider situations where the price leader loses demand to the follower in a staggered price increase. This intro-

[8]Similar results hold in settings where firm size is measured by the base of loyal customers (Deneckere, Kovenock, and Lee, 1992) and when products are imperfect substitutes (Furth and Kovenock, 1993).

[9]In Cooper (1996), for example, one firm acquires information about the current market conditions and changes its price accordingly while the other firm follows rather than investing in information gathering itself.

duces an additional cost that the cartel has to bear in order to avoid the appearance of collusion.

The remainder of the paper is organized as follows. In the next section, we set up and analyze the benchmark model with one staggered price increase. In section 3, we derive the optimal organization of a cartel that implements two staggered price increases. In the following section we introduce cost asymmetry and discuss the optimal allocation of the price leader role. In Section 5, we introduce an explicit antitrust detection function. Section 6 discusses product differentiation, cartels with multiple leaders and followers and staggered pricing with strategic buyers. The last section concludes. All proofs are relegated to the Appendix.

# 2 Benchmark Model

Consider a Bertrand duopoly model where firms compete in prices in a homogeneous product market. The firms produce at the same constant marginal cost $c$ and face a demand of $D(p)$ with $D'(p) < 0$. Total industry profits at price $p$ are given by $\pi(p) = (p - c)D(p)$ and assumed to be quasi-concave. Let $p^m$ be the monopoly price, that is,

$$p^m = \arg\max_p \pi(p),$$

and the corresponding industry profits are $\pi(p^m)$.

Firms play a repeated game over an infinite horizon in discrete time. Each firm has a discount factor $\delta \in [0, 1]$. Firms set their prices simultaneously in each period. We are interested in coordinated price increases towards the fully collusive level $p^m$. Suppose the industry price in the current period $t$ is $p_1 \in [c, p^m)$ and firms intend to increase the price to $p_2 > p_1$. The initial industry price could be the outcome of a repeated game in which firms partially collude to sustain a price below the monopoly level.[10] In the benchmark model we consider a single price increase while in Section 3 the cartel organizes two consecutive price increases in the industry.

[10]Alternatively, with just minor changes to notation, our results would hold in a situation where a cartel faces a cost increase from $c_1$ to $c_2 > c_1$ and firms fully collude. In this case the cartel wants to adjust the price from $p_1 = p^m(c_1)$ to the new monopoly level $p_2 = p^m(c_2)$.

As discussed in the introduction, firms are aware that their pricing behaviour may raise the suspicion of the competition authority. In order to focus on the optimal organization of a staggered price increase, we take a reduced form approach in the baseline model and do not explicitly assume a cartel detection function like, for example, Harrington (2004, 2005). Instead we posit that, given the potential detection of the cartel, a staggered price increase is the most profitable form of raising the cartel price. In section 5 below, we introduce a cartel detection function with two basic properties. First, investigations are only triggered when firms set uniform prices which is consistent with a sustainable long-term cartel price path. In other words, price undercutting is considered as competitive behaviour by the authority. And second, the probability of an investigation is higher if firms increase prices simultaneously. We use this detection function to derive an explicit condition on the alertness of the competition authority such that the cartel is better off with a staggered price increase.

In order to implement a staggered price increase, firms consider the following strategy in the repeated game. At the beginning of period $t$, firms form a cartel agreement specifying their roles. The leader sets price $p_2$ in the current period. The follower keeps its price at the current level $p_1$. From period $t+1$ onwards, both firms charge the new cartel price $p_2$. Deviations from this strategy are punished with reversion to static Nash equilibrium prices and zero continuation profits.[11] We allow firms to assign different market shares for leader and follower from period $t+1$ onwards. Let $s$ be the market share of the price leader. We do not consider monetary transfers between the firms as such payment increases the probability of detection and antitrust prosecution.

In what follows we derive the conditions under which such a staggered price increase strategy is sustainable in a Subgame Perfect Equilibrium of the repeated game. First, consider the incentives in the continuation game from period $t+1$ onwards. The leader's incentive constraint is

$$s\pi(p_2)\frac{1}{1-\delta} \geq \pi(p_2) \quad \text{or} \quad s \geq 1-\delta. \tag{1}$$

[11] This is the maximum punishment that can be imposed. This strategy also ensures that firms always have an incentive to implement the staggered price increase.

Similarly, the follower's incentive constraint is given by

$$(1-s)\pi(p_2)\frac{1}{1-\delta} \geq \pi(p_2) \quad \text{or} \quad s \leq \delta. \tag{2}$$

Undercutting the cartel price allows the deviator to steal its rival's market share. The allocated market shares thus have to ensure that the potential gain in market share does not exceed the discount factor. It follows that, for $\delta \geq 1/2$, any market share allocation $s \in [1-\delta, \delta]$ can be supported in the continuation game of the staggered price increase. The range of available market shares thus depends on the discount factor. For high discount factors, it is easier to sustain more asymmetric market shares. For discount factors closer but above $1/2$, only cartels with similar market shares are sustainable. Note that a simultaneous price increase from $p_1$ to $p_2$ with equal market share can be sustained under the same condition, that is, for $\delta \geq 1/2$.

Let us now look at the incentives to implement the staggered price increase in period $t$. If the leader increases the price to $p_2$, it loses sales in period $t$ but receives the assigned continuation profit from period $t+1$ onwards. Alternatively, the leader can undercut the follower and capture the entire market at price $p_1$. The leader's constraint in period $t$ is then

$$\frac{\delta}{1-\delta} s\pi(p_2) \geq \pi(p_1). \tag{3}$$

Ceteris paribus, the leader's constraint is easier to satisfy the larger the difference between $\pi(p_2)$ and $\pi(p_1)$, that is, the steeper the price increase is. A large price increase implies that a deviating leader has to cut its price stronger in order to attract demand. This makes a cartel easier to sustain. If the current price is equal to the firms' marginal cost, then the leader has no incentive to deviate. Similarly, if the leader's future market share is sufficiently close to one and the discount factor greater than $1/2$, this condition is always satisfied.

If the follower adheres to the collusive agreement, it will serve the entire market and earn $\pi(p_1)$ in period $t$ before receiving its continuation value from period $t+1$ onwards. The best deviation for the follower is to shave the price of the leader and serve the entire market. The follower's incentive constraint in period $t$ is therefore

$$\pi(p_1) + \frac{\delta}{1-\delta}(1-s)\pi(p_2) \geq \pi(p_2) \tag{4}$$

This condition is satisfied for any price increase if the discount factor and the market share of the follower are sufficiently high. Otherwise, the condition is easier to satisfy if the difference between $\pi(p_2)$ and $\pi(p_1)$ is small, that is, when the price increase is not too steep. A large price increase raises the price a deviating follower can charge. This makes the cartel harder to sustain.

The cartel chooses the market share $s$ to increase the sustainability of the cartel. We call a cartel organization optimal when the choice of the market share minimizes the discount factor threshold above which collusion is sustainable. Our first result explores which of the four constraints are binding in an optimal cartel organization.

**Lemma 1** *Consider the lowest possible discount factor such that conditions (1) to (4) are jointly satisfied. If $\pi(p_1) \leq \pi(p_2)/2$, then the follower's period $t$ and the leader's period $t+1$ constraints are strictly binding. Otherwise, the leader's period $t$ and the follower's period $t+1$ are strictly binding.*

When firms intend to implement a large price increase, the follower has a strong incentive to raise the price just below the leader's price and serve the entire market. This implies that the period $t$ constraint of the follower is more restrictive and the cartel has to assign more market share to the follower. An increase in the follower's market share is, however, limited by the period $t+1$ incentive constraint of the leader. Thus, for large price increases, conditions (1) and (4) are strictly binding. Vice versa, for small price increases, the leader has a strong incentive to lower the price and undercut the follower. Hence, the period $t$ constraint of the leader is more restrictive. Thus, for price increases where $\pi(p_1) > \pi(p_2)/2$, the cartel assigns more market share to the leader and conditions (2) and (3) are strictly binding.

This leads us to the conditions under which a staggered price increase is sustainable.

**Lemma 2** *There exists a $\delta'$ with $1/2 \leq \delta' < 1$ such that if $\delta > \delta'$, then a staggered price increase is sustainable. It holds that $\delta' = 1/2$ if and only if $\pi(p_1) = \pi(p_2)/2$.*

First, reflecting the usual Folk Theorem type result, if firms are sufficiently patient, any staggered price increase is sustainable. Since $\delta' \geq 1/2$, staggered price increases are weakly harder to sustain as simultaneous price increases. A staggered price increase reduces the cartel's profit on the equilibrium path due to the follower's undercutting of the leader. The cartel's ability to adjust market shares to satisfy the more restrictive period $t$ constraint is limited by the fact that the continuation of the cartel beyond period $t+1$ is only feasible if the market shares are not too asymmetric. The leader and follower's period $t$ constraints coincide when the staggered price increase doubles industry profits. In this case symmetric market shares are optimal. A staggered and a simultaneous price increase can be implemented under the same condition.

In what follows we focus on the case of intermediate discount factors ($1/2 \leq \delta \leq \delta'$) and explore the optimal organization of staggered price increases when deviation incentives are strictly binding. To further illustrate the constraints of the cartel, consider FIGURE 1 below. The figure uses a $\pi(p_1) - \pi(p_2)$ diagram for intermediate discount factors. First, consider values such that $\pi(p_1) = \pi(p_2)/2$. We know from Lemma 2 that along those values, a price increase can be sustained for symmetric market shares and any $\delta \geq 1/2$. Now fix a target price $p_2$ and reduce the initial industry price $p_1$. This makes the period $t$ constraint of the leader easier to sustain and that of the follower harder to sustain. At some point, the follower's constraint is strictly binding for symmetric market shares and the cartel needs to assign more market share to the follower in order to sustain the price increase. Eventually, this increase in market share is limited by the $t+1$ constraint of the leader. The line denoted (1) and (4) depicts the pairs of industry profits before and after the price increase at which conditions (1) and (4) are both strictly binding. For lower $\pi(p_1)$, the cartel is unable to satisfy the follower's period $t$ constraint without violating the leader's $t+1$ constraint, and the price increase is not implementable.

