Credible Mechanisms

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A tale of three auctions

Ascending auction

Art, fish, livestock, timber, oil rights, used cars, real estate. . .

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Ascending auction strategy-proof

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First-price auction

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Mechanism Design: The Standard Approach

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> *[The auctioneer] binds himself in such a way that all the bidders know that he cannot change his procedures after observing the bids, even though it might be in his interest ex post to renege. In other words, the organizer of the auction moves as the Stackelberg leader or first mover.*

McAfee & McMillan 1987

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4. Bidder 1 brings a lawsuit and wins.

Second-price auctions by mail

After some time in the business, I ran an auction with some high mail bids from an elderly gentleman who'd been a good customer of ours and obviously trusted us.

Jeff Purser, stamp auctioneer, Connecticut
reported by Lucking-Reiley 2000

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Second-Price Auctions for Online Ads

‘A proverbial black box’: Open-exchange auctions have a transparency problem

MAY 8, 2017 by Yuyu Chen

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In a second-price auction, raising the price floors after the bids come in allows [online auctioneers] to make extra cash off unsuspecting buyers [. . .]

Ross Benes, reporting for *Digiday*, Sep 13 2017

Second-Price Auctions for Online Ads

In a second-price auction, raising the price floors after the bids come in allows [online auctioneers] to make extra cash off unsuspecting buyers [. . .] This practice persists because neither the publisher nor the ad buyer has complete access to all the data involved in the transaction, so unless they get together and compare their data, publishers and buyers won't know for sure who their vendor is ripping off.

Ross Benes, reporting for *Digiday*, Sep 13 2017

"Chandelier Bidding"

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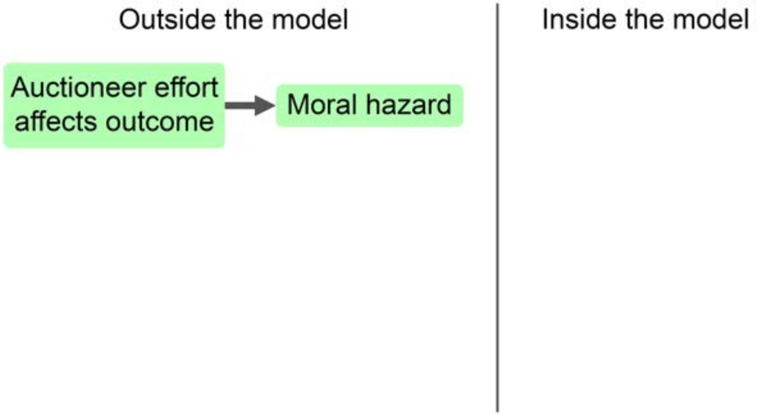
Under New York City regulations auctioneers can fabricate bids up to an item's reserve price. Because a reserve price is secret and not listed in the catalog, bidders have no way of knowing which offers are real.

NYT, April 24, 2000

From the world to the model



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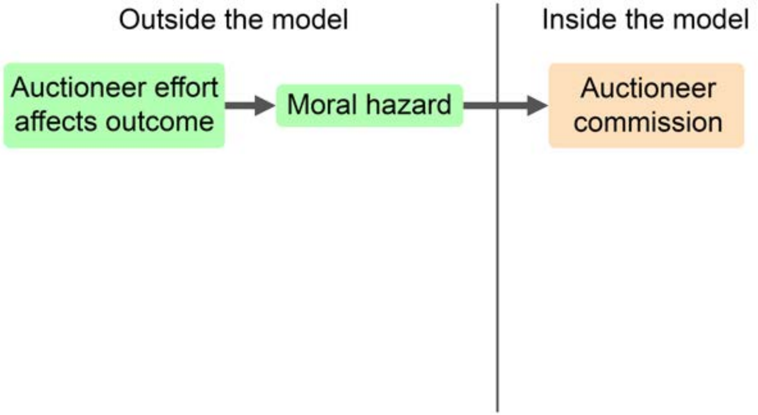

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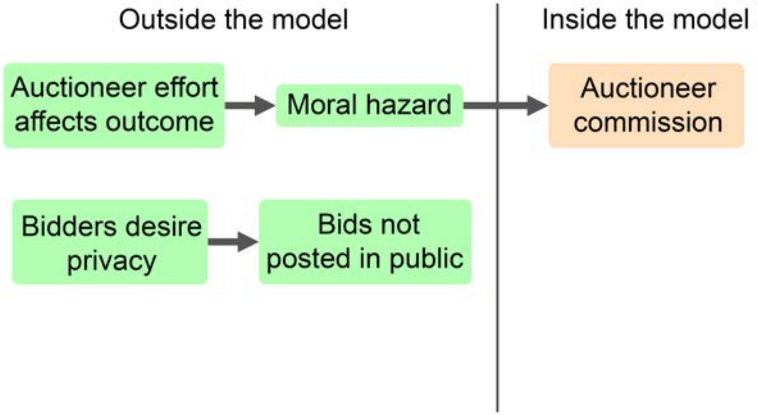

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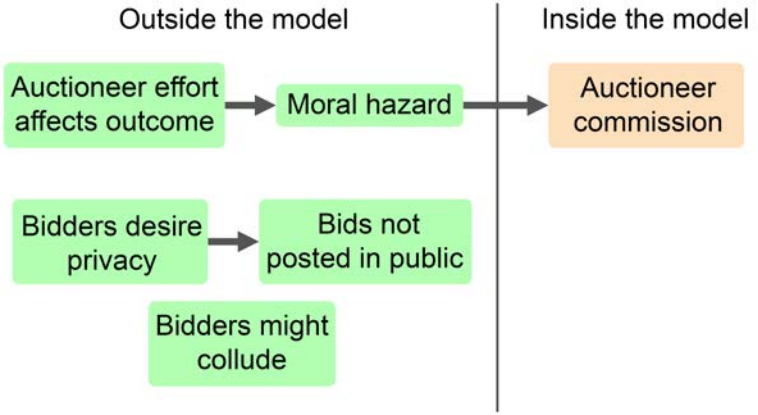

