"What is socialism today? Conceptions of a cooperative economy"

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Abstract. Socialism is back on the political agenda in the United States. Politicians and some economists who identify as socialists, however, do not discuss property relations, a topic that was central in the intellectual history of socialism, but rather limit themselves to advocacy of economic reforms, funded through taxation, that would tilt the income distribution in favor of the disadvantaged in society. In the absence of a more precise discussion of property relations, the presumption must be that ownership of firms would remain private or corporate with privately owned shares. This formula is identified with the Nordic and other western European social democracies.

In this article, I propose several variants of socialism, which are characterized by different kinds of property relation in the ownership of society's firms. In addition to varying property relations, I include as part of socialism a conception of what it means for a socialist society to possess a *cooperative ethos*, in place of the *individualistic ethos* of capitalist society. Differences in ethea are modeled as differences in the manner in which economic agents optimize. With an individualistic ethos, economic agents optimize in the manner of John Nash, while under a cooperative ethos, many optimize in the manner of Immanuel Kant. It is shown that Kantian optimization can decentralize resource allocation in ways that neatly separate issues of income distribution from those of efficiency. In particular, remuneration of labor and capital contributions to production need no longer be linked to marginal-product pricing of these factors, as is the key to efficiency with capitalist property relations. I present simulations of socialist income distributions, and offer some tentative conclusions concerning how we should conceive of socialism today.

Contents

1. <u>Introduction</u>

Socialism is back on the political agenda in the United States. Bernie Sanders and Alexandra Ocasio-Cortez (AOC) are self-declared socialists, and the Democratic Socialist Alliance, to which AOC belongs, has grown exponentially in the last few years. Most of the current crop of Democratic Party would-be presidential candidates support policies that many call socialist – single-payer health insurance, guaranteed employment, massive infrastructural investment, universal pre-school, and state-financed tertiary education.   About one-half of young adults in the United States polled in surveys state they prefer socialism to capitalism[1].

At least five recent books discuss the ills of capitalism, and recommend reforms: Piketty (2015), Atkinson (2015), Corneo (2017), Stiglitz (2019) and Saez and Zucman (2019).   Piketty argues that the period of the *trente glorieuses* , 1945-1975, when income inequality in the advanced capitalist democracies was low by historical standards and the welfare state was ascendant, was not an advanced phase of a more benign capitalism, but rather a pause in the otherwise steady increase in the concentration of wealth and income, brought about by the catastrophes of the 20th century – two world wars and a depression—that set capital on its heels. His central reform proposal is to tax wealth.   Atkinson and Stiglitz propose menus of reform to weaken capital and increase the real income of the working and middle classes – the latter would be funded in the main by taxation—as well as anti-trust and pro-labor legislation that would alter the bargaining power of labor and capital in labor's favor.   Corneo proposes that the state purchase shares of capitalist corporations, eventually taking a sizable share of corporate profits for the public purse. Saez and Zucman are concerned with raising substantially taxes on the very rich.   The reforms proposed by Sanders, AOC, Piketty, Atkinson, Corneo ,Stiglitz , Saez and Zucman would implement a kind of socialism called *social democracy,* whose defining

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[1] The GenForward Survey, conducted by the University of Chicago, whose respondents are between the ages of 18 and 34, reports that 49% hold a favorable view of capitalism and 45% hold a favorable view of socialism. Sixty-two percent think 'we need a strong government to handle today's complex economic problems.' (*Chicago Tribune,* May 18, 2018)

1   characteristic is that capitalist property relations – centrally, the private ownership of firms—

2   would remain largely intact, as would the income allocation rule. Investment in infrastructure,

3   research, and human beings would increase substantially, funded by taxation.   Stiglitz, indeed,

4   calls his design 'progressive capitalism,' rather than social democracy, although this may be a

5   gloss that he adopts in order to increase its appeal to an American audience.   The most

6   advanced examples of social democracy in today's world are the economic regimes in the Nordic

7   countries – as one travels south in Europe, social democracy becomes somewhat attenuated,

8   although in France, the state still collects about one-half of the national income in taxes.   Social

9   democracy has also become attenuated over time, as well as space, in Europe, as in almost all

10 countries, the state's share of national income has fallen in the last twenty years.

11   Social democracy, however, is only one variant of socialism: I will call it Socialism 1. At

12 the other pole on the interval of socialist variants is the regime of central planning, best

13 represented by the Soviet Union and China prior to 1979.   It is fair to say that the architects of

14 the centrally planned economies were attempting to implement what they saw as Karl Marx's

15 vision of socialism, a system in which private ownership of firms (the 'means of production') is

16 abolished and replaced by state ownership.   Combining state ownership with central planning

17 (in place of market allocation) and political control by one party (in place of democracy) turned

18 out to deliver a toxic cocktail, from both the political and economic viewpoints.   While central

19 planning in the Soviet Union engendered rapid industrialization, and in particular enabled the

20 Russians to turn around Hitler's onslaught to the east, economic development eventually

21 atrophied after the low-hanging fruit had been gathered – moving large populations of semi-

22 employed peasants into urban industry (see Allen [2003]).   The absence of democratic political

23 competition, in combination with the absence of decentralization via markets, induced economic

24 atrophy. The Chinese, however, through the introduction of markets and quasi-private property

25 in rural areas, beginning in 1979, developed a dual economy, with a fast-growing private sector,

26 and a slow-growing but still significant state sector.

27   My intention in this paper is to retrieve, from the history of the socialist idea, several

28 alternatives to these two socialist varieties. I will here develop a second variant, and several sub-

29 variants. I call this variant Socialism 2; the rule 'from each according to his ability, to each

30 according to his contribution' is implemented, where the contribution can be one of labor or

31 capital (or both) , and these contributions to production may be treated asymmetrically. These

1 two variants of socialism share with capitalism two features: markets exist for capital, labor and

2 the commodity, and firms maximize profits.

3   Socialism 2 *differs* from capitalism in that firm profits are not distributed to shareholders,

4 but to those who contribute inputs to the firm, of labor and capital.   The background model of

5 capitalism is the Arrow-Debreu model, where there is a distinction between shareholders, who

6 hold a property right in the surplus accruing to the firm after factor payments to labor and capital

7 have been paid, and those who supply capital to the firm.   I will review a simple version of this

8 model in section 2 below.

9   The two variants of socialism that I present differ from capitalism in another way.

10 Capitalism is conventionally defined as a regime characterized by private ownership of firms and

11 productive factors, and competitive markets.   This definition is incomplete.   Capitalism also

12 requires a pervasive ideology, an *individualistic ethos*: that economic behavior is acceptably

13 motived by the accumulation of individual wealth, within the rules of the game.   Those rules

14 include respect for contracts and the law.   It is beyond my scope here to characterize the

15 structure of law under capitalism; surely, law must respect the freedom of individuals to pursue

16 activities that increase their wealth, subject to respecting the rights of others to pursue similar

17 activities.   Marx's view, that I here endorse, is that the 'superstructure' of capitalism (including,

18 principally, its corpus of law) develops in order to support the economic structure, whose

19 principal function, under capitalism, is to engender the accumulation of capital[2].

20   Likewise, socialism – in both its variants – must be characterized by an ideology. I

21 maintain that that ideology is a *cooperative ethos*.   The socialist citizen is not motivated by the

22 pursuit of individual wealth, subject to constraint, but rather by the desire to cooperate with other

23 citizens – in particular, to cooperate in her economic behavior.   She views economic activity as

24 a cooperative venture, rather than an individualistic one. While capitalism's ethos

25 conceptualizes economic activity as a struggle of each against Nature and all other individuals,

26 socialism's ethos conceptualizes it as a struggle of human society, in solidarity, against Nature.

27   That socialism includes a cooperative ethos is an old idea – just as the individualistic

28 ethos of capitalism has been recognized for centuries, for which one need only consult Adam

29 Smith's famous adage about what motivates small businessmen.   Smith, of course, was at pains

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[2] For a modern elaboration, see Cohen (1978). For a recent treatise on how law has evolved to
support the accumulation of capital, see Pistor (2019).

1   to point out that the individualistic ethos would (given certain rules and markets) result, willy-

2   nilly, in an outcome that was good for the many, an idea that is represented today in the first

3   theorem of welfare economics.   *Models* of socialist economies, however, have as yet not

4   incorporated cooperative behavior, except to the extent that one might, trivially, consider non-

5   capitalist property relations in firms to constitute a form of cooperation. I say that non-capitalist

6   property relations alone are insufficient to characterize the cooperative ethos, and moreover, if

7   we include a behavioral model of cooperation as a necessary component of socialism, we can

8   extend Smith's adage, as will be shown – stronger forms of the first theorem of welfare

9   economics will obtain under socialism.

10   Have our models of capitalism incorporated the individualistic ethos that I claim is part of

11   capitalism's definition? I say they have; that ethos is instantiated in *Nash optimization* by

12   economic agents.   Indeed, economists frequently say that the decentralization of economic

13   activity that markets accomplish is illustrated by the fact that economic agents need only know

14   prices and their own preferences or technology to choose their actions.   No agent, in the

15   competitive model, at equilibrium, need pay attention to what other agents are doing or might do.

16   To put it slightly differently, in her optimization behavior (whether she be a firm manager, a

17   worker, investor or consumer) the individual treats all others' behavior as parametric. Nash

18   optimization models a protocol of 'going it alone.' Taking as given what all others are doing, the

19   agent solves a decision problem, involving the choice of only her own action, which optimizes

20   her own preference order given the parameters of the problem.

21   The second way (aside from property relations) in which I distinguish between models of

22   socialism and capitalism is, then, that economic behavior of agents under socialism is no longer

23   ubiquitously characterized by Nash optimization.   I represent cooperative behavior by *Kantian*

24   *optimization.*   Under Kantian optimization, which is defined for an important class of games,

25   agents' optimizing behavior is proscribed by what one could call a social norm, that an action be

26   taken only if the agent would prefer, according to her preferences, that all other (relevant) agents

27   take a similar action to the one she is contemplating.   I have developed a theory of Kantian

28   optimization in Roemer (2019), and will assume the reader has some familiarity with the

29   approach, although I will review it very briefly in section 3.

30   In sum, my task here is expand the conception of socialism as a regime of economic

31   allocation beyond the version that is dominating the current political discussion, the version of

1 social democracy.   That version assumes that firms remain privately owned, and that the

2 redistribution of income contemplated in the socialist alternative is financed through taxation of

3 various kinds – of income, wealth, value added, and estates. I will amend social democracy by

4 substituting Kantian optimization for Nash optimization in one or several markets. I will then

5 propose another socialist variant that represents an older idea, that socialism requires new

6 property relations in firms.   The reader should consider this as an example of a richer landscape

7 of socialist allocation mechanisms, not as exhaustive.   Non-private-ownership in these variants,

8 however, is *not* to be identified with bureaucratic control by the state of the firms' actions. Firms

9 will in all cases maximize profits in a market economy, but the distribution of the firm's income

10 will neither be according to the rules of capitalism nor by bureaucratic diktat, but according to

11 specific rules that are defined for the variant in question.   I will be concerned with the efficiency

12 properties of these socialist variants--to be precise, what form, if any, the first theorem of welfare

13 economics takes.   As important in varying the property relations governing firms from capitalist

14 ones is the incorporation of a cooperative ethos in optimizing behavior.

15   The punch line is that we can substitute non-capitalist property relations for capitalist

16 ones, and preserve and *extend* the result that equilibria are Pareto efficient, even in the presence

17 of redistributive taxation, public bads, and public goods.   But doing so requires incorporating

18 cooperative behavior in the formal models of socialism. As a corollary, we should cease to view

19 Nash optimization as a universal conception of rational behavior in games, but as a representing

20 the individualistic ethos that is part and parcel of capitalism.

21   Finally, I will offer some tentative conclusions regarding what variant of socialism is

22 most appropriate in today's world.

23

24 2. The <u>capitalist</u> <u>economy</u> (Arrow-Debreu)

25

26   Let's begin with a simple economy in which a good is produced from labor and capital.

27 There is a firm with a production function $G(\cdot,\cdot)$, where labor (L) is measured in efficiency

28 units. We assume that G is increasing, differentiable, and concave. A private firm owns the

29 technology G.   The population consists of n individuals; the preferences of individual i are

30 represented by a quasi-concave differentiable utility function u^i, defined on bundles of the

31 consumption good and labor. Individual i possesses an endowment of capital $\bar{K}^i$ and

1   (efficiency units of) labor $\overline{L}^i$. Individual i also owns a share θ^i of the firm. This market

2   economy will display prices, for the consumption good, labor, and capital (r is the interest rate).