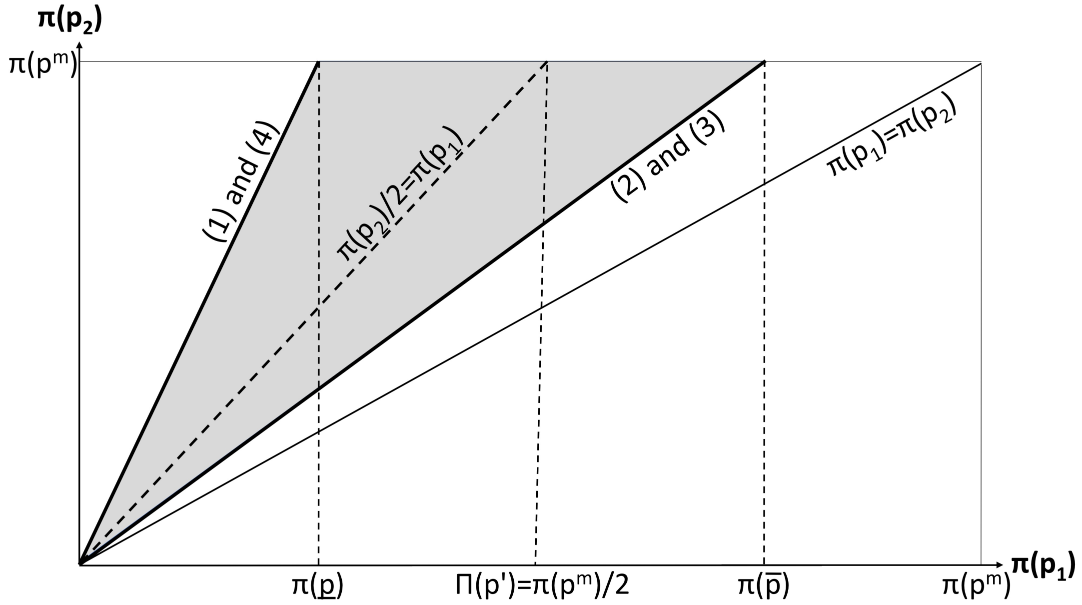


Figure 1: *Implementable staggered price increases for intermediate discount factors*

Vice versa, increasing the initial industry price beyond the level where $\pi(p_1) = \pi(p_2)/2$ relaxes the follower's and tightens the leader's period $t$ constraint. Satisfying the leader's constraint with a higher market share is limited by (2). Any prices yielding profit pairs below the line denoted (2) and (3) are not implementable in a staggered price increase. We can thus conclude the following.

**Proposition 1** *Consider intermediate discount factors and a cartel that intends to implement a single staggered increase to a target price* $p_2$. *There exist price levels* $\underline{p}(p_2)$ *and* $\overline{p}(p_2)$, *with* $c < \underline{p}(p_2) < \overline{p}(p_2) < p_2$, *such that the staggered price increase is sustainable if and only if the initial industry price satisfies* $p_1 \in [\underline{p}(p_2), \overline{p}(p_2)]$.

This proposition has two main implications. First, a staggered price increase can only be sustained if the initial industry price is at an intermediate level relative to the target price. This implies as a special case that a staggered price increase to the monopoly price is only feasible if the current industry price is such that $p_1 \in [\underline{p}, \overline{p}]$ where $\underline{p} \equiv \underline{p}(p^m)$ and $\overline{p} \equiv \overline{p}(p^m)$. A cartel is not able to implement a single staggered price increase from a sufficiently competitive industry level to the monopoly price due to the follower's incentive to undercut the leader. The cartel is also not able to raise the price to the monopoly level if the current price is too high as the leader's constraint in period $t$ would be binding.

At the same time, while some staggered price increase might not be implementable, Proposition 1 also implies that for any $\delta \geq 1/2$, there exist starting prices $p_1$ to reach any target price $p_2$. This result will be useful in the next section. For example, in the case of the monopoly price, let $p'$ be the price such that $\pi(p') = \pi(p^m)/2$. Then it is easy to check that the price thresholds $\underline{p}$ and $\overline{p}$ are both equal to $p'$ at $\delta = 1/2$, but they diverge as $\delta$ increases further.

# 3 Two Staggered Price Increases

There is some case evidence that cartels plan more than one staggered price increase at a time. For example, the *Rubber Chemicals* cartel scheduled two consecutive price increases in anti-degradants and primary accelerators in meetings in early 1999. The first increase was led by Bayer, effective on 1 October 1999 for non-tyre customers and on 1 January 2000 for tyre customers. The second increase on 1 July 2000 was led by Flexsys and implemented on a global scale.[12] In this section we analyze the optimal organization and sustainability of two consecutive staggered price increases. We investigate whether collusion is easier to sustain when using the same price leader in both increases or with an alternating price leadership like in the *Rubber Chemicals* cartel.[13]

We consider the following set-up. In period $t$, the follower charges the current industry price $p_0$ while the leader sets a price $p_1 > p_0$. In period $t+1$, the leading firm sets $p_2$ while the follower charges $p_1$. From period $t+2$ onwards, both firms charge the new cartel price $p_2$. We treat the initial industry price $p_0$ and the target price $p_2$ as parameters. The cartel can choose the intermediate price level to increase the sustainability of the cartel. In the previous section we showed that a single staggered price increase might not be able to raise the cartel's price to the monopoly level. Hence, when considering two price increases we are most interested in the case where the cartel would not be able

[12] See EC Case 38.443, 21 December 2005, para 105-107, 121.

[13] *Flat Glass* (EC Case 39.165, 28 November 2007, para 81-89, 129, 142) is another example of a cartel with two consecutive staggered price increases. Glaverbel was the leader in both rounds, which took place in October 2003 and March 2004. In the *Professional Videotapes* cartel (EC Case 38.432, 20 November 2007, para 82, 114), Sony was the leader of two staggered price increases in October 1999 and August 2000. There were no further price increases by the cartel afterwards.

to sustain a single staggered price increase to the target monopoly price. That is, the current industry price satisfies $p_0 \leq \underline{p}$ and the target price is $p_2 = p^m$. For notational convenience, let $s$ be the market share of the leader of the second price increase from period $t+2$ onwards. We first consider the case where the same firm leads both price increases and then consider alternating price leadership.

## 3.1 Same Price Leader

Suppose the same firm leads both staggered price increases. In period $t+2$, the cartel has reached its target price $p_2 = p^m$ and the incentive constraints for leader and follower are given by (1) and (2). In period $t+1$, after the first price increase from $p_0$ to $p_1$, the same incentive constraints as in the single price increase have to hold. That is, the cartel needs to satisfy condition (3) and (4) for the leader and follower, respectively.

Now consider the leader's incentives to start the first price increase in period $t$. Leading two consecutive price increases means that the firm is losing sales in period $t$ and $t+1$ but then it receives its share of the continuation profits in period $t+2$. Alternatively, the leader could undercut the follower's price and obtain $\pi(p_0)$. Incentive compatibility requires

$$\frac{\delta^2}{1-\delta} s\pi(p^m) \geq \pi(p_0). \tag{5}$$

This condition is easier to satisfy if the current industry price $p_0$ is relatively low compared to the monopoly price level. Similarly, consider the incentives of the follower. The follower makes strictly positive profits in periods $t$ and $t+1$ before obtaining its share of the continuation profits. The best deviation is to shave the price of the leader. It thus has to hold that

$$\pi(p_0) + \delta\pi(p_1) + \frac{\delta^2}{1-\delta}(1-s)\pi(p^m) \geq \pi(p_1). \tag{6}$$

The follower's incentives are easier to satisfy if the initial price level is high relative to the intermediate price. The intermediate price enters the period $t$ deviation profits and the period $t+1$ cartel profits. Hence, a higher $p_1$ makes the constraint harder to sustain. Put together, it is easier to satisfy the follower's incentive when a small profit increase in period $t+1$ is followed by a large profit increase in $t+2$.

The cartel chooses the firms' market shares and the intermediate price $p_1$ to minimize the discount factor threshold above which the cartel is sustainable subject to the constraints (1) to (6).[14] Let $p'' < p'$ be the price such that

$$\pi(p'') = \pi(p^m)/4.$$

The following proposition characterizes consecutive price moves with the same leader.

**Proposition 2** *Consider two consecutive staggered price increases with the same price leader and $\delta \geq 1/2$. If $p_0 \leq p''$, then the cartel is always able to raise the industry price to the monopoly level. Otherwise, there exists a $\delta_S$ with $\delta_S > 1/2$ such that if $\delta \geq \delta_S$, then the staggered price increases are sustainable. The maximum intermediate price satisfies $p_S^* = \overline{p}$ and the market share of the leader is $s = \delta$.*

If the current price level in the industry is sufficiently small, two consecutive staggered price increases to the monopoly price can be sustained for any $\delta \geq 1/2$. To see this, suppose the intermediate price is at $p_1 = p'$ and the incentives constraints (1)-(4) are satisfied such that the second increase to the monopoly price is incentive-compatible. We know from our analysis in the previous section that this is feasible for any $\delta \geq 1/2$. How about the incentives for the first price increase? With the same leader in both increases, the follower in the first increase also serves the market in the second increase. This relaxes the incentives to deviate in period $t$. In fact, the follower's constraint in period $t$ is always satisfied if there is no profitable deviation from period $t+1$ onwards.[15] We can thus drop condition (6) from the cartel's problem. Now consider the leader's constraint (5). The lowest discounted profits for the leader that are consistent with the cartel operating after period $t+2$ occur at $s = 1-\delta$ and $\delta = 1/2$. Hence, if the minimum cartel profit of $\pi(p^m)/4$ exceeds the deviation profits in period $t$, the leader has no incentive to undercut the follower. This implies that the leader's constraint is

[14] If a set of prices $p_1$ is able to sustain the minimum discount factor, then we select the highest intermediate price to maximize cartel profits.

[15] The short-term deviation profits of deviating in $t$ are limited by the leader's incentive constraint in $t+1$. At the same time, incentive compatibility in $t+2$ requires a minimum market share of $1-\delta$ for the follower. Thus, cartel profits exceed deviation profits in $t$. This is demonstrated in the appendix to the next proposition.

always satisfied if $p_0 \leq p''$, that is, when a low current industry price deters deviations during the first price increase.

If the current industry price level is sufficiently high, the leader's period $t$ constraint is binding and the cartel needs to allocate a higher market share to the leader. Hence, the lowest discount factor at which collusion is sustainable satisfies condition (2) and (5) with equality. Additionally, as shown appendix, it depends on the initial price $p_0$ whether the leader or the follower's period $t+1$ is binding as well. In either case, since (2) and (5) are independent of the intermediate price, the cartel can set the highest intermediate price that makes the second increase sustainable, that is, $p_1 = \overline{p} = p_S^*$.

## 3.2 Alternating Leadership

Now consider a situation where one firm leads the first price increase and the other firm leads the second price increase. Again, the incentive constraints (1) to (4) ensure that there is no deviation in periods $t+1$ and $t+2$.

The first price leader is the follower in period $t+1$ and receives a share $1-s$ of the continuation profit in $t+2$. In period $t$ the first leader prefers to set $p_1$ rather than to deviate and shave $p_0$ if

$$\delta\pi(p_1) + \frac{\delta^2}{1-\delta}(1-s)\pi(p^m) \geq \pi(p_0). \tag{7}$$

This constraint is satisfied if the current industry price is sufficiently low relative to the intermediate price $p_1$. This is more likely to hold for a large profit increase in period $t+1$ followed by a small profit increase in $t+2$.

Vice versa, the follower of the price increase in $t$ is the leader in $t+1$. Hence, this firm receives full industry profits at the initial price level, makes no profits as the leader in $t+1$ and receives a share $s$ of the continuation profits. A deviation in period $t$ to undercut the leader at $p_1$ is not profitable if

$$\pi(p_0) + \frac{\delta^2}{1-\delta}s\pi(p^m) \geq \pi(p_1). \tag{8}$$

This condition is easier to satisfy the smaller the first price increase and the larger the second price increase is. Hence, a lower intermediate price alleviates the incentive constraint for the first follower.