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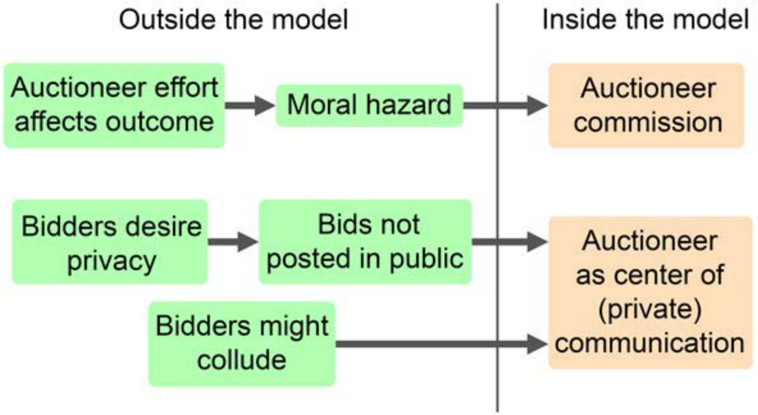
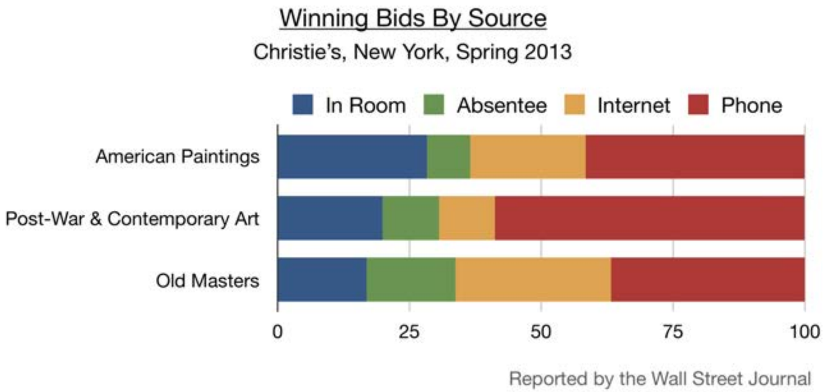


Auctions by telephone



Auctions by telephone

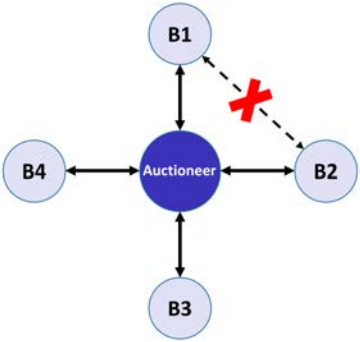
Auctions by telephone

Suppose all the serious bidders are phone bidders.
In which formats does the auctioneer *want* to follow the rules?

Warning: Substantive Assumptions

Auctioneer is the nexus of communication

Private ‘telephone calls’ to bidders.
Can misrepresent to i what j has done.



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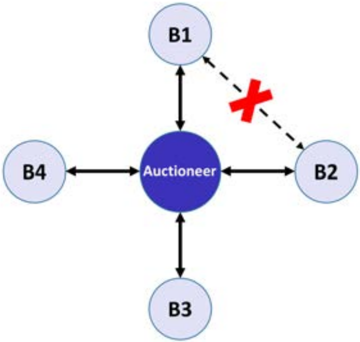

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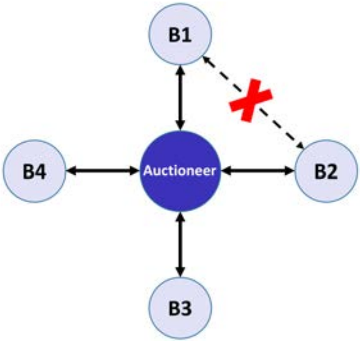
Informal definition

Auctioneer may deviate in ways that no single bidder can detect.
credible $\equiv$ incentive-compatible for auctioneer to follow the rules.



Optimal auctions

regular i.i.d. values
only winners make transfers
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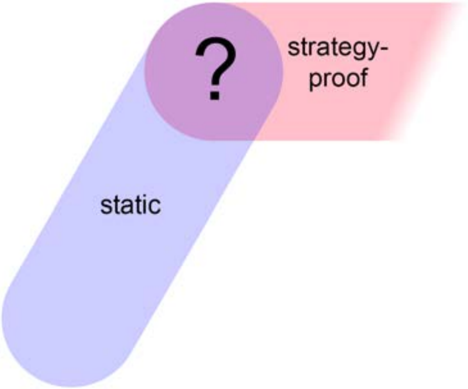

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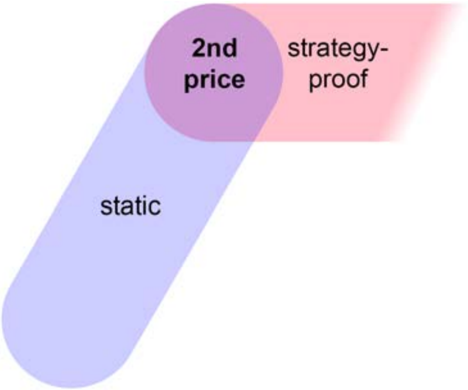

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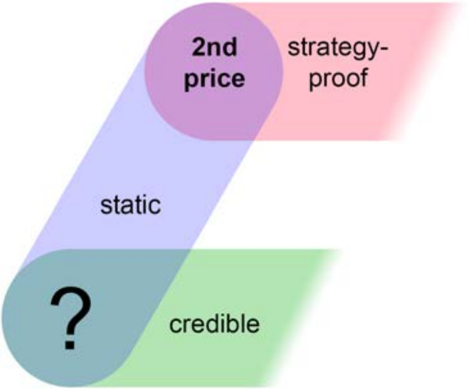

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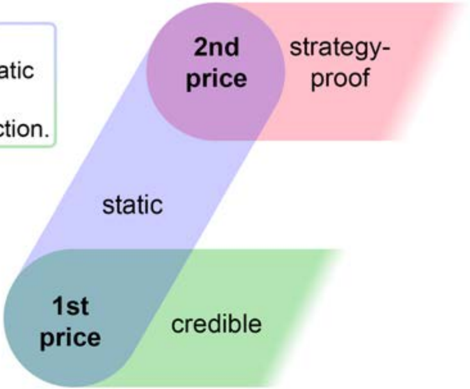

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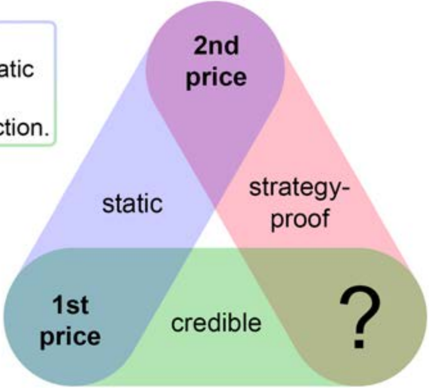