3   We do not explain how capital was produced: it is simply an endowment of individuals, coming

4   from the un-modeled past.

5

6   Definition 1. A *competitive equilibrium* for the economic environment

7    $\{G,\{u^i,\overline{K}^i,\overline{L}^i,\theta^i \mid i=1,...,n\}\}$ comprises a price vector (p,w,r) , demands for capital and labor

8   by the firm (K^*,L^*) , a supply of the good y^* by the firm, demands for the good $(x^1,...,x^n)$ by

9   the n agents , supplies of labor $(L^1,...,L^n)$ and capital $(K^1,...,K^n)$ by the worker-investors such

10   that:

11   ● (y^*,K^*,L^*) maximizes $py-rK-wK$, subject to $y=G(K,L)$; we denote profits by

12   $\Pi^* = py^* - rK^* - wL^*$;

13   ● (x^i,L^i,K^i) maximizes $u^i(x,L)$ subject to

     $$px = wL + rK + \theta^i\Pi^*$$

14   $$L^i \le \overline{L}^i$$

     $$K^i \le \overline{K}^i$$

15   ● Markets clear: $y^* = \sum x^i, L^* = \sum L^i$, and $K^* = \sum K^i$.

16

17   The first-order conditions for profit-maximization by the firm are:

18   $$G_1(K,L)=\frac{r}{p} \text{ and } G_2(K,L)=\frac{w}{p}, \qquad (2.1)$$

19   where G_j is the jth partial derivative of G.   At equilibrium, it makes sense to say that worker

20   i's contribution to production is $\frac{w}{p}L^i$, if L^i is small compared to $L^S \equiv \sum L^i$, since if i

21   withdraws her labor, the product falls by approximately this amount.   Likewise, the

22   (approximate) contribution of investor i's capital to production is $\frac{r}{p}K^i$. Thus the *total*

23   *contribution* of the factor owners to production is:

24   $$G_1(K^*,L^*)K^* + G_2(K^*,L^*)L^* < G(K^*,L^*), \qquad (2.2)$$

1. where the strict inequality holds if G is strictly concave. That is, after the factor owners are paid

2. for their contributions, a surplus remains, which is the firm's profit.

3.   The average product of the firm is:

4.
$$\frac{G(K^*,L^*)}{G_1(K^*,L^*)K^* + G_2(K^*,L^*)L^*}\,; \qquad\qquad (2.3)$$

5. this is output per unit of input contribution. Because the average product is greater than unity for

6. a strictly concave production function, production in general yields a surplus – output is greater

7. than the sum of factor contributions.

8.   Often, neoclassical economists say that profits are not a surplus, but a return to

9. entrepreneurial or managerial talent. But this is a just-so story. Entrepreneurial talent does not

10. exist in this model. If it did, we should write the production function as $\hat{G}(M,K,L)$, where M

11. entrepreneurial or managerial labor. If m were the wage of such labor, then the firm would

12. maximize profits by maximizing:

13.
$$p\hat{G}(M,K,L) - mM - wL - rK\ . \qquad\qquad (2.4)$$

14. If the managerial input were really the missing input that explains profits, then it must be that at

15. the solution to (2.4), profits are zero: that is, we would have

16.
$$p\hat{G}(M^*,K^*,L^*) = mM^* + rK^* + wL^* = (p\hat{G}_1)M^* + (pG_2)K^* + (pG_3)L^*, \qquad (2.5)$$

17. where $\hat{G}_j$ is the jth partial derivative of $\hat{G}$, and I have used the fact that each factor is paid its

18. marginal value product at the profit-maximizing solution. Now dividing (2.5) by p gives us:

19.
$$\hat{G}(M^*,K^*,L^*) = \hat{G}_1 \cdot M^* + \hat{G}_2 \cdot K^* + \hat{G}_3 \cdot L^*, \qquad\qquad (2.6)$$

20. and so profits are zero if the function $\hat{G}$ is homogeneous of degree one.

21.   However, as I said, it is a fiction to claim that profits are a return to managerial labor[3].

22. Certainly, in the modern corporation, managers are paid salaries (wages), and if the firm is

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[3] In their classic article, Arrow and Debreu (1954, p. 267) write, "The existence of factors private to the firm is the standard justification in economic theory for decreasing returns to scale." They in turn cite similar statements in earlier papers by Hicks and Samuelson. This view, however, conflicts with the postulate that commodities are goods (including labor) that trade on markets. Surely managerial labor is a commodity, but it is distinct from profits, and paid a salary. The entrepreneurial input, on the other hand, often does not trade on markets, and it is only a metaphor to say that profits equal the value of the entrepreneurial input. It is an ethically loaded metaphor that disguises the more intuitive view that profits are the surplus that remains after factor inputs are paid for, which redound to the residual claimant.

1 viable, profits are positive after those salaries are paid.   And there is no market for

2 entrepreneurial labor, although metaphorically, one might think of venture capitalists as

3 attempting to create one.

4   It is certainly commonplace in economics to argue that viewing production functions as

5 characterized by decreasing returns is short-sighted, in the sense that McKenzie (1959) and

6 others argue. My claim is that this view is a tautology, and should not be used to justify profits

7 as a *de facto* payment for an invisible input.   Surely, one can contract concerning property rights

8 to the firm's profits, or the firm's value, but it would be mystical to write contracts concerning

9 the ownership of an invisible production factor. Viewing profits as a return to an invisible factor

10 is an 'as if' statement.   It is more concrete to view profits as a surplus that arises in a market

11 economy, rights to which may be assigned in various ways.

12   There are three remarks:

13   (A1) As is well-known, the competitive equilibrium is Pareto efficient, a fact known as

14 the first theorem of welfare economics;

15   (A2) The price system decentralizes the competitive allocation, in the sense that:

16   o The firm need only know prices and its production function G, but not the

17   preferences of consumers;

18   o Consumers need only know prices, their preferences, and their profit

19   remittances from the firm.

20

21   It is these attributes that summarize the *main virtues* of the capitalist system, viewed as a

22 resource-allocation device. To be somewhat more circumspect, the *dynamic* efficiency of

23 capitalism – its tendency to foster innovation and productivity increases -- is not modeled here.

24 The Pareto efficiency of the equilibrium is a stand–in for that important aspect of capitalism.

Perhaps the most militant defender of the claim that neoclassical profits in a decreasing-returns world are in fact the return to an unstated entrepreneurial input is McKenzie (1959, p. 66). Indeed, in is general-equilibrium work, McKenzie derives the case of decreasing returns as a corollary to the case of constant returns, where an 'entrepreneurial factor' is fixed. He writes, "To bring this model [i.e., decreasing returns] within the linear model we have described, we must introduce the entrepreneurial factor which is private to the firm and *not marketed* (my italics, JER)."

1   The informal view is that profit-maximization induces innovation and technological advance, as

2   capitalists seek to survive in competitive markets.

3   To these, I add a third remark:

4   (A3) Workers and investors receive *precisely their contributions* to production, while the

5   firm owners receive the entire surplus.   The fairness of this allocation is questionable.   For is it

6   not arguable that workers and investors should share in the surplus that emerges in production?

7   The legal structure of capitalism allocates profits to owners, but that is not necessarily fair or

8   ethical.   It is a tradition in neoclassical theory to say that workers are not exploited if they

9   receive wages equal to their marginal (value) products.   Marxists, however, say that workers

10   who receive marginal-product wages *are* exploited because they do not share in the surplus from

11   production. In our present model, investors should probably also be viewed as exploited (by

12   Marxists) for they, too, receive only their contributions to production and do not share in the

13   surplus.[4]

14   The model of this section is too sparse to enable us come to a definitive conclusion

15   concerning whether workers and investors are exploited, or unfairly treated, for it does not report

16   the history whereby individuals became owners, workers, and/or investors of the firm. Therefore,

17   I will not press the case here that workers and investors are exploited, but will be satisfied with

18   the more neutral statement that they are paid precisely their contributions to production, and do

19   not share in the surplus produced, which legally is distributed to the firm's owners.

20   Let us review another important point about this simple capitalist model.   Suppose

21   society wishes to redistribute income from the Arrow-Debreu equilibrium, or to produce a public

22   good.   The simplest policy would be imposing a linear income tax, and distributing the proceeds

23   as an equal demogrant to all citizens. If the income-tax rate were t, then the budget constraint of

24   the worker-investor becomes:

25 $$px = (1-t)(wL + rK + \theta^i(pG(K^*,L^*) - wL^* - rK^*)) + \frac{t}{n}pG(K^*,L^*), \qquad (2.7)$$

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[4] Marx argued that capital did not come about, in its original form, from honest labor, and so he would have laughed at the thought that providers of capital to the firm should be considered exploited. But if some capital accumulation does emerge through honest activity (such as savings from labor income), it might well be appropriate for a Marxist to consider those who provide capital to a firm exploited, if they are paid precisely their contribution to production and do not share in the economic surplus.

1   subject to which the individual chooses his plan (x^i, K^i, L^i) in order to maximize $u^i(x^i, L^i)$.

2   The last term in (2.7) is the value of the demogrant. Treating profits and the size of the

3   demogrant as fixed, as is rational if the individual is a Nash optimizer, and if she is small

4   compared to the size of the population, her first-order conditions for optimization are:

5
$$(1-t)\frac{w}{p} = -\frac{u_2^i(x^i, L^i)}{u_1^i(x^i, L^i)} \text{ and } K^i = \bar{K}^i. \tag{2.8}$$

6   Along with (2.1), this implies that :

7

8
$$(1-t)G_2(K^*, L^*) = -\frac{u_2^i}{u_1^i} = MRS^i, \tag{2.9}$$

9   and a necessary condition for Pareto efficiency is violated – that the marginal rate of substitution

10   between income and labor must equal the marginal rate of transformation between output and

11   labor. Equation (2.9) displays the deadweight loss due to income taxation when $t > 0$.

12   What is salient for us is that the deadweight loss follows from the *Nash optimizing*

13   *behavior* of the agent, who considers the choice of his optimal plan under the assumption that all

14   other agents' actions remain fixed at the equilibrium plans. This observation suggests that it may

15   be incorrect to view the deadweight loss of taxation as a *market* failure – it is, more precisely, a

16   failure of Nash optimization as a coordination device. This observation will turn out to be the

17   key to achieving Pareto efficiency in our socialist variants, when individuals will be assumed to

18   optimize in a Kantian fashion. If the use of markets does not require agents to maximize in the

19   Nash manner, perhaps the deadweight loss of taxation can be circumvented in market economies.

20   A question that is suggested by this analysis is the following. How unique is the

21   capitalist allocation mechanism, in possessing the two desirable attributes (A1) and (A2)? Are

22   the Pareto efficiency of equilibrium and the decentralization of resource allocation necessarily

23   associated with marginal-product remuneration of factors, and private ownership of firms?

24

25   3.   <u>Kantian</u> optimization: Modeling <u>cooperation</u>[5]

26   Let $\mathbf{V} = (V^1, ..., V^n)$ be a game in normal form with n players, where the payoff functions

27     $V^i : I^n \to \Re$ and I is an interval in $\Re_+$, the strategy space for each player. We call the

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[5] This reviews material discussed more thoroughly in Roemer (2019).

1   strategies $E^i \in I$ 'contributions' or 'efforts.'   A game is *strictly monotone increasing*

2   *(decreasing)* if each payoff function V^i is a strictly increasing (decreasing) function of the

3   contributions of the players other than i.

4   <u>Definition</u> 2

5   a) A constant strategy profile $(E, E, ..., E)$ is a *simple Kantian equilibrium* if:

6   $$(\forall i)(E = \arg\max_{x \in I} V^i(x, x, ..., x)) \; ; \tag{3.1}$$

7   b) A strategy profile $(E^1, ..., E^n)$ is an *additive Kantian equilibrium* if:

8   $$(\forall i)(0 = \arg\max_{\rho} V^i(E^1 + \rho, E^2 + \rho, ..., E^n + \rho)) \; ; \tag{3.2}$$

9   c) A strategy profile $(E^1, ..., E^n)$ is a *multiplicative Kantian equilibrium* if

10 $$(\forall i)(1 = \arg\max_{\rho} V^i(\rho E^1, \rho E^2, ..., \rho E^n)) \; ; \tag{3.3}$$

11   The appellation 'Kantian' is derived from the 'simple' case: here, E is the contribution

12 that each player would like all players to make. In Immanuel Kant's language, each player is

13 taking the action he would will be universalized.   In an additive Kantian equilibrium, no player

14 would desire to *translate* the strategy profile by any constant vector. In multiplicative Kantian

15 equilibrium, no player would desire to *re-scale* the strategy profile by any factor.