Again we are able to drop one of the period $t$ incentive constraints. Here the first leader's constraint (7) is always implied by the same firm's incentive constraint as follower in the second increase, that is, condition (4). If this firm has no incentive to deviate in period $t+1$ when it is the follower, then it also has no incentive to deviate in period $t$ as leader.[16] This means the cartel chooses the market share $s$ and the intermediate price $p_1$ to maximize sustainability subject to the conditions (1) to (4) and (8). We get the following result.

**Proposition 3** *Consider two consecutive staggered price increases with alternating price leadership and* $\delta \geq 1/2$. *If* $p_0 \geq p''$, *then a cartel is always able to raise the industry price to the monopoly level. Otherwise, there exists a* $\delta_A$ *with* $\delta_A > 1/2$ *such that if* $\delta \geq \delta_A$, *then the staggered price increases are sustainable. The maximum intermediate price satisfies* $p_A^* \leq \overline{p}$ *and the market share of the second leader satisfies* $s \leq \delta$.

In order to implement staggered price increases with alternating leadership, the cartel needs to provide incentives for the second increase from $p_1$ to $p^m$ and prevent the first follower from deviating. The follower's incentives to deviate in period $t$ are lower, the higher the initial industry price. In fact, if the initial price is sufficiently large, $p_0 \geq p''$, then the cartel is able to implement the consecutive price moves for any $\delta \geq 1/2$. The reason is that as $\delta$ approaches 1/2, the second price increase requires an intermediate price $p'$ which satisfies $\pi(p_1) = \pi(p^m)/2$. This is the deviation profit in the follower's constraint (8) while the future collusive profit approaches $\pi(p^m)/4$. Hence, if $\pi(p_0) \geq \pi(p^m)/4$, the cartel can sustain alternating price leadership for any $\delta \geq 1/2$.

For lower current price levels, $p_0 < p''$, this is no longer possible as the first follower's constraint is strictly binding in the cartel's optimal organization for $\delta$ sufficiently close to 1/2. The cartel can strengthen the follower's constraint (8) by reducing the intermediate price and/or increasing the firm's market share $s$. As to whether the cartel uses market

[16] The reason is that, for any $\delta \geq 1/2$ and initial prices $p_0 \leq p'$, the relative incentives to deviate are higher as follower in period $t+1$ and there is a longer wait for the steady-state cartel profits in period $t$. This is demonstrated in the appendix.

share allocation or a reduction of the intermediate price depends on the level of the initial industry price (see appendix to Proposition 3). For sufficiently low values of $p_0$, the intermediate price is reduced to $p_1 < \overline{p}$ while the market share $s$ is set below the maximum sustainable level of $\delta$ from condition (2). For higher values of $p_0$, the highest feasible intermediate price $\overline{p}$ is implemented while the market share is $s = \delta$.

Let us summarize and compare the analysis of staggered price increases with the same leader and alternating leadership.

**Proposition 4** *Consider two consecutive staggered price increases and $\delta \geq 1/2$. A cartel can always implement a move from $p_0 \leq \underline{p}$ to the monopoly price. If the initial price is relatively low, $p_0 \leq p''$, staggered price increases are easier to sustain with the same price leader. Otherwise, they are easier to sustain with alternating price leadership. Staggered price increases with the same leader allow for a (weakly) higher intermediate price and require a (weakly) higher market share for the leader of the second increase.*

As seen above, one single staggered price increase can be implemented if and only if it is an intermediate step size. With two consecutive staggered price increases, the monopoly price can be reached from any starting level for any $\delta \geq 1/2$, that is, under the same condition as one simultaneous price increase can be sustained. With consecutive increases, the first price move targets an intermediate price that allows the second price increase to the monopoly price to be implementable. If the initial industry price is low and the move to the intermediate price is large, the follower of the first move is more tempted to deviate. In this case, it is easier to sustain the increases using the same leader which relaxes the follower's incentives in the first increase. Vice versa if the move is small, the leader is more tempted and alternating leadership alleviates the incentives constraint of the leader in the first increase. Hence, the optimal organization of consecutive increases is a function of the level of current industry price.

Another difference in the optimal cartel organization is that, with the same leader, firms' incentives during the first increase do not impose additional constraints on the level of the intermediate price. By contrast, with alternating leaders, the corresponding incentives become weakly easier to satisfied when the intermediate price is lower. This means that when both types of increases are sustainable, using the same price leader yields weakly higher industry profits. Finally, since alternating leadership requires each firm to suffer the loss of sales only once, the implied market share of the second leader is weakly smaller under this cartel organization.

Let us return to the *Rubber Chemicals* cartel from the beginning of this section. In line with our result that increases are easier to sustain with a lower intermediate price, there is evidence suggesting the cartel did not want the resulting price from the first increase to be too high. During implementation of the first increase, the cartel decided to lower the increase in Europe from 7% to 5% while the second move was scheduled to be a 10% increase worldwide.[17]

# 4 Cost Asymmetries and Price Leadership

In this section we explore the effect of cost asymmetries on the sustainability of staggered price increases and the optimal allocation of the leadership role within the cartel. To fix ideas, suppose the cartel members face different constant marginal cost of production such that firm 1 is the low-cost firm with $c_1$ and firm 2 faces a cost of $c_2 \geq c_1$. Let us assume that the cost advantage of firm 1 is non-drastic, that is, $c_2 < p^m(c_1)$. Furthermore, adjust notation and let $\pi(p, c) = (p - c)D(p)$ be the industry profit at price $p$ when producing with a marginal cost level of $c$.

Suppose the cartel intends to implement a staggered price increase from the competitive level $p_1 = c_2$ to $p_2 > c_2$ using trigger strategies with reversion to the static Nash equilibrium.[18] This means that, after a deviation, firm 2 receives zero continuation profit while the low-cost firm gets $\pi(c_2, c_1) > 0$. In what follows, we consider the sustainability of a staggered price increase with a low-cost and high-cost leader, respectively. Then, we compare and derive the optimal leadership allocation and discuss the role of asymmetry.

Consider the sustainability of the cartel at the new industry price $p_2$ from period $t + 1$ onwards and suppose the low-cost firm 1 is assigned a market share $s$. This firm

[17]See EC Case 38.443, 21 December 2005, para 118 and 125.

[18]The qualitative nature of the results would not change if firms would use continuation strategies which impose a harsher punishment on the more efficient firm. Note that the analysis focuses on intermediate discount factors and optimal punishment strategies as in Abreu (1988) might not be able to implement the minmax continuation value of zero.

has no incentive to deviate if the difference between future cartel and punishment profits exceed the gains in market share in the current period,

$$\frac{\delta}{1-\delta}[s\pi(p_2, c_1) - \pi(c_2, c_1)] \geq (1-s)\pi(p_2, c_1). \tag{9}$$

This condition is easier to satisfy, the higher the price $p_2$ and the larger the market share for firm 1. Similarly, for firm 2, collusion is sustainable in the long run if

$$\frac{\delta}{1-\delta}(1-s)\pi(p_2, c_2) \geq s\pi(p_2, c_2) \quad \text{or} \quad s \leq \delta. \tag{10}$$

For a given market share, the low-cost firm 1 gains more from deviating in period $t+1$ and earns higher profits in the punishment phase. From the point of view of period $t+1$, the cartel can increase its sustainability and relax firm 1's constraint by allocating a higher market share that is still compatible with the high-cost firm 2's incentive constraint.

Let us analyze the two price leadership scenarios in turn. First assume the low-cost firm 1 is the price leader. In the first period of the price increase, firm 1 has an incentive to raise the price to the new collusive level $p_2$ rather than undercutting the current price level at $p_1 = c_2$ and triggering punishment if and only if

$$\frac{\delta}{1-\delta}[s\pi(p_2, c_1) - \pi(c_2, c_1)] \geq \pi(c_2, c_1). \tag{11}$$

The higher the cartel price target and the higher the market share of firm 1, the easier it is to sustain this constraint. The high-cost firm 2 follows by charging the competitive price in period $t$ and not making any profits. This is sustainable if the future cartel profits exceed the gains from shaving the price $p_2$ charged by firm 1, that is, if

$$\frac{\delta}{1-\delta}(1-s)\pi(p_2, c_2) \geq \pi(p_2, c_2) \quad \text{or} \quad s \leq \frac{2\delta - 1}{\delta}. \tag{12}$$

It is clear that if firm 2 has no incentive to deviate in period $t$, it will not deviate in period $t+1$ either. Hence, collusion is sustainable as long as there exists a market share that jointly satisfies (9), (11) and (12). In order to increase sustainability, the cartel shifts market share to firm until firm 2 constraint is binding. The two constraints of firm 1 have the same future value on the equilibrium and deviation path. However, the current gains from deviation in period $t+1$ are larger than in period $t$ if the cartel's price increase is sufficiently large. Hence, the binding constraint for firm 1 is either (11) if the increase is small or (9) if the target price $p_2$ is large.

**Lemma 3** *A staggered price increase with the low-cost firm leading is sustainable if and only if*

$$\frac{\pi(c_2, c_1)}{\pi(p_2, c_1)} \leq \min\{\frac{\delta^2 + \delta - 1}{\delta^2}, 2\delta - 1\}.$$

A staggered price increase with the low-cost firm leading is more likely to be sustainable if the price increase is sufficiently high and the cost asymmetry small.

Now consider the high-cost firm 2 as the price leader. In the first period of the price increase, firm 2 always has an incentive to raise the price to $p_2$ rather than to undercut the current price level of $c_2$ charged by the follower. By contrast, the low-cost firm 1 could deviate by undercutting leading firm 2 at the collusive price level and trigger punishment. The low-cost follower, firm 1, has no incentive to deviate if

$$\frac{\delta}{1-\delta}[s\pi(p_2, c_1) - \pi(c_2, c_1)] \geq \pi(p_2, c_1) - \pi(c_2, c_1). \tag{13}$$

In a sustainable price move it has to hold that $s\pi(p_2, c_1) - \pi(c_2, c_1)$. Hence the RHS of (13) is larger than the RHS (9). Firm 1 stands to gain more from deviating as a follower in period $t$ while the future gain from the cartel is the same in both periods of the staggered price increase. The increase with firm 2 as the leader is thus sustainable if there exist market shares that jointly satisfy firm 2's period $t+1$ constraint, (10), and firm 1's period $t$ constraint, (13).

**Lemma 4** *A staggered price increase with the high-cost firm leading is sustainable if and only if*

$$\frac{\pi(c_2, c_1)}{\pi(p_2, c_1)} \leq \frac{\delta^2 + \delta - 1}{2\delta - 1}.$$

Again the move is easier to sustain with a higher target price and less cost asymmetry between the firms.

Comparing the results in Lemma 3 and 4, it is easy to check that a staggered price increase is easier to implement when the high-cost firm 2 is the leader. A high-cost firm stands to gain less when it deviates as a leader by undercutting the follower and increasing production. In fact, the high-cost firm only imposes a constraint in the first period of the price move if it is a follower. Now consider the first period incentive constraints of the low-cost firm 1, conditions (11) and (13). The gain in current period profits on the RHS is higher with firm 1 as a follower if $\pi(p_2, c_1) \geq 2\pi(c_2, c_1)$. However, for such large price increases the incentive constraint (13) is not binding as the cartel is able to increase firm 1's market share to sustain the move. Hence, the low-cost firm's incentives to deviate in period $t$ are also alleviated when the high-cost firm is the leader and we get an unambiguous result.