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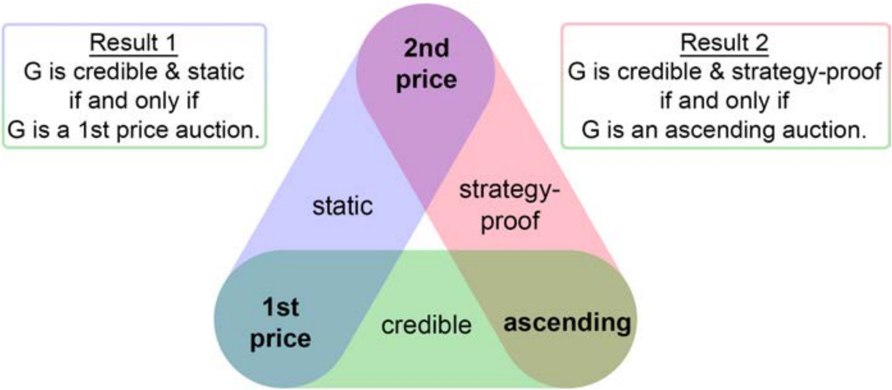

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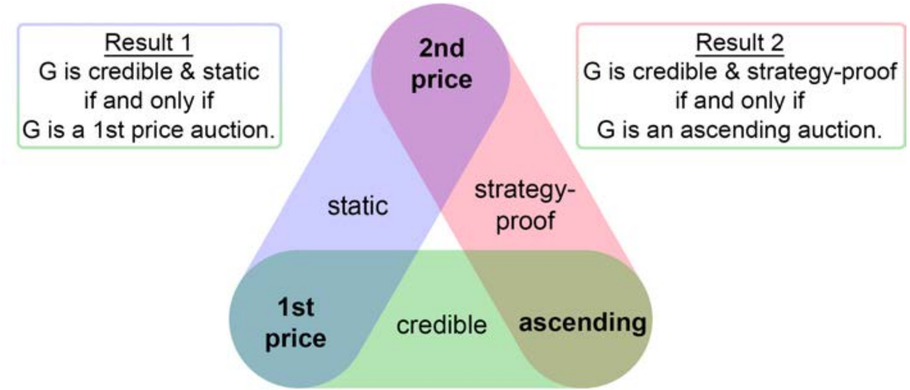

A Mechanism Design Framework

(auctions will be a special case)