16

17 <u>Remark.</u> The concepts of additive and multiplicative Kantian equilibrium are nested in simple

18 Kantian equilibrium.   That is, any simple Kantian equilibrium is an additive and multiplicative

19 Kantian equilibrium.

20   If the game **V** is symmetric (for example, there is a function $\hat{V}$ such that for all i,

21 $V^i(E^1, ..., E^n) = \hat{V}(E^i, E^{-i})$) then a simple Kantian equilibrium exists. For games with

22 heterogeneous payoff functions, simple Kantian equilibria generally do not exist, but additive

23 and multiplicative Kantian equilibria often do.

24   The important result is:

25

26 <u>Proposition</u> 1.   *In any strictly monotone game, simple and additive Kantian equilibria are*

27 *Pareto efficient, and any strictly positive multiplicative Kantian equilibrium is Pareto efficient.*

28

1   Strictly increasing games are games with positive externalities, where contributions

2   create a public good.   Strictly decreasing games are games with negative externalities – games

3   with congestion effects.   Proposition 1 justifies calling Kantian optimization a protocol of

4   'cooperation', for it resolves efficiently the free rider problem (in monotone increasing games)

5   and the tragedy of the commons (in monotone decreasing games) that characterize Nash

6   optimization in the presence of externalities.

7   In what follows, we embed Kantian optimization of various kinds in simple general-

8   equilibrium models of socialism.

9

10   4. Socialism 1: Social democracy

11   As defined in section 1, social democracy is an economic mechanism in which firms

12   remain privately owned, individuals contribute factor inputs to firms, but taxation redistributes

13   incomes, perhaps substantially. In this section, we show that social democracy, conceived as a

14   mechanism where citizens optimize according to a Kantian protocol, separates the issue of

15   income distribution from that of efficiency.   Pareto efficient allocations are achievable with any

16   degree of income taxation.

17   We modify the utility functions of citizens, now, to be of the form $u^i(x,L,K)$ where K

18   is the agent's contribution of the capital input to production, and $\dfrac{\partial u^i}{\partial K} < 0$.[6] Otherwise, the

19   economic environment is exactly as in section 2.   The reason for this modification is so that

20   labor and capital contributions are symmetrically treated. If we did not associate any disutility

21   with an investor's lending capital to the firm, we could still carry out the analysis of this section,

22   but the definition of additive Kantian equilibrium would have to be modified to treat corner

23   solutions properly.   This would render the analysis less transparent, and so I have elected instead

24   to modify the utility function, so that (we may assume that) solutions to the investor's problem

25   will be interior, just as we will assume this is the case for labor contributions.

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[6] Perhaps the inclusion of K in the utility function represents a concern to save for the unmodeled future.

1   We first define two games for the economic environment $\{G,\{u^i,\overline{K}^i,\overline{L}^i,\theta^i \mid i=1,...,n\}\}$.

2   The *workers' game* is given by payoff functions, which are defined on the vector of labor

3   supplies:

4

5    $$W^i(L^1,L^2,...,L^n) = u^i\left(\frac{(1-t)(wL^i+rK^i+\theta^i\Pi(K^*,L^*))}{p} + \frac{t}{n}\frac{wL^S+rK^S+\Pi(K^*,L^*)}{p},L^i,K^i\right) , (4.1)$$

6   where for any variable $z,$ $z^S = \sum z^i$ and $\Pi(K^*,L^*) = pG(K^*,L^*) - wL^* - rK^*$. The term

7    $\dfrac{t}{n}\dfrac{wL^S+rK^S+\Pi(K^*,L^*)}{p}$ is the amount of the consumption good that can be purchased with the

8   demogrant from taxation that is returned to each individual.   Note that workers and investors

9   treat profits parametrically, but take into account the effect of their contributions on the

10   demogrant.

11   The *investors' game* is given by the same payoff functions, but defined on the vector of

12   capital investments:

13   $$V^i(K^1,K^2,...,K^n) = u^i\left(\frac{(1-t)(wL^i+rK^i+\theta^i\Pi(K^*,L^*))}{p} + \frac{t}{n}\frac{wL^S+rK^S+\Pi(K^*,L^*)}{p},L^i,K^i\right). (4.2)$$

14   The payoff functions W^i and V^i are identical in these two games, but the strategy spaces on

15   which they are defined differ.   In the workers' game, the parameters are

16   $(p,w,r,K^1,...,K^n,K^*,L^*)$, while in the investors' game, the parameters are

17   $(p,w,r,L^1,...,L^n,K^*,L^*)$.

18   To clarify, each person is (in general) both a worker and an investor. She participates as

19   a player in both the above games, where in one her strategy is a supply of labor, and in the other

20   her strategy is a supply of capital.

21

22   Definition 3.   A *social democratic (Socialist* 1*) equilibrium for the economic environment*

23   $\{G,\{u^i,\overline{K}^i,\overline{L}^i,\theta^i \mid i=1,...,n\}\}$ *at tax rate t*, comprises a price vector (p,w,r) , demands for labor

24   and capital by the firm (K^*,L^*) , a supply of the good y^* by the firm, demands for the good

25   $(x^1,...,x^n)$ by the *n* agents, supplies of labor $(L^1,...,L^n)$ and capital $(K^1,...,K^n)$ by the worker-

26   investors such that:

1   • (y^*, K^*, L^*) maximizes $py - rK - wL$, subject to $y = G(K, L)$; we denote profits by

2       $\Pi^* = pG(K^*, L^*) - rK^* - wL^*$;

3   •   The vector $(L^1, ..., L^n)$ is an additive Kantian equilibrium of the workers' game

4       $\mathbf{W} = \{W^i\}$, given $(K^1, ..., K^n)$;

5   •   The vector $(K^1, ..., K^n)$ is an additive Kantian equilibrium of the investors' game

6       $\mathbf{V} = \{V^i\}$, given $(L^1, ..., L^n)$;

7   •   For all i, $x^i = \dfrac{(1-t)(wL^i + rK^i + \theta^i \Pi(K^*, L^*))}{p} + \dfrac{t}{n} G(K^*, L^*)$;

8   •   All markets clear: $x^S = y^*, L^S = L^*, K^S = K^*$.

9

10

11 The tax rate t is exogenous. Clearly, what differentiates social-democratic equilibrium from

12 capitalist equilibrium is that workers and investors choose their contributions in a cooperative

13 manner, according to the additive Kantian protocol. The consequence of using this protocol is:

14

15 <u>Proposition</u> <u>2</u> *Let* $(K^*, L^*, y^*, \{K^i, L^i, x^i\})$ *be the allocation at a social-democratic equilibrium at*

16 *any tax rate* $t \in [0,1]$. *The equilibrium is Pareto efficient.*

17 <u>Proof:</u>

18 1. By profit-maximization, $pG_1(K^*, L^*) = r$ and $pG_2(K^*, L^*) = w$.

19 2. I state what it means for $(L^1, ..., L^n)$ to be an additive Kantian equilibrium of the game $\mathbf{W}$,

20 given $(K^1, ..., K^n)$:

21

22 $(\forall i) \dfrac{d}{d\rho}\bigg|_{\rho=0} u^i \left(\dfrac{(1-t)(w(L^i + \rho) + rK^i + \theta^i \Pi(K^*, L^*))}{p} + \dfrac{t}{n} \dfrac{w(L^S + n\rho) + rK^S + \Pi(K^*, L^*)}{p}, L^i + \rho, K^i \right) = 0.$

23 Calculate that this reduces to:

24 $$(\forall i) u_1^i \cdot \left((1-t)\dfrac{w}{p} + \dfrac{t}{n}\dfrac{wn}{p} \right) + u_2^i = 0 .$$

25   But this says:

$$(\forall i)(\frac{w}{p} = -\frac{u_2^i}{u_1^i}).$$

3. In like manner, the condition that $(K^1,...,K^n)$ be an additive Kantian equilibrium of the game

V given $(L^1,...,L^n)$ is, for all i, $\quad \frac{r}{p} = -\frac{u_3^i}{u_1^i}$.

4. From steps 1, 2, and 3, we have:

$$(\forall i)\left(G_1(K^*,L^*) = -\frac{u_3^i}{u_1^i} \text{ and } G_2(K^*,L^*) = -\frac{u_2^i}{u_1^i} \right) .$$

Given concavity, these are precisely the conditions that the equilibrium allocation be Pareto efficient. ∎

Existence of social-democratic equilibrium at any tax rate is proved in the Appendix.

The key to this 'first theorem of welfare economics' in social democracy can be seen by comparing the proof of Proposition 2, to equations (2.8) and (2.9), which are the first-order conditions of optimality for a Nash optimizing factor owner. The 'wedge' $(1-t)$ that renders unequal the marginal rate of transformation and the consumer's marginal rate of substitution in these equations appears because the Nash optimizer's counterfactual is that *only he alters his labor supply,* while others' labor supplies remain fixed.   The additive Kantian optimizer's counterfactual, in contrast, is that the *entire vector* of labor supplies is translated by a common constant.   It then turns out that the reduction of the wage through taxation is exactly compensated for by the addition to income from the demogrant, and there is no wedge between the marginal rate of transformation and the consumer's marginal rate of substitution.

Five remarks are in order. The first concerns the information the optimizing agent (say, the worker) needs to compute her optimal labor supply in equilibrium. Under Nash optimization, she needs to know prices and the tax rate.   The Kantian optimizing worker needs to only prices. She need not know the tax rate, because with additive Kantian optimization, if she assumes all workers alter their labor supplies by ε , she computes at equilibrium her total income will change by $w\varepsilon$ regardless of the value of t (because $(1-t)w\varepsilon + \frac{tnw\varepsilon}{n} = w\varepsilon$).

The second remark concerns price illusion.   If the Nash optimizer's contribution (of labor or investment) is small compared to the total, he can reasonably assume that prices remain

1   fixed as he considers his counterfactual contributions, holding all others' constant. For the

2   Kantian optimizer, this is not so, because if all agents increase their labor supplies by a small

3   amount, the market-clearing wage would change. However, in the proof of Proposition 2, I held

4   prices fixed. Thus the price-taking assumption must be strong for the efficiency result to hold.

5   Third, it should be remarked that the ownership structure of the firm – that is, the vector

6     $(\theta^1,...,\theta^n)$ -- is here taken as given, but it may also be viewed as a policy variable.   Corneo

7   (2017) proposes that the state purchase shares in the large firms of the country.   This proposal is

8   easily represented in the social-democratic model.   Suppose the state purchases a share θ^0 of

9   the firm, and distributes its share of profits equally to all households.   This changes the effective

10   shares of individuals from θ^i to $\hat{\theta}^i = \theta^i(1-\theta^0)+\dfrac{\theta^0}{n}$.   Otherwise, the formal model remains as

11   in Definition 3.   There may be political reasons to favor the policy of creating a 'federal

12   shareholder,' as Corneo calls it, to income taxation, as a method of reducing income inequality,

13   but they are not modeled at the level of abstraction adopted here. A polar case of the Corneo

14   model is one where $\theta^0 = 1$.   In this case, profits are equally divided among the whole

15   population.   We would, however, lose the monitoring advantages that might accrue to having

16   firms be in part privately owned. And having the state own a large share of firms introduces the

17   issue of political interference in firm decisions.

18   Fourth, note that although workers' after-tax *wage* is not equal to the marginal product of

19   labor, the allocation is Pareto efficient.

20   Finally, I remark on what the equilibria look like when the utility functions are quasi-

21   linear (that is, linear in consumption).   Examination of the first-order conditions in steps 2 and 3

22   of the proof of proposition 2 shows that all factor supplies remain invariant as we change the tax

23   rate in this case.   It follows that the price vector does not change as we vary *t*.   In other words,

24   production plans remain invariant as we change *t* – all that happens is that income (consumption)

25   is redistributed via the changing demogrant.   Therefore, any Gini coefficient of consumption

26   between the laissez-faire Gini (when $t = 0$) and zero (when $t = 1$) can be achieved *efficiently*.

27   Society can completely separate the issues of equity and efficiency.   For an example, see section

28   8 below.