**Proposition 5** *Consider firms with asymmetric constant marginal cost. A staggered price increase is easier to sustain if the firm with the higher marginal cost is the leader.*

Market share information - which could be used to proxy asymmetric cost positions - is often not available in published cartel case decisions. However, in the *EMCG* cartel mentioned in the introduction, there was at least one staggered price increase that was led by a smaller competitor. On December 14, 1999, the cartel decided that Schunk would lead a price increase on March 10, 2000 and Morgan would follow one month later. According to the market share ranges in the case documentation, Morgan was a significantly larger competitor.[19]

# 5 Endogenous Detection Probabilities

So far we have focused on the cartel's strategy and incentives of staggering price increases. The implicit assumption was that the threat of antitrust detection after a simultaneous price increase is sufficiently large to make such moves unprofitable. In this section we introduce an explicit detection function and compare the sustainability of simultaneous and staggered price increases.

[19]See EC Case 38.359 - Electrical and mechanical carbon and graphite products, 3 December 2003, para 37, 107.

Consider a cartel's price increase from $p_0 \geq c$ to $p_1$, with $p_0 < p_1 \leq p^m$ and, for simplicity, assume that cartels use symmetric market shares in the long run. Let us start by introducing a simple cartel detection function for this context. First, investigations are only triggered when firms are on a sustainable cartel price path, that is, when their current period prices are the same. The competition authority thus interprets price undercutting as normal competition. Second, in any given period $t$, if firm reach a uniform industry price after one firm increased its price, an investigation is triggered with probability $\rho_1$, $0 \leq \rho_1$. By contrast, if both firms simultaneously raise their prices to the new uniform level, then an investigation starts with probability $\rho_2$, where $\rho_1 \leq \rho_2 \leq 1$. In general, the parameter $\rho_1$ is a function of how aware and active the competition authority is in fighting cartels. The difference between these two probabilities additionally measures how sensitive the competition authority is with respect to simultaneous price increases. Once an investigation is triggered, the cartel is convicted and each member has to pay an antitrust fine $F > 0$. Moreover, collusion is impossible and firms are forced to compete from this period onwards.

First consider a simultaneous price increase by both firms. In the period of the price increase to $p_1$, the probability of detection is $\rho_2$. This increase is sustainable if no firm has an incentive to compete and undercut the target price, that is,

$$(1 - \rho_2)\frac{\pi(p_1)}{2(1-\delta)} - \rho_2 F \geq \pi(p_1). \tag{14}$$

Intuitively, a simultaneous price increase is easier to implement, the less active the competition authority is and the lower the antitrust fine. Note that a simultaneous price move is not sustainable for any level of antitrust fine $F > 0$ if and only if $\rho_2 > 2\delta - 1$. This is the assumption that has been implicitly maintained in the analysis thus far. For the remainder of this section we focus on parameters such that $\rho_2 \leq 2\delta - 1$ and $\delta \geq 1/2$.

Now consider a staggered price increase. Suppose firm 1 is the leader and increases its price in period $t$ from $p_0$ to $p_1$. If firm 2 follows suit in period $t+1$, the probability of detection is $\rho_1$. The follower has an incentive to increase its price rather than undercut

$p_1$ and break the cartel if

$$(1-\rho_1)\frac{\pi(p_1)}{2(1-\delta)} - \rho_1 F \geq \pi(p_1). \tag{15}$$

The same condition also ensures that the leader is willing to stick to $p_1$ rather than to compete. Note that this second period incentive constraint only differs from the simultaneous move constraint by the lower probability of antitrust detection. Hence, condition (15) is weakly easier to satisfy than (14).

In period $t$, the first period of the staggered price increase, there is no threat of antitrust detection as firms end up setting different prices. However, firms anticipate that an investigation could be triggered in the following period. The leader has an incentive to raise its price (and lose current period demand) rather than undercutting the follower if

$$\delta[(1-\rho_1)\frac{\pi(p_1)}{2(1-\delta)} - \rho_1 F] \geq \pi(p_0). \tag{16}$$

Meanwhile the follower has no incentive to deviate and increase its price just under $p_1$ if

$$\pi(p_0) + \delta[(1-\rho_1)\frac{\pi(p_1)}{2(1-\delta)} - \rho_1 F] \geq \pi(p_1). \tag{17}$$

It is straightforward to compare the first and second period constraints for staggered price increases. If the price increase is small, $\pi(p_1) \leq \pi(p_0)/\delta$, then the binding constraint is the leader's period $t$ constraint. For large price increases, $\pi(p_1) \geq \pi(p_0)/(1-\delta)$, the follower's period $t$ constraint is most restrictive. For intermediate increases, the second period condition (15) is binding.

This allows us to compare the sustainability of simultaneous versus staggered price increases. The possibility of antitrust investigations introduces a trade-off for cartel members. Simultaneous price increases bear a higher risk of antitrust detection. However, it may be harder for cartels to induce individual firms to adhere to staggered price increases during the first period. First consider intermediate price increases where $\pi(p_0)/\delta \leq \pi(p_1) \leq \pi(p_0)/(1-\delta)$. In this case, the most restrictive condition for a staggered increase is the second period constraint which ensures both firms set the price at the target level. In this case, there is no additional cost in providing incentives for staggered price increases and it is strictly harder to sustain simultaneous increases as $\rho_2 > \rho_1$.

Suppose the price increase is small, $\pi(p_1) \leq \pi(p_0)/\delta$, then the main restriction for a staggered price increase is to ensure the leader does not deviate in the first period. It is easier to sustain condition (16) rather than (14) if and only if

$$(\rho_2 - \rho_1)[\frac{\pi(p_1)}{2(1-\delta)} + F] \geq \frac{1}{\delta}[\pi(p_0) - \delta\pi(p_1)].$$

The LHS is the gain in profits and avoidance of fines from a reduced exposure to cartel detection when using staggered price increases. The RHS is the higher cost in terms of providing cartel discipline to the leader of a staggered price move. It is shown in the appendix to the next proposition that there exists a price level $p_1'$ with $\pi(p_0) < \pi(p_1') < \pi(p_0)/\delta$ such that if $p_1 \geq p_1'$, it is easier to sustain a staggered price increase.

For large price increases, $\pi(p_1) \geq \pi(p_0)/(1-\delta)$, staggered price increases require to satisfy the follower's constraint. Similarly, comparing

$$(\rho_1 - \rho_2)[\frac{\pi(p_1)}{2(1-\delta)} + F] \geq \frac{1}{\delta}[(1-\delta)\pi(p_1) - \pi(p_0)].$$

In this case it can be shown that there exists a maximum price level $p_1''$ with $\pi(p_1'') > \pi(p_0)/(1-\delta)$ below which a staggered price increase is easier to sustain. We thus get the following result.

**Proposition 6** *Consider the possibility of endogenous cartel detection and assume $\rho_2 \leq 2\delta - 1$. There exist target prices $(p_1', p_1'')$ such that if $p_1 \in [p_1', p_1'']$, then a staggered price increase is easier to sustain than a simultaneous price increase.*

Price staggering is harder to implement when the raise is very small (large) and the leader (follower) has a strong incentive to deviate. For intermediate increases, price staggering is always easier to sustain as simultaneous price increases. Furthermore, as shown in the appendix, the higher the competition authority's sensitivity to simultaneous price increases, that is, the larger the difference $\rho_2 - \rho_1$, the lower is $p_1'$ and the higher is $p_2''$, which implies that the cartel uses staggered price increases more often.

These results naturally carry over to price-sensitive detection functions. For example, consider the following extension where, after an increase in the transaction price from $p_0$ to $p_1$, the probability of detection is given by

$$\rho(p_0, p_1) = \rho_1 + (\rho_2 - \rho_1)\mathbb{I} + \rho_3(p_1 - p_0)^2,$$

with $\mathbb{I} = 1$ if firms use simultaneous price increases and $\mathbb{I} = 0$ if prices are staggered. The parameter $\rho_3 > 0$ measures how sensitive the competition authority is with respect to an increase in the price level in the market (see e.g. Harrington (2005)). It is clear that this extension would not change the results of the analysis for intermediate price increases. However, as shown in the appendix, the threshold value of $p_1'$ would increase while the value $p_1''$ would decrease in the parameter $\rho_3$.

# 6 Extensions

## 6.1 Differentiated Products

In this extension we analyze the effect of product differentiation on the sustainability of a staggered price increase. To fix ideas, suppose the two cartel members produce horizontally differentiated products and consumers' utility is given by

$$U(q_1, q_2) = \alpha(q_1 + q_2) - \frac{2}{1+\gamma}[\sum_{i=1}^{2} q_i^2 + \frac{\gamma}{2}(\sum_{i=1}^{2} q_i)^2]$$

where $q_i$ are quantities, $\alpha > 0$ measures market size, and $\gamma \in [0, \infty)$ represents the degree of substitutability (see Shubik and Levitan (1980)).[20] Products are independent when $\gamma = 0$ and perfect substitutes when $\gamma$ goes to infinity. Standard utility maximization results in symmetric demand functions

$$D_i(p_i, p_j; \gamma) = D(p_i, p_j; \gamma) = \frac{1}{2}[\alpha - (1+\gamma)p_i + \frac{\gamma}{2}\sum_{j=1}^{2} p_j].$$

[20]This framework has the nice property that aggregate demand does not depend on the degree of product substitutability or the number of products. See chapter 8 in Motta (2004) for a discussion. The same qualitative results would obtain with the linear-quadratic model of Singh and Vives (1984).

Suppose firms have the same constant marginal cost $c$ and let the profit of firm $i$ be

$$\pi_i(p_i, p_j; \gamma) = \pi(p_i, p_j; \gamma) = (p_i - c)D(p_i, p_j; \gamma).$$

Furthermore, let $p_1 = p_2 = p^{NE}$ be the unique symmetric Nash equilibrium prices. Industry profits are maximized by $p_1 = p_2 = p^M$, yielding each firm half the monopoly profits $\pi^M/2$.[21] We restrict attention to a staggered price increase from the competitive Nash equilibrium price level $p^{NE}$ to the fully collusive price $p^M$ in the absence of market share arrangements. We also focus on grim trigger strategies with reversion to the static Nash equilibrium prices. None of the qualitative effects we discuss below are affected by the punishment strategy of the cartel.

As a benchmark, when products are perfect substitutes, our analysis in Section 2 applies and a staggered price increase from the competitive to the monopoly level with symmetric market shares is sustainable if and only if $\delta \geq 2/3$. In this case, the leader's constraint is always satisfied while the discount factor condition ensures that the follower has no incentive to deviate in the first period of the price move.