1. A set of agents N
2. Finite type spaces $(\Theta_i)_{i \in N}$
3. Joint distribution $D : \Theta_N \to (0, 1]$ (full support)
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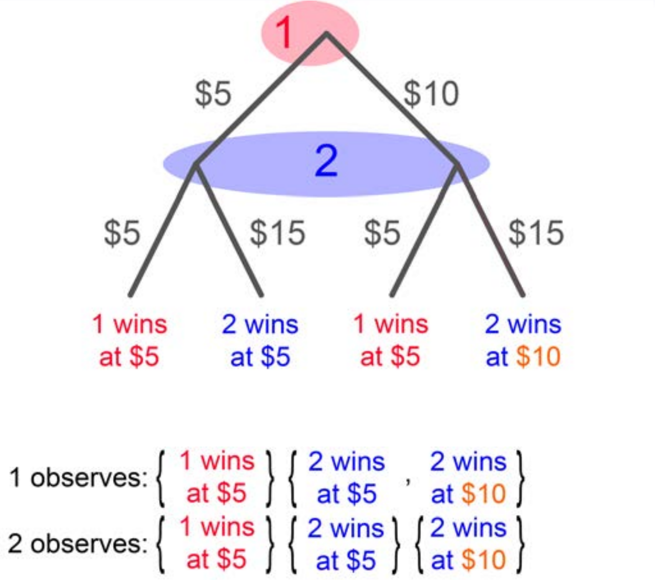
How can we extend this idea to include **auctioneer** deviations?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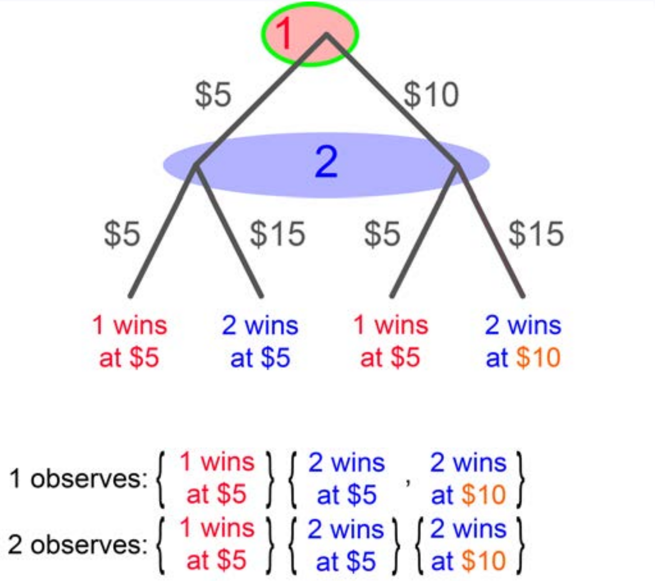
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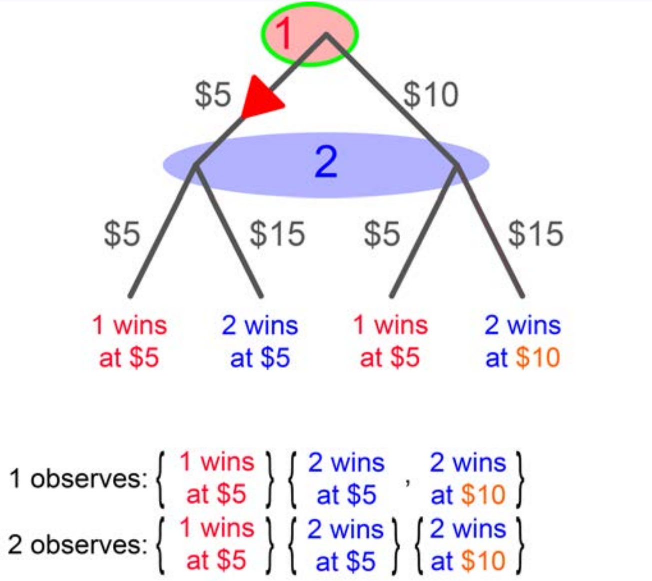
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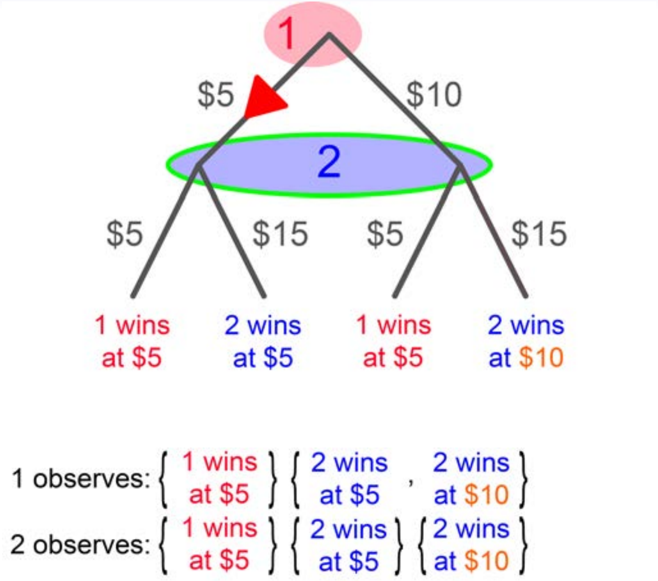
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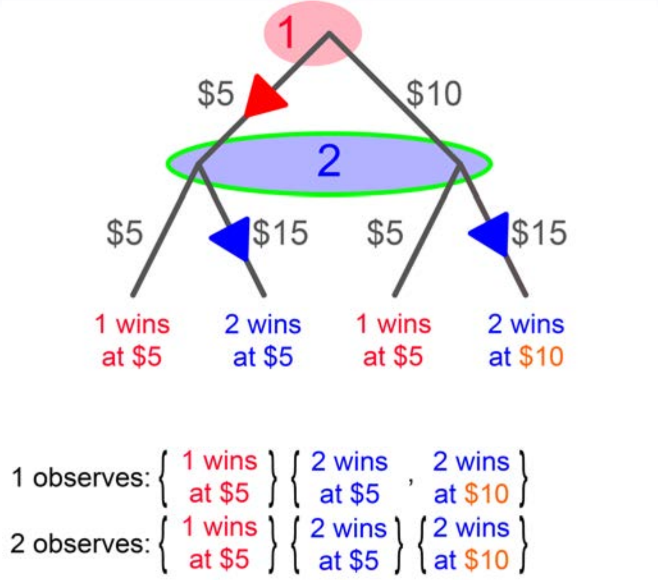
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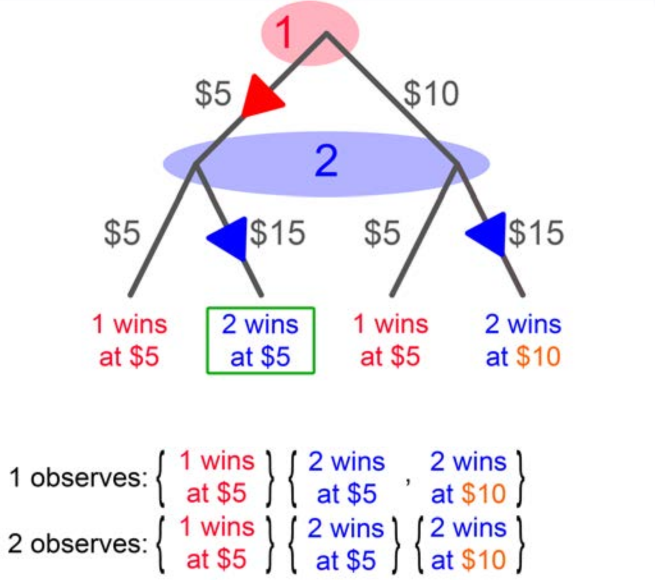
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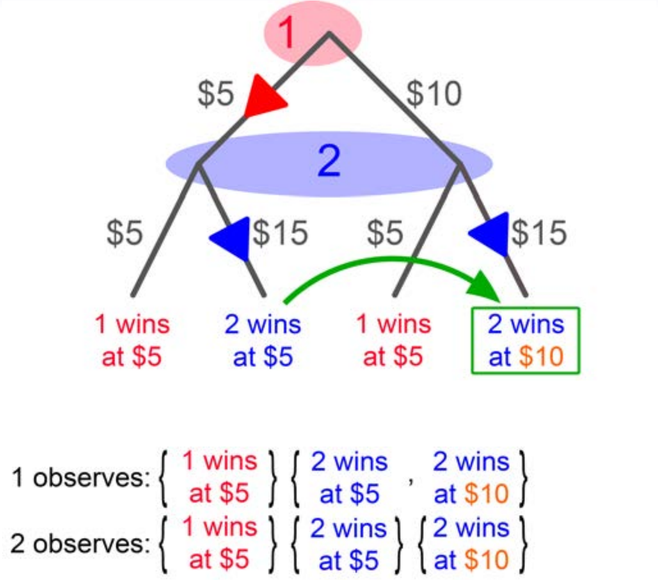
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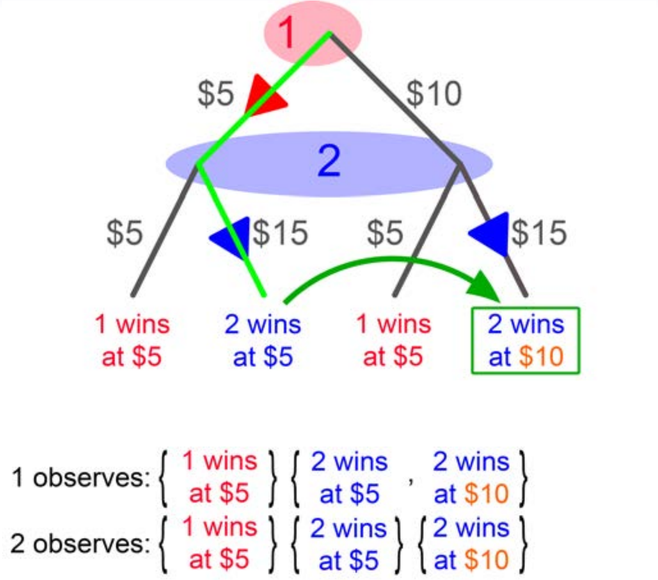
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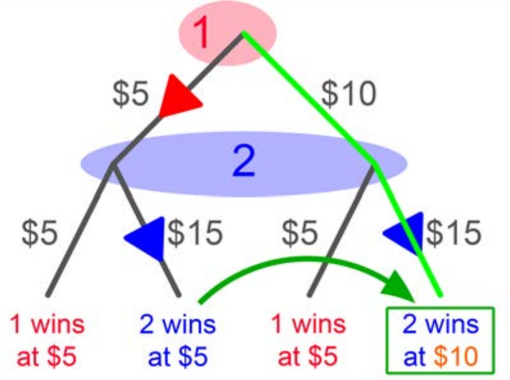