29

30   5. <u>Socialism 2: An asymmetric sharing economy</u>

1   In the variant of socialism proposed next, the entire product of the firm is distributed to

2   workers and investors.   There are no shareholders. A socialist might bridle at the proposal that

3   the sharing economy is a version of socialism, because capital income, in the form of payments

4   to investors, is remunerated according to the same rule as labor income: that is, each

5   contribution, whether it be a capital investment or labor, receives a share of the economic surplus

6   proportional to the size of the contribution.   Isn't socialism supposed to be a system in which

7   the product is distributed in proportion to labor contributions only? I will motivate the proposal

8   to share the firm's product between workers and investors in section 6.

9   I present two versions of this model. In section 5A, I retain the assumption, made until

10   now, that there is a single firm in the economy, an assumption that has simplified the

11   presentation.   There is, however, a significant issue that is not addressed with the single-firm

12   model, and so in section 5B I present a model with many firms. All firms produce the single

13   good , but with different technologies. (It is also possible to generalize to a model with many

14   consumption goods, but that introduces further complexities that, in the end, do not alter the

15   main conclusions.)

16

17

18   5A. The single-firm model

19   Fix a number $\lambda \in [0,1]$. We now define two games. The first is the workers' game; the

20   strategy of a player is her labor supply, and her payoff function is:

21   for $i = 1,...,n$:

22   $$R^i(L^1,...,L^n) = u^i\left(\frac{wL^i + rK^i}{p} + (\lambda \frac{L^i}{L^S} + (1-\lambda)\frac{K^i}{K^S})\frac{(pG(K^*,L^*) - wL^* - rK^*)}{p} +, L^i, K^i \right), \quad (5.1)$$

23   where $L^S = \sum_{i=1}^n L^i$, $K^S = \sum_{i=1}^n K^i$.   The investors' game is given by payoff functions:

    for $i = 1,...,n$:

24   $$I^i(K^1,...,K^n) = u^i(\frac{rK^i + wL^i}{p} + (\lambda \frac{L^i}{L^S} + (1-\lambda)\frac{K^i}{K^S})\frac{pG(K^*,L^*) - wL^* - rK^*}{p}, L^i, K^i) \quad (5.2)$$

25   Consumers who have both labor and capital endowments will be players in both games, as was

26   the case in social democracy. Note the forms of the payoff functions are identical for the games

27   $\mathbf{R} = \{R^i\}$ and $\mathbf{I} = \{I^i\}$: but the strategy spaces are different.

1

2 Definition 4 A $\lambda-$ *sharing equilibrium for the economic environment* $\{G,\{\bar{K}^i,\bar{L}^i,u^i\}\}$

3 comprises an exogenously chosen number $\lambda \in [0,1]$, a price vector (p,w,r) , a supply of the

4 good y^*, firm factor demands (K^*,L^*) , factor supplies $\{(K^i,L^i)\mid i=1,...,n\}$, and consumption

5 demands x^i , such that:

6

7 • (y^*,K^*,L^*) maximizes the firm's profits $py-rK-wL$ subject to $y=G(K,L)$;

8 • Given the capital supplies $(K^1,...,K^n)$), $(L^1,...,L^n)$ is a multiplicative Kantian

9 equilibrium of the game **R**;

10 • Given the labor supplies $(L^1,...,L^n)$), $(K^1,...,K^n)$ is a multiplicative Kantian

11 equilibrium of the game **I**;

12 • For all $i \geq 1$, $\quad x^i = \dfrac{wL^i+rK^i}{p}+\left(\lambda\dfrac{L^i}{L^S}+(1-\lambda)\dfrac{K^i}{K^S}\right)\dfrac{\Pi(K^*,L^*)}{p}$

13 ▪ All markets clear: $y^*=x^S, \quad L^*=L^S, \quad K^*=K^S$.

14

15   In words, each worker is paid wages for her labor, each investor is paid interest for her

16 capital, and then profits are split into a fund for workers and a fund for investors. These funds

17 are distributed to the respective factor suppliers in proportion to their factor supplies. There is a

18 unidimensional family of equilibria, indexed by λ. If $\lambda=1$, all profits go to workers, and

19 investors receive only their contributions to production. If $\lambda=0$, investors get the entire

20 surplus after the factor contributions are paid for.

21

22 Proposition 3 *Any strictly positive[7]* $\lambda-$ sharing equilibrium *is Pareto efficient*.

23 Proof:

24 1. By profit maximization, $\dfrac{w}{p}=G_2(K^*,L^*)$ and $\dfrac{r}{p}=G_1(K^*,L^*)$.

---

[7] That is, an allocation in which every consumer who is endowed with a positive amount of labor
(capital) supplies a positive amount of labor (capital).

2. Note that if a player has zero labor endowment, he is passive in the game **R** – his only feasible strategy is $L^i = 0$. For the set of players with $\bar{L} > 0$, the condition for the labor allocation's being a multiplicative Kantian equilibrium of the game **R** is:

$$\frac{d}{d\rho}\bigg|_{\rho=1} u^i(\frac{w}{p}\rho L^i + \frac{r}{p}K^i + (\lambda\frac{L^i}{L^S} + (1-\lambda)\frac{K^i}{K^S})\frac{\Pi(K^*,L^*)}{p}, \rho L^i, K^i) = 0,$$

which reduces to $u_1^i \cdot (\frac{w}{p}L^i) + u_2^i L^i = 0$. Thus we have:

$$u_1^i \cdot \frac{w}{p} + u_2^i = 0 \tag{5.3}$$

3. In like manner, for the set of players with $\bar{K}^i > 0$, we have:

$$u_1^i \cdot \frac{r}{p} + u_3^i = 0. \tag{5.4}$$

4. By steps 1, 2 and 3, the allocation is Pareto efficient. ■

5B. Labor management with many firms

Suppose there are several firms producing the economy's single consumption good. Workers and investors will not find joining all firms equally attractive, because the profits of firms will generally differ, and so the profit-sharing component of income will vary across firms. Thus, with many firms, all of which must attract workers and investors, something has to be added to the model to solve this problem. One solution is to charge firm-specific membership fees to workers (and, for us, to investors as well, as long as they are sharing in the profits). The second technique, introduced by Drèze (1989), is for firms to pay a rent to the state, where rents are calculated in order to equalize profits per unit labor across firms. I will follow Drèze. The rents will be returned to the citizenry as an equal demogrant.

The economic environment will consist, then, of n consumers, with utility functions u^i as above, and Λ firms, indexed by $l,$ where the lth firm has production function G^l, all producing the single consumption good. As before, consumer i is endowed with a vector of capital and labor $(\bar{K}^i, \bar{L}^i)$. We will represent the supply of labor by consumer i to the firms in the economy as a Λ- vector $\mathbf{L}^i = (L^{il})$ and the supply of capital by consumer i to the set of firms by a $\Lambda-$ vector $\mathbf{K}^i = (K^{il})$. To avoid further complications which add no additional insight, I

1   will restrict this section to a discussion of *labor-managed firms*:   workers will receive a wage for

2   their labor and share in the firms' profits. Investors will receive interest on their loans, but will

3   not share in the profits.   In other works, the parameter λ of section 6A is assumed to be unity.

4   Before stating the definition of equilibrium, we define the following game, played by all

5   workers.   The strategy of each worker is a $\Lambda-$ vector of labor supplies $\mathbf{L}^i$ to the set of firms:

6

$$V^i(\mathbf{L}^1,...,\mathbf{L}^n) = u^i\left(\frac{w\mathbf{L}^i\cdot\mathbf{1}+r\mathbf{K}^i\cdot\mathbf{1}+\sum_{l=1}^{\Lambda}\frac{L^{il}}{L^{Sl}}(\Pi^l(K^{*l},L^{*l})-R^l)+\frac{R^S}{n}}{p},\mathbf{L}^i\cdot\mathbf{1},\mathbf{K}^i\cdot 1\right),\quad (5.5)$$

8   where $\Pi^l(K^{*l},L^{*l})$ is the profit of firm l at the equilibrium, and R^l is a rent paid by Firm l to the

9   center (and $R^S = \sum_l R^l$).   Here, $\mathbf{1}$ is the Λ -vector of 1's, so $\mathbf{L}^i\cdot\mathbf{1}$ is the total labor supply of

10   consumer i and $\mathbf{K}^i\cdot\mathbf{1}$ is her total investment, and $L^{Sl} = \sum_i L^{il}$.   All variables except the

11   arguments of the payoff functions have fixed values when the game is played.

12

13   We will say that $(\mathbf{L}^1,...,\mathbf{L}^n)$ is a *multiplicative Kantian equilibrium of the game* $\mathbf{V}$ if

14       $$(\forall i=1,...,n)(1=\arg\max_{\rho} V^i(\rho\mathbf{L}^1,...,\rho\mathbf{L}^n)) .\qquad (5.6)$$

15   <u>Definition 5</u> *A labor-managed-firm (LMF) equilibrium for an economic environment*

16   $\{(u^1,\bar{K}^1,\bar{L}^1),...,(u^n,\bar{K}^n,\bar{L}^n),G^1,...,G^{\Lambda}\}$ is a price vector (p,w,r), a profit-maximizing plan for

17   each firm $(K^{*l},L^{*l}),l=1,...,\Lambda$, and a vector of firm rents $(R^1,...,R^{\Lambda})$, such that:

18   • (K^{*l},L^{*l}) maximizes $pG^l(K,L)-wL-rK$, for all firms l;

19   •   Given $(\mathbf{K}^1,...,\mathbf{K}^n)$ the matrix $(\mathbf{L}^1,...,\mathbf{L}^n)$ is a multiplicative Kantian equilibrium of the

20   game $\mathbf{V}$ defined in (5.5);

21   •   Given $(\mathbf{L}^1,...,\mathbf{L}^n)$, for each $i=1,...,n$, $\mathbf{K}^i$ maximizes the utility of consumer i (see the

22   right-hand side of (5.5);

23   •   For all $l=1,...,\Lambda$, R^l is defined by the equation $\dfrac{\Pi^l(K^{*l},L^{*l})-R^l}{L^{Sl}}=\min_j\dfrac{\Pi^j(K^{*j},L^{*j})}{L_j^{Sj}}$;

24   •   All markets clear: for all l, $L^{*l}=\sum_i L^{il},K^{*l}=\sum_i K^{il},\sum_i x^i=\sum_l G^l(K^{*l},L^{*l})$, where x^i is

25   the first argument in u^i of equation (5.5).

1

2   <u>Proposition</u> 4 *Any labor-managed firm equilibrium where every worker supplies positive labor is*

3   *Pareto efficient.*

4   The proof follows the method of the proof of Proposition 3.

5   In reality, it may not be advisable to introduce these firm rents, as they would discourage

6   innovation on the parts of the firm's workers and investors, who would have no incentive to cut

7   costs in order to earn above-normal profits.   As in the Arrow-Debreu model, we may elect to

8   view the set of workers and investors in a firm as having a property right in that firm.

9   Introducing financial markets for firm ownership is beyond the scope of this discussion.

10

11 5C. <u>Summary</u>

12   I review the key features of these two socialist allocation mechanisms.

13   (i)   In each mechanism, firms maximize profits, defined as the surplus over factor

14 contributions, where those contributions are evaluated at marginal-product prices. Profit

15 maximization is an essential ingredient in proving Pareto efficiency (the first welfare theorem).

16 But it is also an informal proxy for believing that the mechanism will encourage technological

17 innovation, although this is not modeled in the static environments postulated here.

18   (ii)   In both variants, the equilibria are Pareto efficient. Resource allocation is

19 decentralized by the existence of markets and competitive prices, and optimization by

20 individuals and firms.   Individual optimization might be either in the manner of Nash, or in the

21 manner (of several versions) of Kant.

22   (iii)   Social democracy (Socialism 1) extends the first welfare theorem to apply to

23 equilibrium allocations for any redistribution of income implemented by a linear income tax and

24 demogrant. Avoiding the deadweight loss of taxation is achieved by cooperation, modeled as

25 additive Kantian optimization of workers in the determination of their labor supplies, to be

26 contrasted with the inefficiency of linear taxation under capitalism, which is due to Nash

27 optimization by workers. Except to the extent that incomes are redistributed via taxation, the

28 economic surplus is defined as conventional profits, and is distributed to owners of firms.

29   (iv)   Under Socialism 2, of this section, the firm is conceptualized as owned by workers

30 and investors, who share in conventional profits after interest payments are paid to investors and

31 wages are paid to workers. There is a unidimensional family of equilibria, indexed by the share

of profits that is allocated to workers.   In general, workers and investors may be treated

asymmetrically.   Pareto efficiency is accomplished via cooperation, modeled as multiplicative

Kantian optimization[8]. I do not have a method of income taxation that will be Pareto efficient

for Socialism 2.