Let us now consider the effect of product differentiation on the sustainability of a staggered price increase. We can skip the incentive constraint for sustaining the cartel in the second period, $t+1$, of the price move as it will always be satisfied when the period $t$ constraint of the follower holds (see condition (19) below). In the first period, the leader increases its price to $p^M$ while the follower prices at $p^{NE}$. The leader's best deviation is to match the Nash equilibrium price of the follower which triggers punishment. Hence, the leader has no incentive to deviate if and only if

$$\frac{\delta}{1-\delta} \geq \frac{\pi(p^{NE}, p^{NE}; \gamma) - max\{\pi(p^M, p^{NE}; \gamma), 0\}}{\pi^M/2 - \pi(p^{NE}, p^{NE}; \gamma)}. \tag{18}$$

The numerator is the current period gain from deviating while the denominator is the per period long term loss due to punishment. There exists $gamma^*$ such that the RHS is increasing for values of the substitutability parameter $0 \leq \gamma \leq \gamma^*$ and decreasing for higher values. This is due to two opposing effects. First, suppose products are close

[21]For more details see the proof of the next proposition in the appendix.

substitutes ($\gamma > \gamma^*$) such that the leader is not making any profits in period $t$ on the equilibrium path. In this case, increasing $\gamma$ only lowers the deviation profits of the leader, $\pi(p^{NE}, p^{NE}; \gamma)$, which makes the constraint easier to satisfy. In fact, as the parameter goes to infinity, the RHS approaches zero and - like in the benchmark model - only the follower's constraint may be binding. By contrast, if the leader makes strictly positive profits on the equilibrium path, more substitutability reduce these profits more than it lowers the deviation profits. This means, for low degrees of substitutability ($\gamma \leq \gamma^*$), the RHS is increasing in $\gamma$. In particular, as $\gamma$ goes toward zero, the Nash equilibrium prices approach the monopoly level and the RHS is equal to 1. This implies that condition (18) is satisfied if and only if $\delta \geq 1/2$. Hence, overall, the RHS is maximized, and the sustainability is minimized, at $\gamma = \gamma^*$ where the leader's current period demand is exactly zero. It is shown in the appendix that at this point the leader's condition is satisfied if and only if $\delta \geq 3/4$. From this and our discussion follows that there must exist a range of values $\gamma \in [\gamma', \gamma'']$, with $0 < \gamma' < \gamma^* < \gamma''$ such that a staggered price increase is harder to sustain compared to our benchmark model in Section 2.

Now consider the follower's incentives. The optimal deviation price in the current period solves $p^D = \arg\max_p \pi(p, p^M; \gamma)$. Hence, the follower's constraint in period $t$ can be written as

$$\frac{\delta}{1-\delta} \geq \frac{\pi(p^D, p^M; \gamma) - \pi(p^{NE}, p^M; \gamma)}{\pi^M/2 - \pi(p^{NE}, p^{NE}; \gamma)} \tag{19}$$

The numerator on the RHS is again the difference between current period deviation and equilibrium profits. More substitutability always increases the deviation profits of the follower. More importantly, this effect is always stronger than the effect of $\gamma$ on the equilibrium profits. Thus, the RHS increases in $\gamma$ from an initial value of zero when products are independent to its highest value when products are perfect substitutes. In other words, product differentiation always alleviates the incentive constraint of the follower. We can thus conclude as follows.

**Proposition 7** *Product differentiation makes staggered price increases easier to sustain if products are either sufficiently close substitutes or rather independent. For interme-*

*diate degrees, $\gamma \in [\gamma', \gamma'']$, product differentiation makes it harder to sustain staggered price increases.*

Product differentiation has a non-monotonic effect on the sustainability of a staggered price increase. In the neighbourhood of our benchmark case where products are sufficiently close substitutes, product differentiation relaxes the follower's constraint and allows for more collusion. By contrast, if products are less close substitutes, the leader's constraint becomes binding, which can make it more difficult to sustain the price increase.

## 6.2 Price Staggering with Multiple Firms

In this extension, we analyze the optimal organization of a staggered price increase from $p_1$ to $p_2$ with $n \geq 2$ firms. Let $k \in \{1, 2, .., n\}$ be the number of firms who lead the price increase and charge price $p_2$ in the first period $t$ while the remaining $n - k$ firms charge price $p_1 < p_2$. From period $t+1$ onwards, all firms charge the new cartel price $p_2$. Suppose that the firms are ordered such that firms $i \in \{1, ., k\}$ are leaders while firms $i \in \{k+1, .., n\}$ are followers. Let $s_i$ be the market share of firm $i$ such that $\sum_{i=1}^{n} s_i = 1$.

In period $t + 1$, the cartel is sustainable if, for each firm $i$, it holds that

$$\frac{1}{1-\delta} s_i \pi(p_2) \geq \pi(p_2) \quad \text{or} \quad s_i \geq 1 - \delta \tag{20}$$

In the first period of the staggered price increase, a leading firm $i \in \{1, ., k\}$ has no incentive to lower its price and shave $p_1$ if

$$\frac{\delta}{1-\delta} s_i \pi(p_2) \geq \pi(p_1). \tag{21}$$

Next consider a price follower $i \in \{k + 1, ., n\}$ who has been assigned a market share $\sigma_i$ in period $t$, where $\sum_{i=k+1}^{n} \sigma_i = 1$. The deviation profits of a follower depend on the number of co-followers. If there is only one follower, the optimal deviation is to a price that shaves $p_2$. With more followers, deviation requires undercutting co-followers at $p_1$.

The incentive constraint for a follower is then

$$\sigma_i \pi(p_1) + \frac{\delta}{1-\delta} s_i \pi(p_2) \geq \begin{cases} \pi(p_2) & \text{if } k = n-1, \\ \pi(p_1) & \text{otherwise.} \end{cases} \tag{22}$$

We restrict attention to symmetric market shares for followers in period $t$, that is, $\sigma_i = 1/(n-k)$. This assumption ensures that if the leaders and/or the followers have a binding constraint in an optimal market share allocation, then all firms in the same group must have the same market share. In particular a leading firm obtains a market share of $s_i = s/k$ while a follower gets $s_i = (1-s)/(n-k)$, where $s$ is the total market share of all leaders from period $t+1$. In an optimal cartel organization, firms adjust the overall market share of the leaders in order to maximize sustainability of the cartel. The analysis is similar to the benchmark model and we focus on the comparative statics with respect to the number of leaders and firms.

**Proposition 8** *Consider a staggered price increase with $k \in [1, n-1]$ leaders. There exists a $\delta'_n$ with $(n-1)/n \leq \delta'_n < 1$ such that if $\delta \geq \delta'_n$, then a staggered price increase from $p_1$ to $p_2$ is sustainable. The threshold value $\delta'_n$ (weakly) increases in $n$ and $k$.*

Increasing the number of cartel firms makes collusion harder to sustain as the potential market share gain from undercutting is higher. This standard result carries over to staggered price increase as it applies to both types of period $t$ and $t+1$ incentive constraints. The second comparative statics result with respect to the number of leaders is more subtle. First, if there is more than one follower, the followers' period $t$ constraint is always implied by the followers $t+1$ constraint. In period $t+1$, a deviating follower can increase his market share from $(1-s)/(n-k)$ to 1, which is larger than the increase from $1/(n-k)$ to 1. Moreover, the total market value is higher in period $t+1$ as all firms charge $p_2$.

As a consequence, there are two possible regimes for the optimal organization of the cartel. Either, the period $t+1$ constraints are binding which allows firms to implement a price increase under the same condition as a simultaneous price increase, that is, for $\delta \geq (n-1)/n$. Or, the binding constraints are the period $t$ constraint of the leaders and the period $t+1$ constraint of the followers. Raising the number of leaders $k$ increases the market share of an individual follower and makes condition (20) easier to sustain. At the same time, the market share of an individual leader decreases which makes condition (21) harder to sustain. However, the absolute effect on the discount factor is stronger for the leaders as the stream of discounted cartel profits starts one period later. This implies that the more price leaders there are, the harder it is to sustain collusion. Or, a staggered price increase is easiest to sustain with exactly one leader.

## 6.3 Intertemporal Demand and Strategic Buyers

So far we have assumed that demand is static and not responding to the cartel's staggered pricing. In this subsection we consider strategic buyers who anticipate the cartel's price increase and are able to bring forward their purchase. We investigate how such intertemporal demand patterns affects the sustainability of price staggering.

To fix ideas, we introduce buyers who can purchase the product one period ahead of consumption. In particular, we consider an overlapping generations model where in each period a cohort of buyers of size one enters the market. A proportion $\rho$ of these consumers have to buy and consume the product in the same period. A proportion $1-\rho$ can either buy immediately and keep the product or wait and buy in the next period. These patient buyers consume the product at the end of the second period and then leave the market. We assume that patient buyers purchase in the first period of their life if and only if it yields a strictly larger pay-off. When expected prices are the same across the two periods, they buy in the period of consumption. Suppose that buyers are differentiated with respect to their willingness to pay $\theta$ which is distributed according to a log-concave cumulative density $F(\theta)$. Demand at a price $p$ from a given set of patient or impatient buyers of measure 1 is then simply $D(p) = 1 - F(p)$.

We consider a staggered price increase from $p_1 > c$ to $p_2 \in (p_1, p^m]$. The leader increases its price in period $t$, the other firm follows in $t+1$, and both firms charge $p_2$ from that period onwards. The initial price increase is unexpected which means that the proportion of patient buyers from period $t-1$ are present in period $t$. Those consumers plus the impatient buyers entering in period $t$ buy in this same period. The proportion

$1-\rho$ patient buyers entering in $t$, however, buy in period $t+1$ if and only if there is a weakly lower price next period.

First consider the constraints in period $t+2$ on the equilibrium path. The industry prices in period $t+1$ and $t+2$ are the same which implies that in $t+2$, the patient buyer from $t+1$ plus the impatient buyers from $t+2$ buy. Total demand in this and in any future period is therefore $D(p_2)$. This means the binding constraints for the leader and follower are given by (1) and (2).

In period $t+1$, there is a proportion of $\rho$ impatient buyers willing to buy on the equilibrium path. The patient buyers of period $t$ have bought in the previous period while the current patient buyers wait for period $t+2$. This means that the leader's period $t+1$ incentive constraint is given by

$$s\rho\pi(p_2) + s\frac{\delta}{1-\delta}\pi(p_2) \geq \rho\pi(p_2).$$

Note that the deviation profits are lower compared to our benchmark model. If the leader undercuts the follower, consumers anticipate lower, competitive prices in period $t+2$ and patient buyers delay their purchase. Similarly, the follower's period $t+1$ constraint is given by

$$(1-s)\rho\pi(p_2) + (1-s)\frac{\delta}{1-\delta}\pi(p_2) \geq \rho\pi(p_2).$$

In both constraints, the current period gains from deviation are less than in period $t+2$ while the future gains from cooperation are the same. Hence, if conditions (1) and (2) are satisfied, then the period $t+1$ constraints always hold.

Now consider incentives in period $t$. The leader has no incentive to deviate if

$$\delta s\rho\pi(p_2) + s\frac{\delta^2}{1-\delta}\pi(p_2) \geq \pi(p_1). \tag{23}$$

The leader's deviation profits are the same as in the benchmark model. Shaving the price of the follower means that prices are competitive from period $t+1$. This implies that only patient consumers from $t-1$ and impatient consumers from $t$ buy after a deviation. At the same time, future cartel profit are less than in the benchmark model for any $\rho < 1$ since patient buyers anticipate the price increase and purchase in the current period. Hence, the leader's incentives are harder to sustain.