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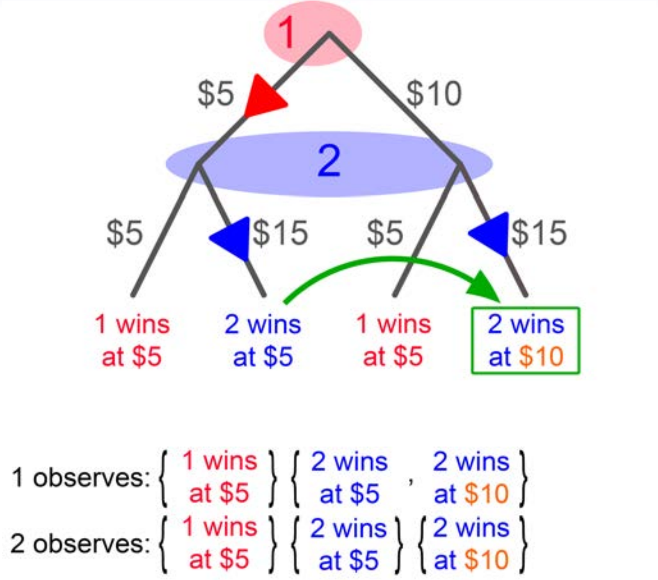
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1. Auctioneer can:
   1.1 Either: Choose an outcome and end the game.
   1.2 Or: Go to Step 2.
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An isomorphism

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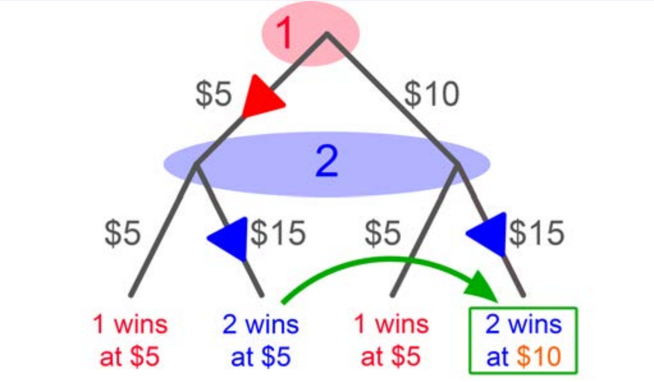
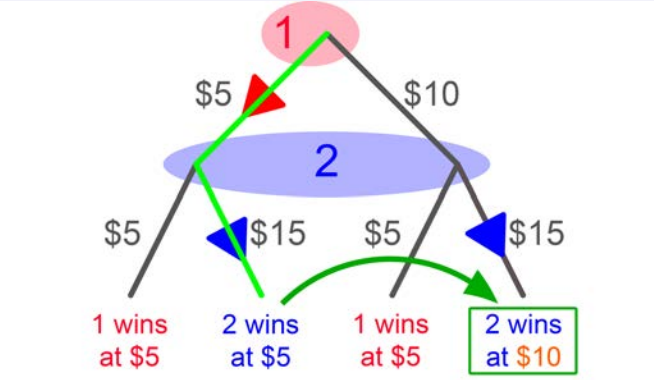
S_0 is **safe** if $\forall i : \forall \theta_N : o_i(S_0, S_N, \theta_N)$ has an innocent explanation.



Auctioneer's deviation

1 observes: {1 wins at $5} {2 wins at $5 , 2 wins at $10}

2 observes: {1 wins at $5} {2 wins at $5} {2 wins at $10}



Innocent explanation for 1's observation

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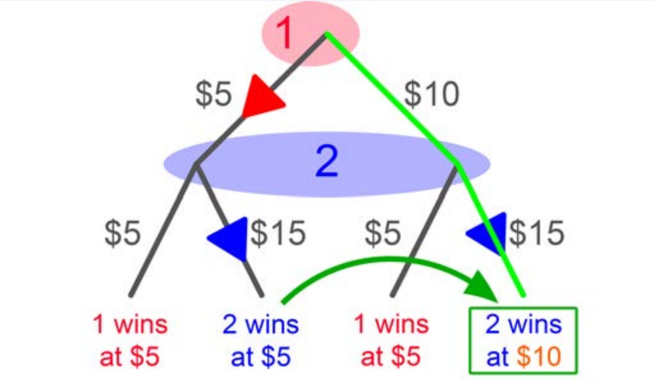

Innocent explanation for 2's observation

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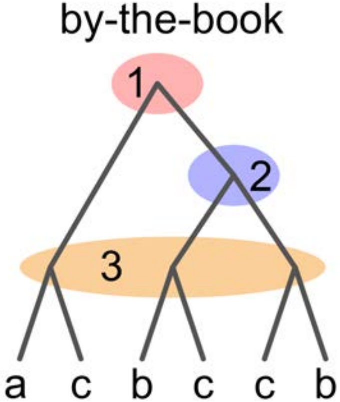
The auctioneer can deviate 'midway'.



1 observes: $\{a, b\}\{c\}$
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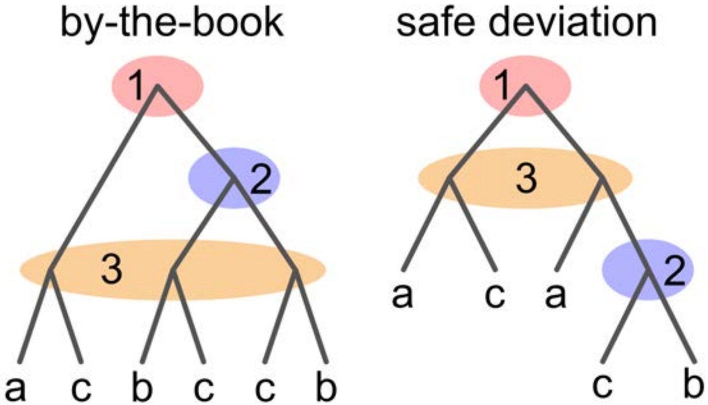

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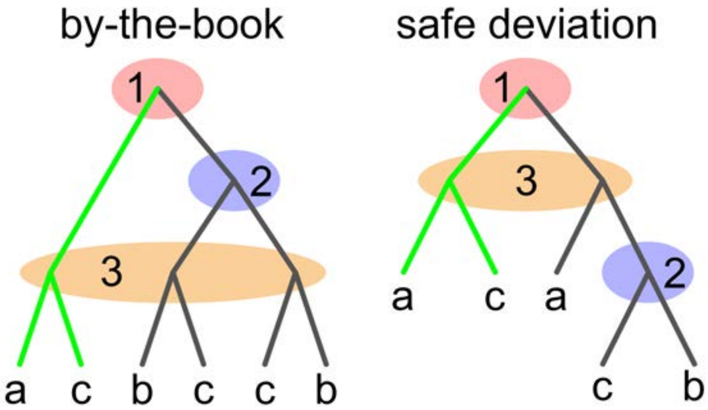