6. On the treatment of capital owners

In defining these socialist variants, I have respected the distinction made in the Arrow-

Debreu model between owners of firms and suppliers of capital to the firm. Both profits and

factor payments to capital suppliers appear as capital income in the US national accounts,

although they are different kinds of income, both legally and conceptually. Their different legal

status is shown by the fact that firm owners only receive their shares of profits after factor

payments have been made.   Owners are the residual claimants, who stand behind factor

suppliers, in the queue whose members divvy up firm income.

One might wish to respect a distinction, in thinking about socialism, between firms that

are created by individuals, and are not incorporated, and publicly-held corporations.   For the

first kind of firm, one might be more inclined to think of profits accruing to the owner as an

entitlement, a return to entrepreneurial talent.   Owners of corporate shares, however, have not in

general contributed any entrepreneurial talent to the firm – indeed, whether a corporate investor

buys shares or bonds, and thereby becomes either an owner or a factor contributor to the firm,

may be due to preferences for risk rather than to having any particular role in the firm's actions.

One possibility for a conception of socialism would be as a regime that encourages the formation

of small firms, which would remain privately owned until a certain level of sales is reached, at

which time the firm must be transformed into a public firm of the kind described in the λ-sharing

economy.   When that level of sales is reached, the firm would be purchased from the private

owner by the state: after that, the distribution of firm income would change as described in

section 5, but the former owner might well be hired to manage the new public firm, given her

superior knowledge of the firm's technology and market.

The distinction between firm owners and suppliers of capital is probably also important

historically.   At the time Marx wrote, the distinction may not have been as important as it is

---

[8] An earlier formulation of a worker-ownership equilibrium is due to Jacques Drèze (1993). In his model, workers maximize in the Nash manner.   The equilibrium is also Pareto efficient.

today, because the middle class was much smaller in the early nineteenth century.   It was likely

the case that firm owners were largely entrepreneurs, and investors were members of the landed

gentry.   The more undeserving of these two groups would appear to be the aristocrats, who

were searching for profitable returns on incomes that came from landed property ultimately

derived from regal distributions to nobility in times past.   The twentieth century saw the advent

of a patrimonial middle class, as described by Piketty (2015), a middle class he defines as

comprising the fiftieth to ninetieth, or perhaps ninety-ninth centiles of the distribution of income

or wealth.   The income and wealth of this class are due more to the productive contributions of

its members than was the income of the aristocracy a return to its members' productive

contributions.   Of course, the wealth of the middle class must be invested productively in any

efficient economy, and returns to owners will accrue. Thus, unless one conceives of socialism as

coming about through a revolution in which the wealth of citizens is confiscated by the state –

and very few of those who call themselves socialist today would advocate this – one must pay

serious attention to providing incentives for citizens to invest their wealth productively.   These

incentives exist in the models that I have proposed.

Given that a large class of citizens will be investors (roughly speaking about 50% of the

households in an advanced economy, because in most advanced capitalist countries, those in the

top half of the wealth distribution own virtually all the financial wealth), the extent to which

(these variants of) socialism would redistribute income from capital to labor is uncertain and

important.   The uncertainty is clear; the importance derives from the fact that surely the most

disadvantaged in society are those with little or no wealth, whose incomes come solely from

labor. Although socialism, with its cooperative ethos, should give priority to investment that will

augment the skills and earning power of the disadvantaged, we can suppose that class differences

will continue to remain between those whose incomes come primarily from labor, and those

whose incomes have a significant capital component, and membership in these classes will

therefore continue to be closely correlated to social and economic advantage in family

background.   Although I have not here discussed here what constitutes *socialist justice* – my

views on that are presented in Roemer (2017) --that justice is roughly defined by the elimination

of disadvantage due to the luck of the birth lottery. See section 10 below. It is for this reason

that the partition of income between capital and labor income will remain important.   That

1 partition will cease to be of ethical concern only when there is little correlation between the

2 source of a person's income and the degree of social/economic disadvantage of his background.

3

4 7. Is Kantian optimization credible behavior, or simply a mathematical curiosum?

5   The three pre-requisites for a group of individuals to optimize in the Kantian manner are

6 desire, understanding, and trust.   People must *desire* to cooperate, because they see their

7 situation as one of solidarity, meaning they face a common economic problem (the struggle

8 against Nature) whose solution will require cooperation.   Secondly, they must *understand* that

9 Kantian optimization can lead to good (efficient) solutions to the economic problem.   Third,

10 each must *trust* that others will optimize in the Kantian manner if he/she does, so that the

11 Kantians will not be taken advantage of by Nash optimizers, who can always benefit as

12 individuals, at least in the short run, by playing Nash against the Kantian crowd. If desire,

13 understanding and trust exist, groups of economic agents may entrust decisions (such as optimal

14 investments or supplies of labor) to organizations that represent them, such as unions, which can

15 carry out the Kantian optimization for them. Indeed, the success of the Nordic social

16 democracies depended on strong centralized labor unions, which in their tripartite negotiations

17 with capitalists and government may have proposed Kantian-optimal strategies for workers (this

18 is a conjecture for further research).

19   We know that ethnic, linguistic, and religious heterogeneity frustrate the realization

20 among individuals that they face a situation of solidarity, and many have argued that the

21 homogeneity of Nordic populations along these dimensions contributed to the success of social

22 democracy, because of the *relative* ease of establishing trust in a homogeneous group.

23   The mathematical *similarity* between Nash and Kantian equilibrium of agents is that each

24 agent chooses a preferred action in a set of counterfactual strategy profiles in the game, and

25 equilibrium obtains when all agents agree upon what the most preferred strategy profile is. The

26 *difference* between Nash and Kant protocols is in the specification of the counterfactual sets of

27 strategy profiles.   In Nash optimization, each agent inspects a *differen*t set of counterfactual

28 profiles, while in Kantian optimization, all agents inspect the *same* counterfactual set. Thus,

29 Kantian optimization builds in symmetry that does not exist in Nash optimization.   It is this

30 symmetry that holds the ethical appeal for the Kantian: for fairness, in our minds, is deeply

31 associated with symmetrical treatment.   It is for this reason that I suggest that *if* citizens acquire

1   an understanding of their solidaristic situation, and thereby desire to cooperate, the technology of

2   Kantian optimization may become an ethically attractive optimization protocol.

3

4   8. <u>Simulations</u> of <u>socialist</u> <u>income</u> <u>distributions</u>

5   It is important to emphasize that the advantage, in terms of reduction of income

6   inequality, of the laissez-faire socialist variant of social democracy, and of the asymmetric

7   sharing economies, only exists when the production function of the firm is strictly concave.   For

8   suppose that, to the contrary, G is homogeneous of degree one – that is, production enjoys

9   constant returns to scale.   Then the asymmetric sharing equilibrium and the worker-ownership

10   equilibrium are identical to the capitalist equilibrium with zero taxation.   In other words, what

11   those variants of socialism do is distribute *profits* in proportion to factor contributions; but when

12   there are constant returns to scale, profits are zero in these models, there is no surplus to

13   distribute, and so capitalism with zero taxation is equivalent to both socialist variants.

14   Democratic-socialist equilibria will differ, however, from capitalist equilibrium *if the tax rate is*

15   *positive*, because workers optimize in different ways (Nash and Kant) in their labor-supply

16   decisions in the socialist and capitalist models.

17   I am unsure how best to characterize returns to scale for the economy as a whole.   There

18   is certainly a tradition of assuming that returns are constant[9].   In this section I will simulate three

19   models in which I assume decreasing returns so that profits are positive. These are: capitalism

20   with a positive tax rate, social-democracy with various positive tax rates, and the sharing

21   economy with various distributions of the endowment of capital.

22   I assume a Cobb-Douglas production function:

23 $$G(K,L) = AK^{\gamma} L^{\varphi-\gamma} \quad \text{, some } \gamma < \varphi < 1 \text{ ,} \qquad\qquad (8.1)$$

24   and a quasi-linear utility function

---

[9] It would be a false inference to argue that, because reported profits in the national accounts are positive, therefore returns to scale must be decreasing.   Reported profits are different from neoclassical profits. One important reason for the difference is that most firms own some of their capital stock – it is not rented from households as in the Arrow-Debreu model – and interest payments on that capital are zero.   Were we to subtract imputed interest payments on owned capital from firm profits, it is conceivable profits would be zero, in line with the constant-returns assumption. Of course, the main reason that profits are positive is monopoly pricing by firms, perhaps related to their having increasing returns. For instance, Stiglitz (2019), argues that non-competitive pricing is a central cause of inequality today.

1
$$u(x,\ell) = x - b\frac{\ell^{1+1/\eta}}{1+1/\eta} \qquad\qquad (8.2)$$

2   where x is income measured in thousands of dollars per annum, ℓ is labor *time* expended in a

3   calendar year measured in (full-time equivalent) years, and η is the elasticity of substitution of

4   labor time with respect to the wage[10].   Workers differ in the efficiency units of their labor.   The

5   labor efficiency of a worker is s, measured in some normalized amount of output that the worker

6   can produce with one year's work.   I assume a lognormal distribution of s in the population,

7   with a mean of unity and a median of 0.85; that is

8
$$\int_0^\infty s\, dF(s) = 1, \quad F(0.85) = 0.5 \ , \qquad\qquad (8.3)$$

9   where F is lognormal. It will be assumed that the share of society's capital endowment owned

10   by a worker of type s is given by an increasing function $k(\cdot)$: in reality, this may be only

11   approximately correct. It is assumed every member of the population is a worker.   Thus:

12
$$\int_0^\infty k(s)\, dF(s) = 1 \ , \qquad\qquad (8.4)$$

13   and the amount of a worker's capital endowment is $k(s)\overline{k}$, where $\overline{k}$ is capital per worker. The

14   number of workers is n, total capital stock is $K^{tot} = n\overline{k}$.

15

16   The parameters of the model are the functions $F(\cdot)$ and $k(\cdot)$, and the numbers

17   $(A, b, \varphi, \gamma, \eta, n, K^{tot})$.

18

19   A. Calibration

20

21   Total wealth will be total financial wealth in the US in 2016.   The distribution of total wealth

22   is computed from the data set of G. Zucman (2017).   The data series provides a cumulative

23   distribution function of financial wealth of US adults[11].   Total financial wealth is $55.6 trillion.

---

[10] We need not suppose that preferences have capital lent to firms as an argument. Individuals supply their entire capital endowment to the firm.
[11] Total financial wealth is the sum of equities, fixed-income assets, pensions, and life insurance. The file from Zucman (2017) is "USdina2016.dta," which gives an empirical distribution function of financial wealth.

1   I assume this is the value of capital invested in the corporate sector.   Value added in the

2   corporate sector in 2017 was $y =$ \$9.5 trillion[12]: this is the 'GDP' of the economy that I study[13].

3   Conventionally, I take $\gamma = 0.333$.   (Capital's share appears to be increasing, perhaps

4   implying this value is too small.) I take $\varphi = 0.93$. [14] I will suppose $\eta = 0.1$, although there is

5   much debate about the appropriate value.   Finally, the average US worker works 44 hours per

6   week (amortized over 52 weeks).   If we take one FTE year of labor to be 2080 hours (that is,

7   40×52 hours), then total labor time expended is:

8
$$\ell^{tot} = \frac{44}{40} n = 1.1n. \tag{8.5}$$

9   In the continuous version, the set of workers is a continuum of size 1; however, to calibrate the

10 model I take the number of workers to be $n = 127$ million (workers in private industry, Bureau of

11 Economic Analysis). I will assume that the observed values are those of a competitive capitalist

12 economy with a tax rate of $t = 0.30$.   The calibration task is to compute the function $k(\cdot)$ and

13 the numbers (A,b) ; other parameters have been set above. We assume the price of output is

14 unity, and the wage for one unit of labor in efficiency units, and the interest rate for capital, are

15 (w,r) , respectively.

16   (i) The firm's problem is to demand capital and labor to

17
$$\max G(K,L) - wL - rK, \quad, \tag{8.6}$$

18 where L is labor in efficiency units. The f.o.c.'s are:
19
20

21
$$w = \frac{(\varphi - \gamma)y}{L}, \quad r = \frac{\gamma y}{K}. \tag{8.7}$$

22 Denote the solution by (K^*, L^*) . Profits are $(1 - \varphi)y$.
23

---

[12] From National Income and Product Accounts (NIPA), Table 1.14, line 17.
[13] Zucman's data on financial wealth are for a sample of the population of US adults.   The total population of US adults in 2016 was 238 million.   Thus, average capital per adult was \$233 thousand.   However, in the model I take the population of *workers* to be 127 million, and I assume they own all the capital.   Capital per worker is thus \$438,000. This renders the worker in the model wealthier than he or she is reality, and this should be recalled when I present below Gini coefficients of the distribution of income with various tax rates.
[14] Thus capital's share of national income will be $1 - \varphi + \gamma = 0.40$, approximately in line with recent estimates in the United States.