The follower's incentive constraint in period $t$ is given by

$$\delta(1-s)\rho\pi(p_2) + (1-s)\frac{\delta^2}{1-\delta}\pi(p_2) \geq \pi(p_2) - (2-\rho)\pi(p_1). \tag{24}$$

Again, the future cartel profits on the LHS are less relative to the benchmark due to patient buyers pre-empting the price increase. However, the follower also benefits as his current period demand increases by $(1-\rho)D(p_1)$. It thus depends on the relative size of these two effects as to whether strategic buyers weaken or strengthen the follower's incentives. We consider the overall effect in the next proposition.

**Proposition 9** *The presence of strategic buyers and intertemporal demand makes staggered price increases easier to sustain if and only if*

$$\delta^2 \leq \frac{\pi(p_1)}{\pi(p_2)} \leq \frac{1+\rho}{4}.$$

*Otherwise, they are strictly harder to sustain relative to the benchmark model.*

The effect of strategic buyers on the sustainability of staggered price increases is ambiguous. On the one hand, strategic buyers anticipate higher prices and purchase the products right at the beginning of the cartel's moves. This reduces future demand and profits of the cartel and weakens incentives for both the leader and follower. However, at the same time, the follower benefits from the increased demand from patient buyers. This allows the cartel to relax the follower's constraint when it is binding in the optimal cartel scheme. This occurs when the initial price is sufficiently low relative to the target price. If the initial price is too low though, that is, the step size is large, then the value of the follower's current demand gains is outweighed by the loss in future cartel profits from impatient buyers in $t+1$. As a consequence, strategic buyers increase the potential for collusion if the step size is intermediate and the fraction of those strategic buyers is relatively small.

# 7 Conclusions

The literature on cartels is usually concerned with the sustainability of collusion at some long run target conditions such as the industry monopoly price. There is very little research into the initiation and gradual formation of cartels despite the fact that antitrust cases suggest that firms face various obstacles on their way to a long term stable collusive scheme. One important hurdle is the potential detection by buyers and competition authorities due to collusive patterns in firms' market behaviour.

This paper considers staggered price increases, which are a widely observed strategy by cartels to avoid the suspicion of price collusion. Staggering a price increase introduces a cost as the leader risks losing sales before the follower raises its price. We discuss various forms of cartel organization and their effects on the sustainability of staggered price increases. As such, our results are of immediate interest to competition authorities in their assessment as to whether a given staggered price increase is potentially a sign of collusion or the outcome of competitive behaviour. We find that a single staggered price increase is more likely to be sustainable by a cartel if the step size is neither too small nor too large. When firms implement two consecutive price increases, a cartel may use the same leader twice if the initial industry price is low. By contrast, if the starting price level is higher, it is optimal to use alternating price leadership. When cartel members take turns in leading, they implement a small price increase followed by a larger price increase. Furthermore, when firms face asymmetric cost structures, our analysis suggests that cartels would choose the small, high-cost firm as a leader. Staggered price increases are also easier to sustain when products are not perfect substitutes and there is some degree of product differentiation. Finally, cartels find it easier to implement price staggering when the number of leaders is small.

# Appendix

## Proof of Lemmas 1 and 2

As (1) and (2) are never jointly satisfied with $\delta < 1/2$, we only consider $\delta \geq 1/2$. Condition (3) is satisfied if

$$s \geq \frac{1-\delta}{\delta}\frac{\pi(p_1)}{\pi(p_2)} \equiv s_L.$$

The RHS is decreasing and convex in $\delta$. It takes value $\pi(p_1)/\pi(p_2)$ at $\delta = 1/2$ and is equal to 0 at $\delta = 1$. Hence, if $\pi(p_1)/\pi(p_2) \leq 1/2$, then condition (3) is implied by condition (1). Otherwise, if $\pi(p_1)/\pi(p_2) > 1/2$, then conditions (2) and (3) can be jointly satisfied if and only if $\delta \geq \delta'_L$ where

$$\delta'_L = \{\delta | \delta^2 \pi(p_2) = (1-\delta)\pi(p_1)\}.$$

At $\delta'_L$ the RHS of condition (3) is greater than 1/2. It follows that (1) is satisfied for all $\delta \geq 1/2$. Condition (4) is satisfied if

$$s \leq 1 - \frac{1-\delta}{\delta}\frac{\pi(p_2)-\pi(p_1)}{\pi(p_2)} \equiv s_F.$$

The RHS is increasing and concave in $\delta$. It takes value $\pi(p_1)/\pi(p_2)$ at $\delta = 1/2$ and is equal to 1 at $\delta = 1$. Thus, if $\pi(p_1)/\pi(p_2) > 1/2$, then condition (4) is implied by condition (2). Otherwise, if $\pi(p_1)/\pi(p_2) \leq 1/2$, then conditions (1) and (4) are jointly satisfied if and only if $\delta \geq \delta'_F$ where

$$\delta'_F = \{\delta | \delta^2 \pi(p_2) = (1-\delta)(\pi(p_2) - \pi(p_1))\}.$$

At $\delta'_F$ the RHS of condition (4) is less than 1/2. This implies that (2) is satisfied.

From the above follows that if $\pi(p_1) > \pi(p_2)/2$, then the minimum discount factor is given by $\delta'_L$ and the binding constraints are (2) and (3). Otherwise, the discount factor threshold is $\delta'_F$ and conditions (1) and (4) are binding. Lemma 1 follows. Furthermore, using total differentiation, we get

$$\frac{d\delta_L'}{d\pi(p_1)} = \frac{1-\delta}{2\delta\pi(p_2)+\pi(p_1)} > 0$$

and

$$\frac{d\delta_F'}{d\pi(p_1)} = -\frac{1-\delta}{2\delta\pi(p_2)+\pi(p_1)} < 0.$$

Note that $\delta_L' = \delta_F' = 1/2$ for $\pi(p_1) = \pi(p_2)/2$. This means it holds that $\delta_L' > 1/2$ if $\pi(p_1) > \pi(p_2)/2$ and $\delta_F' > 1/2$ if $\pi(p_1) < \pi(p_2)/2$. Lemma 2 follows. QED.

## Proof of Proposition 1

The highest price $\overline{p}(p_2)$ is obtained when conditions (2) and (3) are jointly satisfied which implies

$$\overline{p}(p_2) = \{p_1 | \delta^2\pi(p_2) = (1-\delta)\pi(p_1)\}.$$

From our analysis in the proof of Lemma 2 follows that $\overline{p}$ is increasing in $\delta$ and $\overline{p}(p_2)$ is defined by $\pi(p_1) = \pi(p^2)/2$ at $\delta = 1/2$. The lowest price is achieved when conditions (1) and (4) are jointly satisfied, that is,

$$\underline{p}(p_2) = \{p_1 | \delta^2\pi(p_2) = (1-\delta)(\pi(p_2) - \pi(p_1))\}.$$

Note that $\underline{p}(p_2)$ is decreasing in $\delta$ and $\underline{p}(p_2) = \overline{p}(p_2)$ at $\delta = 1/2$. The proposition follows. QED.

## Proof of Proposition 2

Show that if conditions (1) and (2) hold and $p_0 \leq p''$, then the leader's constraint (5) is satisfied. The leader's constraint in period $t$ is given by

$$s \geq \frac{1-\delta}{\delta^2}\frac{\pi(p_0)}{\pi(p^m)} \equiv s_L^S.$$

This condition is satisfied for any $s \geq 1-\delta$ if and only if $\delta^2 \geq \pi(p_0)/\pi(p^m)$. If $\pi(p_0) \leq \pi(p^m)/4$, then this holds for any $\delta \geq 1/2$. Since conditions (1) and (2) can only be jointly satisfied if $\delta \geq 1/2$, the above statement follows.

Show that if conditions (2) and (3) hold, then the follower's constraint (6) is satisfied. Note that (6) is hardest to satisfy for $\pi(p_0) = 0$ which yields

$$\frac{\delta^2}{1-\delta}(1-s)\pi(p^m) \geq (1-\delta)\pi(p_1).$$

Condition (3) puts an upper bound on the value of $\pi(p_1)$. Plugging this value into the above condition gives

$$\frac{\delta}{1-\delta} \geq \frac{s}{1-s}$$

which holds for any value that satisfies (2). This establishes the first part of point (i).

To prove the rest of the proposition, consider $p_0 > p''$. There are two cases to consider. If $p_1 \leq p'$, then the binding constraints for the cartel is (5) and either (2) or (4). In the former case, the cartel can be sustained if and only if $s_L^S \leq \delta$ or $\delta \geq \delta_{S1}$ with

$$\delta_{S1} = \{\delta | \delta^3 \pi(p^m) = (1-\delta)\pi(p_0)\}.$$

In the latter case, the minimum discount threshold is given by

$$\delta_{S2} = \{\delta | \delta(1-\delta)\pi(p_1) + \delta(2\delta - 1)\pi(p^m) = (1-\delta)\pi(p_0)\}.$$

It holds that the binding discount factor is $\max\{\delta_{S1}, \delta_{S2}\}$. By total differentiation

$$\frac{d\delta_{S2}}{d\pi(p_1)} = -\frac{(1-\delta)\delta^2}{(2-\delta)\pi(p_0) + \delta(\pi(p^m) - \pi(p_1))} < 0.$$

At $\pi(p_1) = \pi(p^m)/2$ we have

$$\delta_{S2} = \{\delta | \frac{1}{2}\delta(3\delta - 1)\pi(p^m) = (1-\delta)\pi(p_0)\}.$$

The LHS of this definition is larger than the LHS in the definition of $\delta_{S1}$ whereas the RHS are identical. Since $\delta_{S1}$ is independent of $\pi(p_1)$, it follows that the lowest collusive discount factor is $\delta_{S1}$.

If $p_1 > p'$, then the binding constraints for the cartel is (2) and either (3) or (5). The resulting minimum threshold ist thus given by $\max\{\delta_{S1}, \delta'_L\}$. Note that $\delta'_L$ takes value $1/2$ at $\pi(p_1) = \pi(p^m)/2$ and increases in $\pi(p_1)$ whereas $\delta_{S1}$ is independent of $\pi(p_1)$ and strictly larger than $1/2$ for $\pi(p_0) > \pi(p^m)/4$. It follows that the overall lowest collusive discount factor is given by $\delta_{S1}$. Increasing $\pi(p_1)$ raises cartel profits without affecting sustainability up to the point where condition (3) holds with equality. The proposition follows. QED.

## Proof of Proposition 3

First show that if condition (4) hold, then the first leader's constraint (7) is always satisfied. For the first part of the lemma, check that (7) holds if and only if

$$s \leq 1 - \frac{1-\delta}{\delta^2} \frac{\pi(p_0) - \delta\pi(p_1)}{\pi(p^m)}.$$

The RHS is larger than $s_F$ from condition (4) if

$$\frac{1-\delta}{\delta^2} \frac{\delta\pi(p^m) - \pi(p_0)}{\pi(p^m)} > 0$$

which holds for any $\pi(p_0) \leq \pi(p^m)/2$ and $\delta \geq 1/2$.