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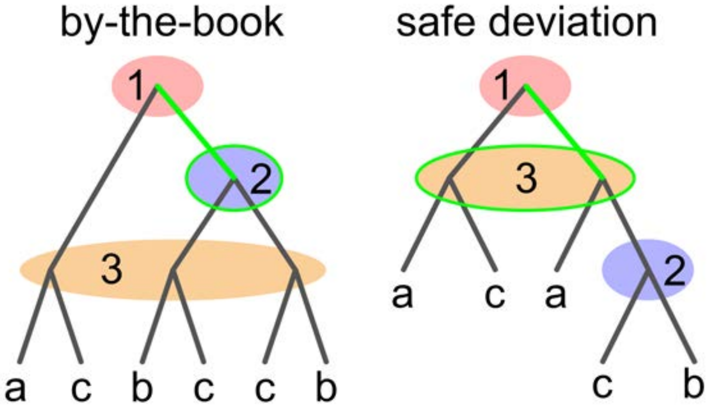

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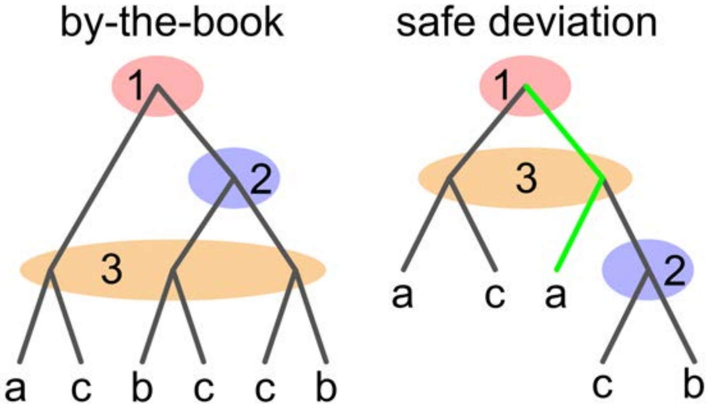

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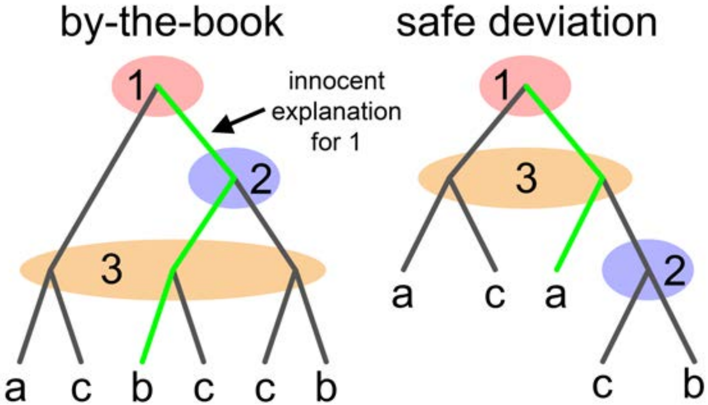

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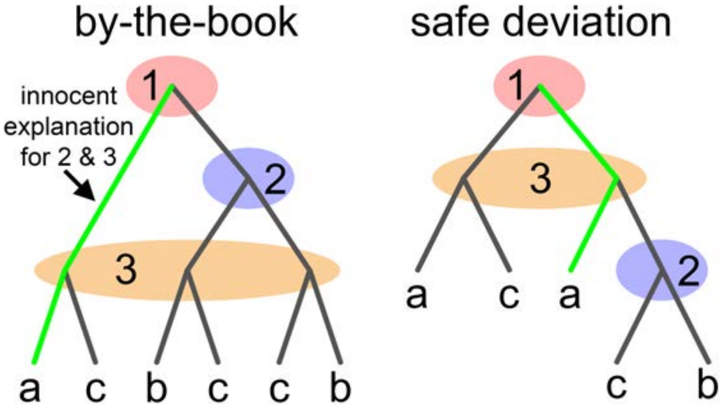

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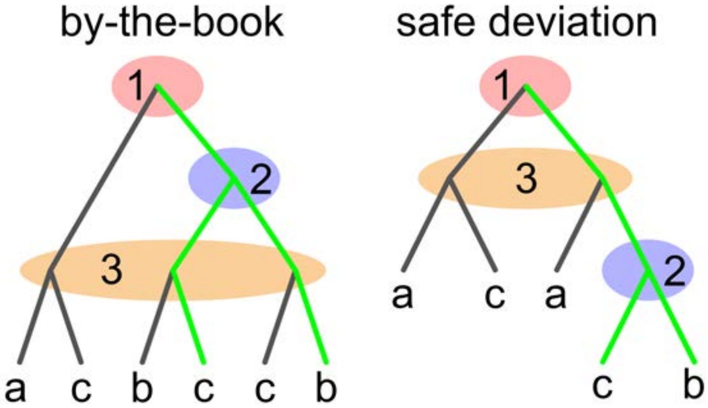

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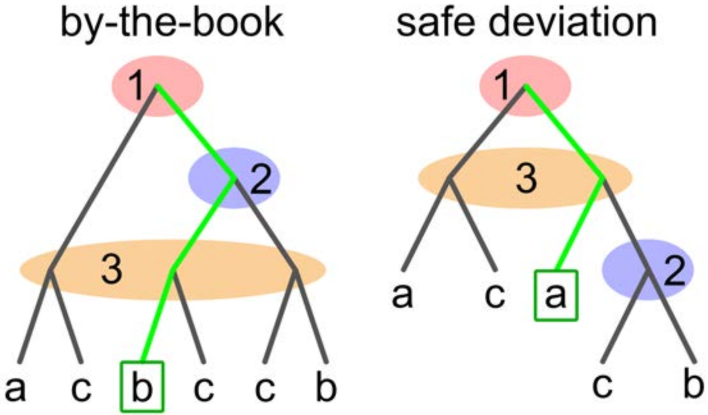

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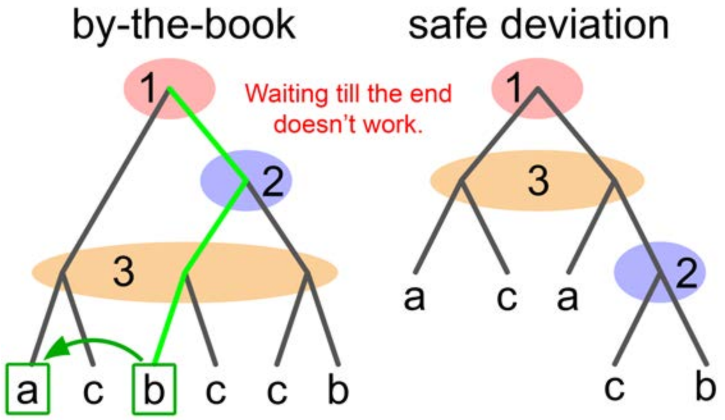