1   (ii) The problem of a worker of type s is to choose $(x(s), L(s))$ to :

$$\max u(x(s), \frac{L(s)}{s})$$

2       $s.t.$ (8.8)

$$x(s) = (1-t)(wL(s) + rk(s)\bar{k}) + \frac{t}{n}G(K^*, L^*)$$

3 The f.o.c. under Nash optimization by the worker is:

4     $$L(s) = \left(\frac{(1-t)w}{b}\right)^{\eta} s^{1+\eta} \text{ and } \ell(s) = \frac{L(s)}{s} = \left(\frac{(1-t)w}{b}\right)^{\eta} s^{\eta}.$$ (8.9)

5 Thus average (per-worker) units of efficiency labor supplied are :

6     $$\int L(s)dF(s) = \left(\frac{(1-t)w}{b}\right)^{\eta} \mu_{1+\eta} \text{ where } \mu_{1+\eta} \equiv \int s^{1+\eta} dF(s).$$ (8.10)

7 Furthermore, from (8.5) we have:

8     $$1.1n = \int \frac{L(s)}{s}dF(s) = \left(\frac{(1-t)w}{b}\right)^{\eta} \mu_{\eta} \text{ where } \mu_{\eta} \equiv \int s^{\eta} dF(s).$$ (8.11)

9   We next compute the function $k(\cdot)$. For this we use the distribution of wealth in 2016,

10 computed (by the author) from Zucman (2017):

11

Wealth fractile	Fraction of total financial wealth owned by fractile
Bottom half	0.025
.50 - .90	0.261
.90 - .99	0.303
.99 -.999	0.177
.999 -.9999	0.105
.9999 -1.0	0.129

12

13 Table 1. Wealth shares for various fractiles of the population, computed by the author from

14 Zucman (2017)

15

16 Denote quantile q of the distribution of F by s_q . (For example, the median is $s_{0.5} = 0.85$.)

17 We compute the values $(s_{0.5}, s_{0.9}, s_{0.99}, s_{0.999}, s_{0.9999})$ from postulate (8.3) . We now define the

18 function $k(s)$ by a piece-wise linear approximation:

19

1

$$k(s) = \begin{cases} a_0 s, \ s \in [0, s_{0.5}) \\ a_0 s_{0.5} + a_1(s - s_{0.5}), \quad s \in [s_{0.5}, s_{0.9}) \\ a_0 s_{0.5} + a_1(s_{0.9} - s_{0.5}) + a_2(s - s_{0.9}), \quad s \in [s_{0.9}, s_{0.99}) \\ a_0 s_{0.5} + a_1(s_{0.9} - s_{0.5}) + a_2(s_{0.99} - s_{0.9}) + a_3(s - s_{0.99}), \quad s \in [s_{0.99}, s_{0.999}) \\ a_0 s_{0.5} + a_1(s_{0.9} - s_{0.5}) + a_2(s_{0.99} - s_{0.9}) + a_3(s_{0.999} - s_{0.99}) + a_4(s - s_{0.999}), \quad s \in [s_{0.999}, s_{0.9999}] \\ a_0 s_{0.5} + a_1(s_{0.9} - s_{0.5}) + a_2(s_{0.99} - s_{0.9}) + a_3(s_{0.999} - s_{0.99}) + a_4(s_{0.9999} - s_{0.999}) + a_5(s - s_{0.9999}), \quad s \geq s_{0.9999} \end{cases}$$

2

3 $\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (8.12)

4

5 We compute the values of $(a_0, a_1, a_2, a_3, a_4, a_5)$ so that in each interval (s_q, s_p), we have the

6 wealth share equals the estimated wealth share from Table 1, and $\int_0^\infty k(s) dF(s) = 1$. Thus

7 $k(s) dF(s)$ is the 'share' of total capital owned by workers of type s.

8 This calibration gives:

9 $\qquad\qquad (a_0, a_1, a_2, a_3, a_4, a_5) = (0.088, \ 1.68, \ 3.94, \ 23.04, \ 105.12, \ 916.49)$. (8.13)

10 $\qquad\quad$ We now finish the calibration as follows. From (8.11), we compute that:

11 $\qquad\qquad\qquad\qquad\qquad \dfrac{w}{b} = 4.289$. $\qquad\qquad\qquad\qquad\qquad$ (8.14)

12 Substituting this ratio into (8.10), we compute $L^{ave} = \int L(s) dF(s)$, and hence $L^{tot} = n L^{ave}$. In

13 equilibrium, we have $L^{tot} = L^*$. Of course, $K^{tot} = K^*$. Now, from the equation:

14

15 $\qquad\qquad\qquad\qquad\qquad y = A(K^*)^\gamma (L^*)^{\varphi - \gamma} \qquad\qquad\qquad\qquad$ (8.15)

16 , we compute $A = 3384$. From equations (8.7), we compute $(w, r) = (\$39299., 0.057)$. That is,

17 the wage for one unit (year) of efficiency labor is about \$39,300. Finally, from (8.14) we

18 compute $b = 9163$. This completes the calibration.

19 $\qquad\quad$ Income is defined by the constraint in program (8.8). We check the calculation by

20 checking that incomes add up to GNP, that is, that:

21

22 $\qquad\qquad\qquad\qquad\qquad \int x(s) dF(s) = G(K^*, L^*). \qquad\qquad\qquad\qquad$ (8.16)

1   Average income per worker is $74,803, and $y = \$9.5$ trillion, as stated above.

2

3   B. Gini coefficient

4   The Gini coefficient of income at this equilibrium is 0.374[15].   The capitalist equilibrium

5   is Pareto inefficient because of the deadweight loss at positive taxation.

6

7   C.   How the Gini coefficient changes with the tax rate in social-democratic equilibrium

8

9   In calibrating the model in section 8A, I took the equilibrium to be that of capitalism with

10   taxation.   In particular, we assumed that workers choose their factor supplies according to Nash

11   optimization. Thus, the equilibrium calculated in section 8A above is Pareto inefficient due to

12   the deadweight loss of taxation.

13   We next compute the capitalist equilibrium when $t = 0$ for the parameterized model.

14   This allocation will be Pareto efficient.   As I pointed out earlier[16], because the utility function is

15   quasi-linear, as we vary the tax rate in social-democratic equilibrium, none of prices change, nor

16   do any labor supplies--all that occurs is a redistribution of income among citizens.   Thus, if the

17   capitalist equilibrium when $t = 0$ is described by the functions and prices $\{L(s),k(s),w,r\}$, then

18   income in the social-democratic equilibrium with a tax rate of t is given by:

19

20   $$x(s;t) = (1-t)(wL(s) + rk(s)\overline{k}) + \frac{t}{n}G(K^{tot}, L^{tot}) \ . \qquad (8.17)$$

21   Thus, we easily compute the Gini coefficients of income in social-democratic equilibrium as we

22   vary the tax rate. We also present the "99:10 ratio," "90:10 ratio," and the "75:10 ratio," the

23   ratios of total income of workers at various pairs of quantiles.

---

[15] The Gini coefficient is defined as $\dfrac{1}{2\mu}\iint |x(s) - x(\tau)| dF(s)dF(\tau),$ where

$\mu = \int x(s)dF(s).$

[16] See the last paragraph of section 4.

```
tax rate    Income Gini    99:10 ratio    90:10 ratio    75:10 ratio
0.          0.544858       21.5369        7.46106        4.48255
0.1         0.490372       14.9259        5.3812         3.36149
0.2         0.435886       10.9302        4.12411        2.68391
0.3         0.3814         8.25408        3.28219        2.23011
0.4         0.326915       6.33655        2.67892        1.90495
0.5         0.272429       4.89508        2.22542        1.66051
0.6         0.217943       3.77197        1.87208        1.47006
0.7         0.163457       2.87223        1.58902        1.31748
0.8         0.108972       2.13526        1.35716        1.19251
0.9         0.0544858      1.52054        1.16377        1.08827
1.          0.             1.             1.             1.
```

Table 2.   Income Gini coefficients in (Pareto efficient) social-democratic equilibrium as the tax rate varies

As I pointed out, the total product is invariant with respect to the tax rate.   In these social-democratic equilibria, it is $9.70 trillion. Thus the deadweight loss of output due to taxation in the capitalist model with $t = 30\%$ is $\dfrac{9.70 - 9.5}{9.70} = 2.0\%$.   Doubtless the true inefficiency, due to oligopolistic price setting and rent seeking (see Stiglitz [2019])), is considerably greater.

Note that the Gini coefficient in (the efficient) social-democratic equilibrium when the tax rate is 30% is slightly larger than the Gini coefficient in (the inefficient) capitalist equilibrium at that tax rate (which is 0.374).   It is interesting to observe what the distribution of welfare (utility) is in latter economy, in comparison with the distribution of welfare in the social-democratic equilibrium at various tax rates.   See Figure 1[17].

[place Figure 1 about here]

The allocations in the three social-democratic equilibria plotted in figure 2 are Pareto efficient. The value $s = 4$ sits at quantile 0.997 of the skill distribution F, and hence at the same quantile of the income distribution, since capital ownership is monotone increasing in s as well. The figure tells us that at a tax rate of 30% and even 50% the social-democratic equilibrium Pareto dominates the capitalist equilibrium well into the 99th centile ($F(3.6) = 0.994$).   At a tax rate of 90% the social-democratic equilibrium is massively better for the low skilled than the capitalist equilibrium at 30%, but those with skill level in the top 7% fare worse than in the

---

[17] The kinks in the graphs of figure 1 are due to the piece-wise linear approximation to the distribution of capital.

1 capitalist equilibrium at 30%. Recall that, due to linear taxation, in all these equilibria, utility is

2 strictly monotone increasing in *s*. Although taxation can sharply reduce income inequality, it

3 never alters the rank of any individual in the income distribution.

4   Finally, I simulate equilibria for the $\lambda-$ sharing economy. By virtue of the quasi-

5 linearity preferences, factor supplies are invariant with λ , and are exactly the same as those in

6 the social-democratic equilibria: all that changes with λ is the distribution of income. Table 3

7 presents the Gini coefficient of income as λ varies.

λ	Income Gini	99:10 ratio	90:10 ratio	75:10 ratio
0.	0.533818	21.5369	7.46106	4.48255
0.1	0.530496	21.2145	7.39428	4.45357
0.2	0.527174	20.8985	7.3288	4.42515
0.3	0.523852	20.5885	7.26458	4.39727
0.4	0.52053	20.2844	7.20158	4.36993
0.5	0.517208	19.9861	7.13978	4.34311
0.6	0.513886	19.6933	7.07914	4.31678
0.7	0.510564	19.406	7.01962	4.29095
0.8	0.507242	19.124	6.96119	4.26559
0.9	0.503919	18.8471	6.90383	4.24069
1.	0.500597	18.5752	6.8475	4.21624

9 <u>Table 3</u>. Gini coefficients of income in the λ-sharing economy

11   The Gini coefficient is high, and quite insensitive to the value of λ .

12   As a second exercise, I suppose that the financial wealth of the top 5% of the wealth

13 distribution is taxed away (before the model starts), and redistributed equally to everyone. To

14 be precise, I alter the distribution of wealth from $k(s)$ to $\tilde{k}(s)$, where:

16
$$\tilde{k}(s) = \begin{cases} k(s)+\kappa, & 0 \le s \le s_{0.95} \\ k(s_{0.95})+\kappa, & s_{0.95} \le s < \infty \end{cases} \tag{8.18}$$

17 where $\kappa = \int_{s_{0.95}}^{\infty} (k(s)-k(s_{0.95}))dF(s) = 43.7\%$; the distribution of capital is substantially leveled

18 by this assumption. The Gini coefficients are now as given in Table 4.

```
                    Gini with top 5% levelling of wealth
        λ       Income Gini    99:10 ratio    90:10 ratio    75:10 ratio
        0.      0.359061       8.72719        4.6853         2.9864
        0.1     0.358775       8.74093        4.69005        2.99301
        0.2     0.358488       8.75472        4.69482        2.99964
        0.3     0.358202       8.76856        4.6996         3.00629
        0.4     0.357915       8.78245        4.7044         3.01297
        0.5     0.357628       8.7964         4.70922        3.01967
        0.6     0.357342       8.81039        4.71406        3.02639
        0.7     0.357055       8.82443        4.71891        3.03315
        0.8     0.356768       8.83853        4.72378        3.03992
        0.9     0.356482       8.85268        4.72867        3.04672
        1.      0.356195       8.86688        4.73358        3.05355
```

Table 4. Gini coefficients of income in the λ-sharing model with capital levelling at the top

Of course, the allocations from which Table 4 is derived are all Pareto efficient by Proposition 3.