Note that condition (8) holds if

$$s \geq \frac{1-\delta}{\delta^2} \frac{\pi(p_1) - \pi(p_0)}{\pi(p^m)} \equiv s_F^A.$$

The price increases are not sustainable if and only if there exists no pair $(\pi(p_1), s)$ such that (1), (4) and (8) hold jointly. There always exist values $s$ such that conditions (1) and (4) are jointly satisfied if

$$\pi(p_1) \geq \pi(\underline{p}) = \frac{1-\delta-\delta^2}{1-\delta}\pi(p^m).$$

Since

$$\frac{ds_F^A}{d\pi(p_1)} = \frac{1-\delta}{\delta^2\pi(p^m)} > \frac{1-\delta}{\delta\pi(p^m)} = \frac{ds_F}{d\pi(p_1)} > 0$$

there exist values $s$ such that the conditions (4) and (8) are jointly satisfied if

$$\pi(p_1) \leq \frac{\pi(p_0)}{1-\delta} + \frac{2\delta - 1}{(1-\delta)^2}\delta\pi(p^m).$$

Both conditions can hold simultaneously if

$$\frac{\pi(p_0)}{1-\delta} + \frac{2\delta - 1}{(1-\delta)^2}\delta\pi(p^m) \geq \frac{1-\delta-\delta^2}{1-\delta}\pi(p^m)$$

or

$$\frac{\pi(p_0)}{\pi(p^m)} \geq 1 - \frac{\delta^2(2-\delta)}{1-\delta}. \tag{app-1}$$

Let $\delta_A$ be the discount factor that satisfies this constraint with equality. Totally differentiating yields

$$\frac{d\delta_A}{d\pi(p_0)} = -\frac{1}{2\delta - 1 + \frac{1}{(1-\delta)^2}} < 0$$

for all $\delta \geq 1/2$. Check that at $\pi(p_0) = \pi(p^m)/4$ we get $\delta_A = 1/2$. Hence, if $\pi(p_0) \geq \pi(p^m)/4$, the cartel's price increases can be sustained for any $\delta \geq 1/2$. Otherwise, the price increases are sustainable if $\delta \geq \delta_A > 1/2$.

The maximum sustainable price $p_1$ is bounded from above by (3) and (8) while the maximum market share $s$ is limited by (2) and (4), that is, $s \leq \min\{s_F, \delta\}$. Since the slope of $s_F^A$ is steeper than the slope of $s_F$, three cases are possible. The first one is where (2) and (3) are jointly satisfied. In this case the maximum sustainable price has to satisfy

$$\pi(p_1) = \frac{\delta^2}{1-\delta}\pi(p^m) \equiv \pi_1 = \pi(\overline{p}).$$

The second case is where (2) and (8) hold and the maximum sustainable price satisfies

$$\pi(p_1) = \pi(p_0) + \frac{\delta^3}{1-\delta}\pi(p^m) \equiv \pi_2.$$

Finally, when (4) and (8) hold strictly, $s_F^A = s_F$, and the maximum price satisfies

$$\pi(p_1) = \frac{\pi(p_0)}{1-\delta} + \frac{\delta(2\delta - 1)}{(1-\delta)^2}\pi(p^m) \equiv \pi_3.$$

The maximum sustainable price is $p_A^*$ such that $\pi(p_A^*) = \min\{\pi_1, \pi_2, \pi_3\}$. It is easy to check that $\pi_3 \leq \pi_2$ if and only if

$$\frac{\pi(p_0)}{\pi(p^m)} \leq \delta^2 - \frac{2\delta - 1}{1-\delta} = 1 - \frac{\delta^2(2-\delta)}{1-\delta} + \delta(2\delta - 1) \tag{app-2}$$

and $\pi_1 \leq \pi_2$ if and only if

$$\frac{\pi(p_0)}{\pi(p^m)} \geq \delta^2. \tag{app-3}$$

Since the RHS of (app-2) is smaller than the RHS of (app-3), it follows immediately that if (app-2) holds, then $\pi(p_A^*) = \pi_3 \leq \pi_2 < \pi_1$ and conditions (4) and (8) are satisfied with equality. Moreover, since the RHS of (app-2) is larger than the RHS of (app-1), there always exist values of $p_0$ such that the cartel is sustainable and $\pi(p_A^*) = \pi_3$. It is easy to check that the maximum price is increasing in $p_0$. At the lowest level of $p_0$ such that the cartel is still sustainable, that is, when (app-1) holds with equality, the maximum price is defined by

$$\pi(p_A^*) = \pi_3 = \frac{1-\delta-\delta^2}{1-\delta}\pi(p^m) < \frac{\pi(p^m)}{2}$$

for all $\delta > 1/2$. The corresponding market share at $\pi(p_1) = \pi_3$ is

$$s_F^A = \frac{(1-\delta)\pi(p_0) + (2\delta-1)\pi(p^m)}{(1-\delta)\delta\pi(p^m)}.$$

This value is increasing in $\pi(p_0)$ and it takes value $s_F^A = 1-\delta$ when (app-1) holds with equality and value $s_F^A = \delta$ when (app-2) holds with equality.

For intermediate values of $\pi(p_0)$ such that both (app-2) and (app-3) are not satisfied, we have $\pi(p_A^*) = \pi_2$ and conditions (2) and (8) hold with equality. The maximum price is again increasing in $p_0$ and the market share is $s = \delta$. Finally, if (app-3) holds, we get $\pi(p_A^*) = \pi_1 = \pi(\overline{p})$ and (2) and (3) are jointly satisfied. This maximum price is independent of $p_0$ and the market share of the second leader is $s = \delta$. The proposition follow. QED.

## Proof of Lemma 3 and 4

The leader firm 1's conditions (9) and (11) hold if and only if

$$s \geq \max\{1-\delta+\delta\lambda, \lambda/\delta\}$$

where $\lambda = \pi(c_2, c_1)/\pi(p_2, c_1)$. The first term in the max expression from (9) is larger if and only if $\lambda \leq \delta/(1+\delta)$. From this we get that (9) and (12) can be jointly satisfied if and only if

$$1-\delta+\delta\lambda \leq \frac{2\delta-1}{\delta} \quad \text{or} \quad \lambda \leq \frac{\delta+\delta^2-1}{\delta^2}.$$

Furthermore (11) and (12) can be jointly satisfied if and only if

$$\frac{\lambda}{\delta} \leq \frac{2\delta-1}{\delta} \quad \text{or} \quad \lambda \leq 2\delta-1.$$

It is easy to check that $(\delta+\delta^2-1)/\delta^2 \leq 2\delta-1$ if $\delta \leq 1/\sqrt{2}$ and the result in Lemma 3 follows. For Lemma 4 notice that condition (13) is equivalent to

$$s \geq \frac{2\delta-1}{\delta}\lambda + \frac{1-\delta}{\delta}.$$

This condition can be jointly satisfied with (10) if its RHS is less or equal to $\delta$ which gives the condition in the lemma. To show the result in the proposition check that

$$\frac{\delta+\delta^2-1}{2\delta-1} - \frac{\delta+\delta^2-1}{\delta^2} = \frac{(1-\delta)^2(\delta+\delta^2-1)}{\delta^2(2\delta-1)} \geq 0$$

for all $\delta \geq 1/2$ and $\lambda \geq 0$. Moreover,

$$\frac{\delta+\delta^2-1}{2\delta-1} - (2\delta-1) = \frac{(1-\delta)(3\delta-2)}{2\delta-1} \geq 0$$

for all $\delta \geq 1/\sqrt{2}$. QED.

## Proof of Proposition 7

The Nash equilibrium and monopoly price and profits are, respectively, given by

$$p^{NE} = \frac{2(\alpha + c) + \gamma c}{4 + \gamma}, \pi^{NE} = \frac{(2 + \gamma)(\alpha - c)^2}{(4 + \gamma)^2}$$
$$p^{M} = \frac{\alpha + c}{2}, \pi^{NE} = \frac{(\alpha - c)^2}{4}.$$

For the leader's incentives check that

$$D(p^M, p^{NE}; \gamma) = \frac{(4 - \gamma)(2 + \gamma)(\alpha - c)}{8(4 + \gamma)} \geq 0$$

if $\gamma \leq 4$. The RHS of condition (18) is then $\psi_L = (2+\gamma)/2$ if $\gamma \leq 4$ and $\psi_L = 8(2+\gamma)/\gamma^2$ otherwise. $\psi_L$ is increasing for $\gamma \leq 4$ and decreasing otherwise. At $\gamma = 4$ the RHS is $\psi_L = 4$ which implies $\delta \geq 3/4$. Check that the leader's constraint is not sustainable for $\delta \leq 2/3$ if and only if $\psi_L \geq 2$ which holds if $\gamma' = 2 \leq \gamma \leq \gamma'' = 2(1 + \sqrt{3})$.

To solve for the follower's optimal deviation price, note that $D(p^M, p; \gamma) \geq 0$ if and only if $p \geq p^M - (\alpha - c)/\gamma \equiv p'$. For lower values of $p$, the follower makes monopoly profits and since $p' < p^m$, the local maximizer is at $p = p'$. For $p \geq p'$, the optimal interior solution satisfies

$$p^* = \arg\max_p (p - c) D(p, p^M) = \frac{4(\alpha + c) + \gamma(\alpha + 3c)}{4(2 + \gamma)}.$$

Check that $p^* \geq p'$ if $\gamma \leq 2(1 + \sqrt{3})$ and the optimal deviation price in the main text is $p^D = \max\{p^*, p'\}$. Furthermore, if $\gamma \geq 4$, then the leader is not selling in the current period. Hence, the profits for the follower are given by

$$\pi(p^{NE}, p^M; \gamma) = \begin{cases} (p^{NE} - c) D(p^{NE}, p^M; \gamma) & \text{if } \gamma \leq 4, \\ (p^{NE} - c) D^m(p^{NE}) & \text{otherwise.} \end{cases}$$

where $D^m(p) = (1 + \gamma)(\alpha + c - 2p)/(2 + \gamma)$. This implies there are three different cases to consider for the RHS of (19). If $\gamma \leq 4$, then the RHS is equal to

$$\psi_F = \frac{\pi(p^*, p^M; \gamma) - (p^{NE} - c)D(p^{NE}, p^M; \gamma)}{\pi^M/2 - \pi(p^{NE}, p^{NE}; \gamma)} = \frac{\gamma^2}{8(2+\gamma)}$$

which takes value 0 at $\gamma = 0$ and increases in $\gamma$. For $\gamma \in [4, 2(1+\sqrt{3})]$, we get

$$\psi_F = \frac{\pi(p^*, p^M; \gamma) - (p^{NE} - c)D^m(p^{NE})}{\pi^M/2 - \pi(p^{NE}, p^{NE}; \gamma)} = \frac{\gamma^2}{8(2+\gamma)} + 2 - \frac{8}{\gamma}$$

which also increases in $\gamma$. Finally, for $\gamma > 2(1+\sqrt{3})$, the RHS is equal to

$$\psi_F = \frac{\pi(p', p^M; \gamma) - (p^{NE} - c)D^m(p^{NE})}{\pi^M/2 - \pi(p^{NE}, p^{NE}; \gamma)} = \frac{1}{\gamma^4}[2(1+\gamma)(\gamma-4)(8+\gamma(2+\gamma))]$$

where

$$\frac{\partial \psi_F}{\partial \gamma} = \frac{1}{\gamma^5}[\gamma^2(4+\gamma) + 32(4+3\gamma)] > 0$$

and, by L'Hôpital's rule, $\lim_{\gamma\to\infty} \psi_F = 2$. Thus, for any finite $\gamma > 0$, there exist values $\delta < 2/3$ such that the follower's constraint is satisfied. QED.