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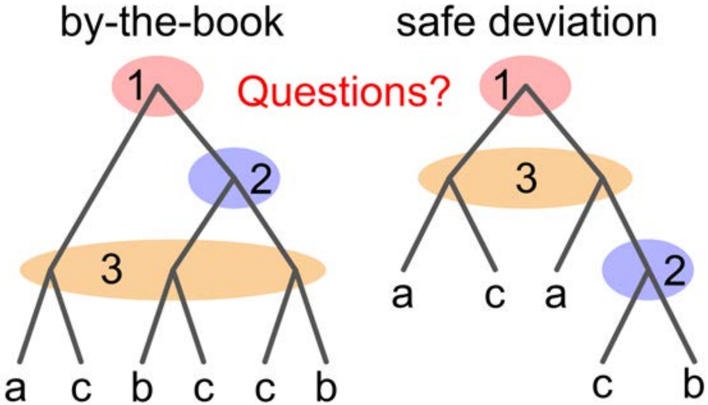

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Defining "Credible"

Definition

(G, S_N) *is* **credible** *if:*

$$S_0^G \in \underset{S_0 \in safe(S_0^G, S_N)}{\operatorname{argmax}} \underbrace{\mathbb{E}_{\theta_N}[u_0(S_0, S_N, \theta_N)]}_{\textit{auctioneer's expected utility}}$$

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(G, S_N) is BIC and credible. $\leftrightarrow$ (S_0^G, S_N) is a Bayes-Nash equilibrium of the messaging game restricted to safe(S_0^G, S_N).

Related literature

As above, but restricted to direct mechanisms

Dequiedt & Martimort 2015

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Auctions as bargaining games

McAdams & Schwarz 2007, Vartiainen 2013, Lobel & Paes Leme 2017

This talk: No 'red-handed' rule-breaking.

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Following Myerson (1981)

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2. N bidders.
3. Only winning bidders make transfers

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Objective

Choose (G, S_N) to maximize revenue subject to BIC and interim participation constraints.

A modeling choice.

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Decision: To use extensive forms, discretize Myerson 1981.

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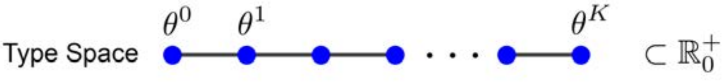

i.i.d. probability mass function $p : \Theta_i \to (0, 1]$

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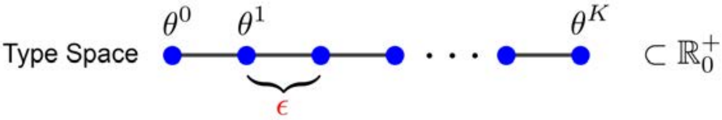

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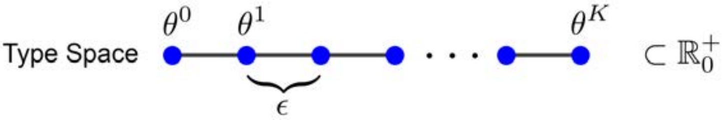

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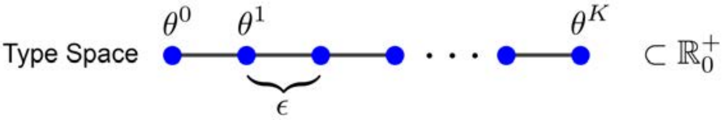

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If (G, S_N) is BIC and bidders with type θ^0 have zero surplus, then

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Assumption. F is regular, *i.e.* $\eta(\cdot)$ is strictly increasing.

Definition

(G, S_N) *is* **orderly** *if, for some reserve* $\rho \leq \theta^K$*, and some strict order* $\rhd$ *on* N*, bidder* i *wins the object iff:*

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Physical: $c \approx 3 \times 10^8$ meters/second

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Conjecture

Assume (G, S_N) is optimal and orderly. (G, S_N) is credible and static if and only if (G, S_N) is a first-price auction.

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Revenue equivalence breaks *slightly* with discrete types.

Sometimes orderly $\cap$ optimal $\cap$ first-price $= \emptyset$ example

credible, static $\leftrightarrow$ quasi-first-price

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(G, S_N) *is a* **quasi-first-price auction** *if it is static, and each* i *either chooses a bid in some feasible set* $B_i \subset \mathbb{R}$ *or declines.*

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We represent a reserve price by restricting B_i.

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Intuition: A very expensive ‘buy it now’ button.

Anomaly vanishes as $\epsilon \to 0$.

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Theorem 1

Assume (G, S_N) is ϵ-optimal and orderly. (G, S_N) is credible and static if and only if (G, S_N) is a quasi-first-price auction.

Proof Sketch

quasi-first-price auction $\rightarrow$ credible and static

By inspection.

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credible and static → quasi-first-price auction

Suppose after i plays a, there are two prices that i might pay.
Safely deviate to charge the higher price.

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Suppose after i plays a, there are two prices that i might pay.
Safely deviate to charge the higher price.

Highest bid must win.
Otherwise deviate to sell to highest bid.
Winning bid must be ≥ 0. Otherwise deviate to sell to no one.

(Plus some extra steps for the corner case.)

Dominant-strategy or credible?

Big Changes Coming To Auctions, As Exchanges Roll The Dice On First-Price

by Sarah Sluis // Tuesday, September 5th, 2017 – 8:00 am

Share:

The second-price auction is crumbling.

"In the next five years, the vast majority of auctions will move to transparent first price," said Criteo's EVP of global supply, Marc Grabowski.

Switching auction dynamics will unleash dramatic changes in the $32.5 billion programmatic market. . .

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Buyers, publishers, and ad tech companies who advocate a switch to first-price auctions say it's because fair second-price auctions don't exist any more. [Online auctioneers] have polluted them with hidden fees and manipulative auction dynamics.

The story so far

regular i.i.d. values, 'in the limit'

Strategy-proof

Definition

(G, S_N) *is* **strategy-proof** *if* $\forall i : \forall S'_{N\setminus i} : S_i$ *best responds to* $S'_{N\setminus i}$.

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No revelation principle.