9. Public bads and public goods

In this section, I show that Kantian optimization in these blueprints may enable us to deal efficiently with the production of public goods and public bads -- without regulation or imposing effluent fees, in the case of public bads, and without state financing, in the case of public goods. I will display two examples.

A. Profit maximization may engender public bads

The socialist blueprints I have offered depend, for their efficiency results, on the maximization of profits by the firm. I have already mentioned that socialists may bridle at the idea that investors should be treated similarly to workers in an advanced socialist economy. They may likewise bridle at the idea that profit-maximization is so central to these models, because we rightly associate profit-maximization with many deleterious practices – employing child labor, emitting carbon dioxide, or running assembly lines at a breakneck pace.

I believe the deleterious practices that accompany profit maximization in capitalist (and twentieth century socialist) economies must be controlled by recognizing that the public bads, like the ones mentioned in the last paragraph, enter the utility functions of citizens, and so it will no longer be the case that unrestricted profit maximization implies Pareto efficiency. One can

1   ask whether Kantian optimization can provide a satisfactory solution to the problem of negative

2   production externalities.

3   To examine this, I postulate a utility function of the form $u^i(x^i, L^i, K^i, y)$, where y is the

4   total product, and utility is decreasing in y.   Thus, think of industrial pollution or the speed of the

5   assembly line as being a monotone increasing function of output. A standard approach would be

6   to regulate firms, or to charge emission fees in the case that the public bad is easily measured.

7   We can also, however, achieve efficiency, in some cases, via Kantian optimization.

8   We first characterize Pareto efficiency for an economy where total output is a proxy for

9   the level of the public bad that firms will produce if they are not otherwise constrained.

10

11   <u>Proposition 5</u>   *Consider the economic environment $\{(u^i), G, n\}$ where production uses labor and*

12   *capital inputs, and preferences are defined over the vectors (x^i, L^i, K^i, y) as above, and*

13   *preferences and production are convex.   Then an interior allocation is Pareto efficient if and*

14   *only if:*

15

16    $$(i) \text{ for all } i, \frac{u^i_3}{u^i_2} = \frac{G_2}{G_1} \text{ and } (ii) \text{ for all } i, \; -\frac{u^i_1}{u^i_3} = \frac{1}{G_1} + \sum_j \frac{u^j_4}{u^j_3}. \qquad (9.1)$$

17   <u>Proof:</u>

18   The claim is proved by solving the program:

19

20
$$
\begin{aligned}
&\max u^1(x^1, L^1, K^1, y)\\
&\text{s.t.}\\
&j > 1 \Rightarrow (u^j(x^j, L^j, K^j, y) \geq k^j)\\
&x^S \leq G(K^S, L^S) \equiv y
\end{aligned}
\qquad (9.2)
$$

21   The (dual) KKT conditions for the solution are precisely (i) and (ii) as stated in (9.1).   ∎

22

23   Let's now insert the public good into a social-democratic economy. We continue to

24   define a social-democratic equilibrium with taxation exactly as in section 4.   Now the game **W**

25   defined in equation (4.1) has payoff functions:

$$\hat{W}^i(L^1,L^2,...,L^n)=u^i(\frac{(1-t)(wL^i+rK^i+\theta^i(pG(K^*,L^*)-rK^*-wL^*)}{p}+$$

1
$$\frac{t}{n}\frac{wL^S+rK^S+\Pi(K^*,L^*)}{p},L^i,K^i,G(K^S,L^S))$$
. (9.3)

2   The condition for $(L^1,...,L^n)$ to be an additive Kantian equilibrium of this game is:

3
$$u_1^i\cdot\frac{w}{p}+u_2^i+u_4^iG_2n=0.$$
(9.4)

4   In like manner, the condition for $(K^1,...,K^n)$ to be an additive Kantian equilibrium of the

5   game **V**, modified from (4.2), is:

6
$$u_1^i\cdot\frac{r}{p}+u_3^i+u_4^iG_1n=0.$$
(9.5)

7   Noting that these equations can be written:

8
$$G_2\cdot(u_1^i+nu_4^i)=-u_2^i \text{ and } G_1\cdot(u_1^i+nu_4^i)=-u_3^i,$$
(9.6)

9   we deduce that condition (i) of Proposition 5 holds.   Next, write (9.5) as:

10

11
$$-\frac{u_1^i}{nu_3^i}=\frac{1}{nG_1}+\frac{u_4^i}{u_3^i}.$$
(9.7)

12   Now suppose that $u^i(x^i,L^i,K^i,y)=x^i-h^i(L^i,K^i)-f(y)$.   Then the second equation in (9.6)

13   becomes $G_1\cdot(1-nf'(y))=-u_3^i$, and so the numbers u_3^i are invariant across i. Consequently

14   (since $u_1^i\equiv1$), we can add equations (9.7) to get $-\frac{u_1}{u_3}=\frac{1}{G_1}+\sum_i\frac{u_4^i}{u_3^i}$.   This is condition (ii) for

15   Pareto efficiency from Proposition 5.   To summarize:

16

17   <u>Proposition 6</u> *If $u^i(x^i,L^i,K^i,y)=x^i-h^i(L^i,K^i)-f(y)$, then social democratic equilibrium*

18   *(Socialism* 1*), amended to include the disutility associated with a public bad that accompanies*

19   *production, is Pareto efficient at any tax rate $t\in[0,1]$.*

20

21   Although Proposition 6 has a restrictive premise, it shows there is a potential for Kantian

22   optimization to eliminate the deadweight loss of taxation *and* the inefficiency associated with

23   production externalities at the same time. What is required is that factor suppliers take into

1   account the negative externality that is a function of the total product that their factor supplies

2   engender. (See (9.3).) In particular, we do not restrict the firm's profit-maximizing behavior

3   through regulation.   Rather, the otherwise deleterious effects of that behavior are controlled by

4   cooperation among workers and investors in their factor supply behavior.

5

6   B. Production of a private and public good

7   Assume that there is a private good produced by the production function G, and a public

8   good, produced by the production function H, also using capital and labor. Preferences of

9   consumers are defined on vectors (x^i, L^i, K^i, z) where z is the amount of the public good

10   produced.   In general, consumers will expend labor and invest capital in both the private-good

11   firm operating G, and the public-good firm operating H.   We first characterize Pareto

12   efficiency.

13   <u>Proposition</u> <u>7</u> *Let $(x^i, L_1^i, L_2^i, K_1^i, K_2^i)$ be consumer i's consumption, supply of labor to the*

14   *private and public good firms, respectively, and her supply of capital to the private and public*

15   *good firms, respectively. Let z be the level of the public good. An interior allocation is Pareto*

16   *efficient if and only if[18]:*

17

18
$$(i)(\forall i)(\frac{u_2^i}{u_3^i} = \frac{G_2}{G_1}), \quad (ii)(\forall i)(\frac{u_2^i}{u_3^i} = \frac{H_2}{H_1}), and \ (iii)\frac{1}{H_1} + \sum_j \frac{u_4^j}{u_3^j} = 0. \qquad (9.8)$$

19   <u>Proof:</u>

20   Pareto efficiency is characterized by the KKT conditions of the following program:

21

22
$$\max u^1(x^1, L_1^i + L_2^1, K_1^1 + K_2^i, z)$$
$$s.t.$$
$$(\forall j > 1) \, u^j(x^j, L_1^j + L_2^j, K_1^j + K_2^j, z) \geq k^j \qquad (9.9)$$
$$x^S \leq G(K_1^S, L_1^S)$$
$$z \leq H(K_2^S, L_2^S)$$

23   These conditions are displayed in (9.8).   ∎

24

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[18] In the utility function, $L^i = L_1^i + L_2^i$ and $K^i = K_1^i + K_2^i$.

1   We will now define an equilibrium for the social-democratic economy with income

2   taxation, and with a public good.   The income tax finances the demogrant which is returned to

3   all consumers as income.   The firm producing the public good operates on a voluntary basis:

4   that is, consumers contribute labor and capital to that firm, but are not paid for these

5   contributions – the reward is the public good produced.   The vectors of labor and capital

6   supplied to the public firm will be additive Kantian equilibria of the appropriate game.

7   Optimizing behavior of consumers, in their supply of labor and capital to the private firm, is

8   likewise governed by additive Kantian optimization.

9   To be precise, I define the *payoff functional form* for consumer i, which is, as always, the

10   utility of the consumer at the allocation:

11

12   $$V^i(\omega^1,...,\omega^n) = u^i((1-t)(\frac{wL_1^i + rK_1^i + \theta^i\Pi(K_1^*,L_1^*)}{p}) + \frac{t}{n}\frac{wL_1^S + rK_1^S + \Pi(K_1^*,L_1^*)}{p}, L^i, K^i, H(K_2^S, L_2^S))$$

13   (9.10)

14   where $L^i = L_1^i + L_2^i$, $K^i = K_1^i + K_2^i$, and $\Pi(K_1^*,L_1^*) = pG(K_1^*,L_1^*) - wL_1^* - rK_1^*$.   The argument of V^i

15   will take four different specifications, where ω^j will take the values L_1^j, K_1^j, L_2^j, and K_2^j,

16   respectively. Thus the form **V** will define :

17   1) the game of labor supply to the private firm, whose strategy profiles are $(L_1^1,...,L_1^n)$,

18   2) the game of investment in the private firm, whose strategy profiles are $(K_1^1,...,K_1^n)$,

19   3) the game of labor supply to the public firm, whose strategy profiles are

20          $(L_2^1,...,L_2^n)$, and

21   4) the game of investment in the public firm, whose strategy profiles are $(K_2^1,...,K_2^n)$.

22   We can now define:

23   <u>Definition 6</u> A *social-democratic equilibrium with taxation and a public good, for a tax rate*

24   $t \in [0,1]$,is an allocation $(x^i, L_1^i, L_2^i, K_1^i, K_2^i)_{i=1,...,n}$, factor demands (K_1^*, L_1^*) for the private firm,

25   a public-good level z, and a price vector (p,w,r) such that:

26   •   the private-good firm demands capital and labor (K_1^*, L_1^*) to maximize profits

27          $pG(K,L) - rK - wL$,

- $(L_1^1,...,L_1^n)$ is an additive Kantian equilibrium of the labor-supply game to the private firm,
- $(K_1^1,...,K_n^1)$ is an additive Kantian equilibrium of the investment game in the private firm,
- $(L_2^1,...,L_2^n)$ is an additive Kantian equilibrium of the labor-supply game in the public firm,
- $(K_2^1,...,K_2^n)$ is an additive Kantian equilibrium of the investment game in the public firm, and
- all markets clear: $K^* = K_1^S, L^* = L_1^S, x^S = G(K^*,L^*)$, and $z = H(K_2^S,L_2^S)$.

Note, in particular, that the public good is produced outside the market.

<u>Proposition 8</u> *At any tax rate $t \in [0,1]$,any interior allocation at a social-democratic equilibrium with a public good is Pareto efficient[19].*

<u>Proof:</u>

The proof proceeds in the manner to which the reader has become accustomed. We calculate the conditions for the four factor-supply vectors to be additive Kantian equilibria of their respective games, and show that these conditions imply the conditions for Pareto efficiency (see Proposition 7).

1. By profit maximization, $\dfrac{w}{p} = G_2$ and $\dfrac{r}{p} = G_1$.

2. The f.o.c.'s for $(L_1^1,...,L_1^n)'s$ being an additive Kantian equilibrium of the labor-supply game to the private firm are:

$$(\forall i)(u_1^i \frac{w}{p} + u_2^i = 0) \ . \tag{9.11}$$

3. The f.o.c.'s for $(K_1^1,...,K_1^n)'s$ being an additive Kantian equilibrium of the labor-supply game to the private firm are:

$$(\forall i)(u_1^i \frac{r}{p} + u_3^i = 0) \ . \tag{9.12}$$

---

[19] Interiority is not needed for this proposition. I assume it in order not to have to amend the definition of additive Kantian equilibrium to encompass the possibility of corner solutions.