## Proof of Proposition 6

A simultaneous price move is sustainable if

$$F \leq \frac{2\delta - 1 - \rho_2}{2(1-\delta)\rho_2}\pi(p_1) \equiv F_{sim}.$$

A staggered price more with a small increase such that $\pi(p_0) \leq \pi(p_1) \leq \pi(p_0)/\delta$ implies that the leader's period $t$ constraint is most restrictive. This move is sustainable if

$$F \leq \frac{1-\rho_1}{2(1-\delta)\rho_1}\pi(p_1) - \frac{1}{\delta\rho_1}\pi(p_0) \equiv F'_{seq}.$$

A staggered price increase is easier to satisfy if $F'_{seq} \geq F_{sim}$ or

$$\pi(p_1) \geq \frac{2(1-\delta)\rho_2}{\delta[\rho_2 - (2\delta - 1)\rho_1]}\pi(p_0) = \pi(p'_1).$$

Check that

$$\frac{\partial \pi(p_1')}{\partial \rho_1} = -\frac{\rho_2}{\rho_1}\frac{\partial \pi(p_1')}{\partial \rho_2} = \frac{2(1-\delta)(2\delta-1)\pi(p_0)\rho_2}{[(2\delta-1)\rho_1+\rho_2]^2} > 0,$$

that is the threshold value is decreasing in $\rho_2$ for $\delta \geq 1/2$, it takes value $\pi(p_0)/\delta$ if $\rho_2 = \rho_1$ and $2(1-\delta)\pi(p_0)/[\delta(1+\rho_1-2\delta\rho_1)] > 0$ if $\rho_2 = 1$. Hence, there exists a unique value $p_1'$ with the properties given in the main text. Moreover, for the extension with price-sensitive detection, let the detection probability with simultaneous price moves be $\rho_2' = \rho_2 + x$ and with price staggering $\rho_1' = \rho_1 + x$ where $x = \rho_3(p_1 - p_0)^2$. Substitute the values $\rho_i'$ for the corresponding $\rho_i$ in the term for $\pi(p_1')$ and taking the derivative with respect to $x$ yields

$$\frac{\partial \pi(p_1')}{\partial x} = \frac{2(1-\delta)(2\delta-1)(\rho_2-\rho_1)\pi(p_0)}{\delta[2(1-\delta)x+(2\delta-1)\rho_1+\rho_2]^2} > 0.$$

Similarly, consider a staggered price move when $\pi(p_1) \geq \pi(p_0)/(1-\delta)$. The follower's period $t$ constraint is most restrictive and satisfied if

$$F \leq \frac{2-3\delta+\delta\rho_1}{2\delta(1-\delta)\rho_1}\pi(p_1) + \frac{1}{\delta\rho_1}\pi(p_0) \equiv F_{seq}''.$$

A staggered price increase is easier to satisfy if $F_{seq}'' \geq F_{sim}$ or

$$\pi(p_1) \leq \frac{2(1-\delta)\rho_2}{(2-3\delta)\rho_2+(2\delta-1)\delta\rho_1}\pi(p_0) = \pi(p_1'').$$

Check that

$$\frac{\partial \pi(p_1')}{\partial \rho_1} = -\frac{\rho_2}{\rho_1}\frac{\partial \pi(p_1')}{\partial \rho_2} = -\frac{2(1-\delta)\delta(2\delta-1)\pi(p_0)\rho_2}{[\delta(2\delta-1)\rho_1+(2-3\delta)\rho_2]^2} < 0$$

which means the threshold value $p_1''$ is increasing in $\rho_2$. Moreover, it takes value $\pi(p_0)/(1-\delta)$ for $\rho_2 = \rho_1$. To verify the comparative statics with price-sensitive detection, use again $\rho_i'$ for the corresponding $\rho_i$ in the term for $\pi(p_1'')$ which yields

$$\frac{\partial \pi(p_1'')}{\partial x} = -\frac{2(1-\delta)(2\delta-1)(\rho_2-\rho_1)\pi(p_0)}{\delta[2(1-\delta)^2x+\delta(2\delta-1)\rho_1+(2-3\delta)\rho_2]^2} < 0.$$

The results in the main text follow. QED.

## Proof of Proposition 8

Note that the period $t+1$ constraint for leaders is given by

$$s \geq (1-\delta)k,$$

and for followers it is

$$s \leq 1-(n-k)(1-\delta).$$

These condition can only be satisfied jointly if $\delta \geq (n-1)/n$. The RHS of the leader's constraint decreases in $\delta$ whereas the RHS of the follower increases. At $\delta = (n-1)/n$, both RHS are equal to $k/n$.

Condition (21) holds if

$$s \geq \frac{1-\delta}{\delta} k \frac{\pi(p_1)}{\pi(p_2)}.$$

The RHS is decreasing and convex in $\delta$. At $\delta = (n-1)/n$, it takes value

$$\frac{k}{n-1}\frac{\pi(p_1)}{\pi(p_2)} \geq \frac{k}{n} \quad \text{or} \quad \frac{\pi(p_1)}{\pi(p_2)} \geq \frac{n-1}{n}. \tag{app-5}$$

If this condition does not hold, then (21) is always satisfied if the leader's period $t+1$ holds. Otherwise, the condition is potentially binding in the optimal cartel arrangement. For $k = n-1$, condition (22) holds if

$$s \leq 1 - \frac{1-\delta}{\delta}\frac{\pi(p_2)-\pi(p_1)}{\pi(p_2)}.$$

At $\delta = (n-1)/n$ the RHS is smaller than $k/n = (n-1)/n$ if

$$\frac{\pi(p_1)}{\pi(p_2)} \leq \frac{1}{n}. \tag{app-6}$$

If this condition does not hold, then (22) is always satisfied if the follower's period $t+1$ holds. For $k < n-1$, condition (22) holds if

$$s \leq 1 - \frac{1-\delta}{\delta}\frac{(n-k-1)\pi(p_1)}{\pi(p_2)}.$$

The RHS is increasing and concave in $\delta$. At $\delta = (n-1)/n$ it takes value

$$1 - \frac{n-k-1}{n-1}\frac{\pi(p_1)}{\pi(p_2)} > \frac{k}{n}$$

which always holds. Hence, (22) is satisfied if the follower's period $t+1$ holds and $\delta \geq 1/2$.

We thus have three distinct parameter constellations. First, suppose (app-5) holds. In this case the binding constraints are (21) and the follower's period $t+1$ constraint. The lowest discount factor to support collusion is then defined by

$$\bar{\delta} = \{\delta | \frac{\delta}{1-\delta}\frac{1-(n-k)(1-\delta)}{k}\pi(p_2) - \pi(p_1) = 0\}$$

which yields, for $\delta \geq (n-1)/n$,

$$\frac{d\bar{\delta}}{dn} = \frac{\delta(1-\delta)^2}{1-(n-k)(1-\delta)^2} \geq 0, \quad \frac{d\bar{\delta}}{dk} = \frac{\delta(1-\delta)}{k}\frac{1-n+\delta n}{1-(n-k)(1-\delta)^2} \geq 0.$$

Next suppose (app-6) and $k = n-1$ hold. The binding constraints are now (22) and the leader's period $t+1$ constraint. The lowest discount factor to support collusion is defined by

$$\bar{\delta} = \{\delta | [\frac{\delta}{1-\delta} - \delta(n-1)]\pi(p_2) - \pi(p_2) + \pi(p_1) = 0\}$$

which yields

$$\frac{d\bar{\delta}}{dn} = \frac{\delta(1-\delta)^2}{2-n+(2-\delta)\delta(n-1)} \geq 0$$

since the denominator increases in $\delta$ and takes value of $(n-1)/n + 1/n^2 > 0$ at $\delta = (n-1)/n$. Finally, for all other parameter values, the period $t$ constraints are satisfied when the period $t+1$ constraints hold. This implies that the price increase is sustainable if $\delta \geq (n-1)/n$. The proposition follows. QED.

## Proof of Proposition 9

Condition (23) holds if

$$s \geq \frac{1-\delta}{\delta(\delta+\rho(1-\delta))}\frac{\pi(p_1)}{\pi(p_2)}.$$

The RHS is convex and decreasing in $\delta$ which implies that there is at most one intersection with $s = \delta$ and $s = 1-\delta$ for $\delta \geq 1/2$. In particular, if the RHS at $\delta = 1/2$ satisfies

$$\frac{2\pi(p_1)}{(1+\rho)\pi(p_2)} \geq 1/2 \quad \text{or} \quad \frac{\pi(p_1)}{\pi(p_2)} \geq \frac{1+\rho}{4}, \tag{app-7}$$

then condition (23) is more restrictive than (1). Similarly, condition (24) holds if

$$s \leq 1 - \frac{1-\delta}{\delta}\frac{\pi(p_2)-(2-\rho)\pi(p_1)}{(\delta+\rho(1-\delta))\pi(p_2)}.$$

As RHS is concave and increasing in $\delta$, we know that (24) is more restrictive than (2) if and only if the RHS at $\delta = 1/2$ is less than 1/2 or

$$\frac{\pi(p_1)}{\pi(p_2)} \leq \frac{3-\rho}{4(2-\rho)}. \tag{app-8}$$

This yields three cases for the parameter values. First, suppose (app-8) is not satisfied. This implies that (app-7) holds and the binding constraints in the optimal cartel arrangement are (23) and (2). The lowest discount factor to support collusion is then defined by

$$\bar{\delta} = \{\delta | \delta^2 \rho\pi(p_2) + \frac{\delta^3}{1-\delta}\pi(p_2) - \pi(p_1) = 0\}.$$

Total differentiation then yields

$$\frac{d\bar{\delta}}{d\rho} = -\frac{\delta^2\pi(p_2)}{\delta\pi(p_2)[(3-2\delta)\delta + 2\rho(1-\delta)^2]/(1-\delta)^2} < 0.$$

Second suppose both conditions (app-7) and (app-8) hold. In this case, the binding constraints are (5) and (6) and the lowest discount factor is given by

$$\bar{\delta} = \{\delta | \delta^2 \rho\pi(p_2) + \frac{\delta^3}{1-\delta}\pi(p_2) - \pi(p_1) + (1-\rho)\pi(p_1) = 0\}.$$

and we get

$$\frac{d\bar{\delta}}{d\rho} = -\frac{\delta\pi(p_2) - \pi(p_1)}{\pi(p_2)[1/(1-\delta)^2 - 1 + \rho]} < 0$$

for all parameter values that satisfy (app-8). Finally, consider the case where (app-7) is not satisfied which implies that (app-8) holds. The binding constraints are (24) and (1). The lowest discount factor to support collusion is then defined by

$$\bar{\delta} = \{\delta|(2-\rho)\pi(p_1) + \delta^2\rho\pi(p_2) + \frac{\delta^3}{1-\delta}\pi(p_2) - \pi(p_2) = 0\}$$

which yields

$$\frac{d\bar{\delta}}{d\rho} = -\frac{\delta^2\pi(p_2) - \pi(p_1)}{\delta\pi(p_2)[(3-2\delta)\delta + 2\rho(1-\delta)^2]/(1-\delta)^2} \geq 0$$

if and only if $\pi(p_1)/\pi(p_2) \geq \delta^2$. The proposition follows. QED.

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