4. The f.o.c.'s for $(L_2^1, ..., L_2^n)'s$ being an additive Kantian equilibrium of the labor-supply game to the private firm are:

$$(\forall i)(u_2^i + u_4^i nH_2 = 0). \tag{9.13}$$

5. The f.o.c.'s for $(K_2^1, ..., K_2^n)'s$ being an additive Kantian equilibrium of the labor-supply game to the private firm are:

$$(\forall i)(u_3^i + u_4^i nH_1 = 0). \tag{9.14}$$

6. Steps 1,2, and 3 above imply condition (i) of Proposition 7. Steps 4 and 5 imply condition (ii) of Proposition 7. Finally, write equations (9.14) as $\dfrac{1}{nH_1} + \dfrac{u_4^i}{u_3^i} = 0$.

Summing these equations over i gives condition (iii) of Proposition 7, proving the stated claim. ∎

The reader should note that factor suppliers must play four different games. We cannot amalgamate this behavior into a single game, or even into two games. As in Proposition 6, Kantian optimization apparently kills two birds with one stone.

10. <u>What does it mean to be socialist today?</u>

Clearly a limitation of my analysis is its classical assumption that technology is characterized by constant or decreasing returns to scale. A proper treatment of what socialism would require when increasing returns to scale (IRS) holds is a project that, I hope, can be informed by this classical analysis. I have not attempted this, for lack of a simple, canonical equilibrium model of IRS.

One could attempt to answer the question posed in this section's title by asking what conception of a cooperative economy best fits the most prominent classical definition of socialism, which I take to be Karl Marx's. Marxian socialism is an economic system in which 'each works according to his ability and is paid according to his work.' Although Marx did not go into the institutional details of how this instruction would be implemented, most Marxists assumed that it would entail state ownership of firms (the means of production), and remuneration of labor in proportion to skill. At least such was the case during the Soviet era. The entire economic product would be so distributed, after a share had been reserved for

1   investment.   Not only firms but capital would be state-owned, so the only privately-owned

2   production factor would be labor power.

3   What was the ethical justification of such a regime? It was that capital comes into being

4   'dripping from head to foot, from every pore, with blood and dirt (Marx[1965, p.760]).' Thus,

5   capital (in the pre-capitalist history of Britain, at least according to Marx's research in the British

6   Museum) was not accumulated through honest work, from a decision to save from earnings, but

7   from plunder, enclosure, royal decree, and conquest. And in the capitalist era, capital grew

8   through the exploitation of labor.   Marx, however, viewed workers as the rightful owners of

9   their labor power, and hence the just or fair division of the economic product was in proportion

10   to labor expended (measured in efficiency units), after the state, presumably, had taken a share of

11   the product for investment.

12   The nature of modern advanced economies today is, however, very different from Marx's

13   *vision* of early capitalist Britain – we need not debate here whether his vision was historically

14   accurate, for it was, in any case, the vision that inspired Marx's conception of socialism.

15   According to my calculation, based upon the financial wealth data in Zucman (2017),   the

16   financial wealth of the Piketty's middle class in the United States, those occupying the 50th to

17   90th centiles of the financial wealth distribution, comprises 26% of total US financial wealth, and

18   if we include the upper middle class, those in the 90th to 99th centile, the financial wealth share

19   rises to 56%.   It cannot be argued that this wealth came about through plunder, conquest, and

20   enclosure: rather, the default assumption must be that most of it came about through investment

21   from saved earnings and inheritance.

22   One can still maintain that this middle-class wealth has not been *justly* acquired, but to do

23   so, one must employ a (Rawlsian) argument quite different from Marx's blood-and-dirt

24   argument. The earning capacity that people acquire in capitalist societies is massively influenced

25   by the families into which they are born, and that circumstance, according to Rawls, is morally

26   arbitrary.   People neither justly benefit nor suffer due to morally arbitrary circumstances that

27   characterize their natural and social environments.   This view is *more* radical than Marx's,

28   because it does not treat even a person's labor power and skill as justly owned by the person, to

29   the degree that that the development of that skill is a consequence of a highly-resourced

30   upbringing and education that the person may have had by virtue of the luck of the birth lottery.

31   It is also, however, *less* radical than Marx's view, because it does not treat all private wealth

1   accumulation as immoral: if a person comes by her skills and earning capacity in an

2   environment of equal opportunity, then her decision to save some of her earnings in order to

3   optimize her lifetime consumption path is ethically protected. I would still argue that inheritance

4   must be sharply restricted, for the differential wealths of the current generation, even if justly

5   acquired, would destroy equality of opportunity for the next generation, were inheritance not to

6   be restricted. See Piketty (2015, chapter 11) for an historical analysis of the importance of

7   inheritance in generating the present distribution of wealth.

8   As G.A. Cohen (2009) has argued, the proper construal of socialist ethics, at the

9   beginning of the twenty-first century, is that income differentials that can be traced to differential

10   luck (in large part, the luck of the birth lottery) should be eliminated, but differential incomes

11   traced to different choices, sterilized of luck, are permissible.   How to construct an economic

12   regime to implement this view is a tricky undertaking. If one wishes to think of Cohen's

13   proposal as a generalization of Marx's view, one would say that for Marx the main circumstance

14   (morally arbitrary luck) was the ownership of capital, and hence (Marx believed) socialism

15   required the elimination of differential capital ownership via state ownership. Marx argued that

16   *elimination* of the private property form was necessary, not redistribution-cum-private

17   ownership.   Perhaps more to the point, he claimed that state ownership was next on the

18   historical agenda.

19   If socialism is to be constructed from the initial conditions of existing capitalism, then

20   one must design rules that view the wealth of the middle class as entitled to remuneration, while

21   at the same time recognizing that wealth has been acquired in a regime characterized by massive

22   inequality of opportunity. Of the variants of socialism that I have presented, Socialism 1 (social

23   democracy with taxation, sections 4 and 8) has the advantage that income taxation can be

24   implemented with Kantian optimization, engendering income equality (or close to it) without

25   sacrificing efficiency.

26   To achieve acceptable income Gini coefficients in the sharing economy (Socialism 2), we

27   need either a significant redistribution of financial wealth, as I have simulated, or income

28   taxation – and the latter, as far as I know, will be inefficient.   However, we should not discard

29   the blueprint of the sharing economy immediately, because of the importance of the cooperative

30   ethos to socialism, and the possible dynamic interaction between that ethos and property

31   relations.

1   Is it psychologically feasible for some members of a society to desire to cooperate with

2   others whom they see have much higher incomes?   G.A. Cohen (2009) writes it is not, and it is

3   hard to disagree. Thus, for workers and investors to cooperate in the sense of Kantian

4   optimization will require a quite substantial redistribution of income. Indeed, equalizing

5   opportunities for the acquisition of earning power, itself a major project, may be insufficient for

6   ensuring the degree of income equality that would be required to generate the trust needed for

7   workers and investors to optimize in the Kantian manner.

8   I am hesitant to discard Socialism 2 because of considerations of *stability* that may

9   recommend it over social democracy.   Stabilizing the cooperative ethos is paramount.   The

10   formalized optimizing behavior upon which I have focused may be only the tip of the

11   cooperative iceberg.   More generally, one can ask whether the cooperative ethos can co-exist

12   with the capitalist allocation rule (of pre-tax income) of Socialism 1.   Socialism 2 has the

13   attractive property that the entire product is distributed to the cooperative producers: no class

14   exists that claims part of the product but whose members do not participate in production.   I

15   certainly do not understand the psychology that will be necessary to maintain the desire,

16   understanding and trust that are the necessary for maintaining a cooperative ethos, but it may be

17   the case that that ethos is more aligned with 'cooperation in production,' as occurs in Socialism 2

18   than with social democracy.

19   Saez and Zucman (2019, chapter 3) relate how, in the period 1930 -1970, a more

20   cooperative ethos existed in the United States than we experience today: the key evidence is the

21   existence of very high, even confiscatory, taxes on the very rich. In 1960, the *average* tax rate

22   applied to 400 richest Americans was close to 60% of their income; today, it is a little over 20%,

23   *lower* than the average tax rate experienced by the poorest 50% of households.[20] This

24   degeneration of social solidarity could not have occurred without massive re-enforcement of the

25   individualistic ethos in America – corresponding historically to the passage from F.D. Roosevelt

26   to R. Reagan[21] and M. Thatcher and its correlated rise in the individualistic ethos.

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[20] Taxes comprise federal, state, local, property, and estate. See Figure 1.4, Saez and Zucman (2019).

[21] Roosevelt said, in a message to Congress, in 1942: "Discrepancies between low personal incomes and very high personal incomes should be lessened; and I therefore believe that in time of this grave national danger, when all excess income should go to win the war, no American

1   To restate my tentative conclusions, thus risking the danger of boring the reader, they are

2   these. Viewing the socialist allocation rule as distribution of the product in proportion to *labor*

3   expended, after subtracting a share for investment, is only justifiable if the accumulation of

4   private financial wealth is viewed as ethically illicit. In socialist society, this cannot be correct.

5   Individual saving must be legitimate, if social mobility (more generally, equality of opportunity)

6   has increased significantly.   To the extent that the distribution of wealth inherited from

7   capitalism is unjust, redistribution either of assets or income should be achieved through

8   taxation.   But the *principle* that private investment of savings is legitimate must be respected.

9   What would be the path to socialism if it were to be defined as requiring confiscation of all

10   private wealth by the state? Certainly, no democratic polity would assent to that. Socialism's

11   rules must respect the legitimacy of private investment, while at the same time, implementing

12   policies – including tax policies but surely much more – that will create a more equal distribution

13   of earning capacities and wealth.

14   Which socialist variant combines optimally the attributes of attainability, sustainability

15   and equality?   Critically, how will property relations affect and be affected by the social ethos?

16   Surely only experience and experiment will tell.

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citizen ought to have a net income, after he has paid his taxes, of more than $25,000 a year
[equivalent to about $1 million in 2019 dollars]." Saez and Zucman (2019, p. 35).

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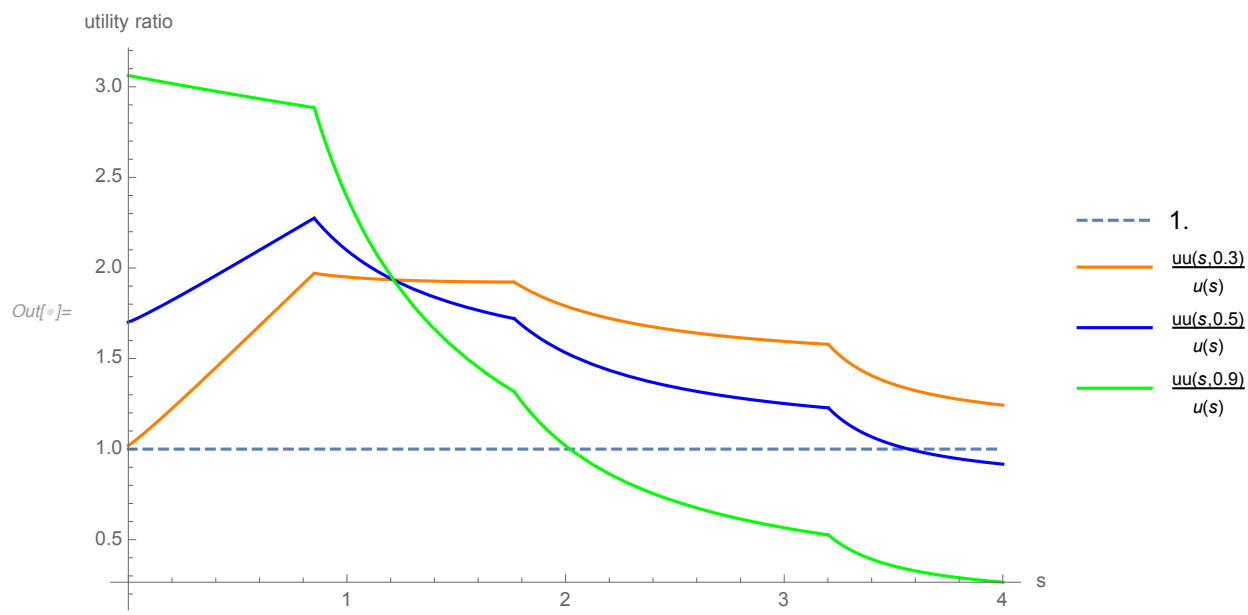
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10   <u>Figure 1.</u>   The ratio of the utility of a worker (as a function of her type s) in social-democratic
11   equilibrium with tax rate t (denoted $uu(s,t)$) to her utility in the capitalist equilibrium at tax rate
12   30% (denoted $u(s)$) for three values of t.

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