1. Auctioneer could make any queries in any order.
2. Agents may receive information when called to play.

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$\Theta_i^h \equiv \{\theta_i \mid i\text{'s play up to history } h \text{ was consistent with } S_i(\cdot, \theta_i)\}$

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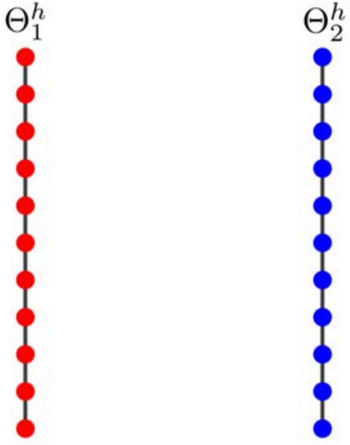

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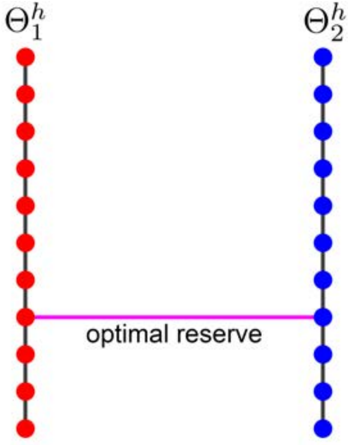

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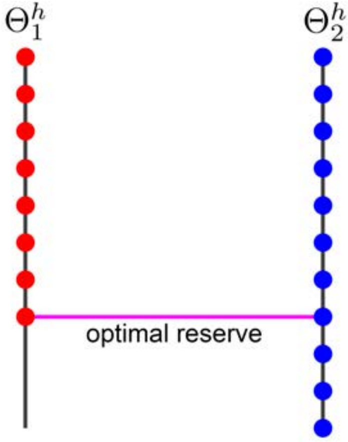

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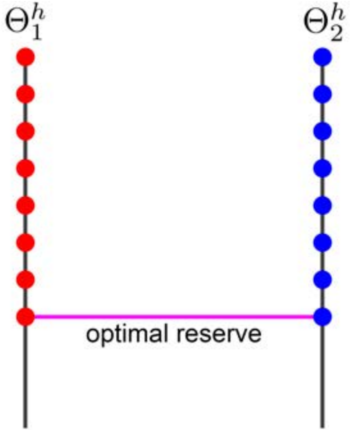

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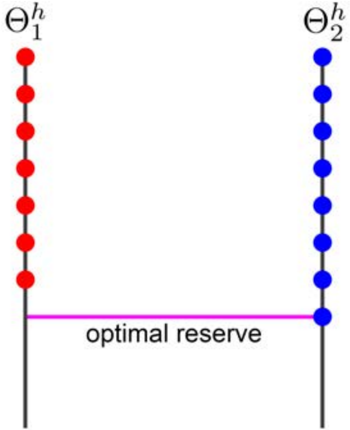

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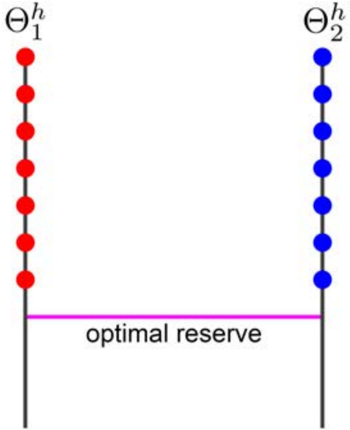

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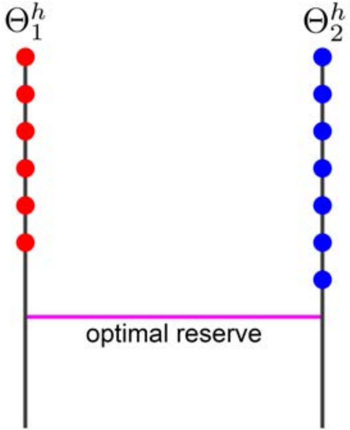

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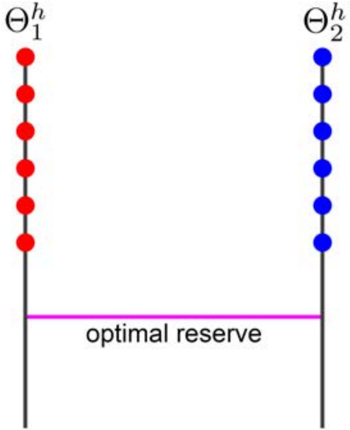

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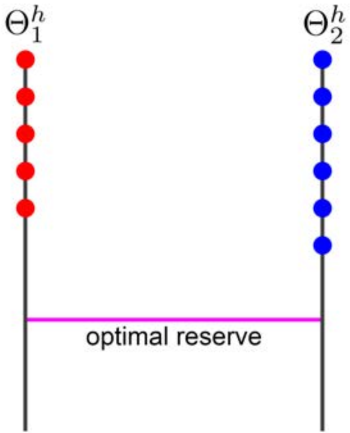

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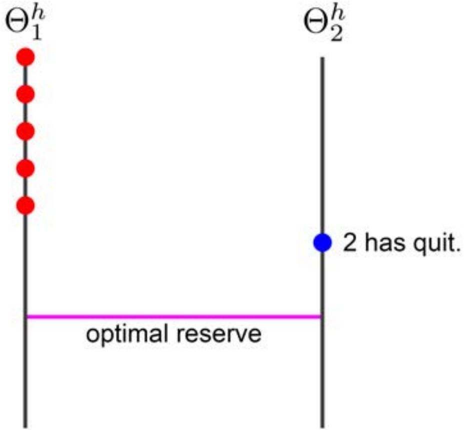

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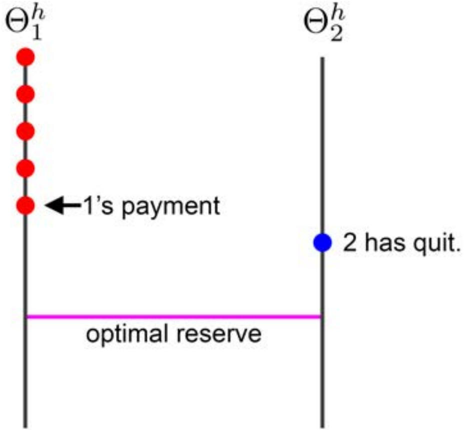

Feasible bids $= \Theta_i$

The **high bidder** has placed the highest bid so far that is (weakly) above the reserve. (break ties with $\triangleright$)

Definition

(G, S_N) *is an* **ascending auction** *if:*

1. *At each history, some active bidder chooses to:*
   1.1 *EITHER raise his bid to b, where b is no more than is necessary to become the high bidder.*
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   1.1 *EITHER raise his bid to b, where b is no more than is necessary to become the high bidder. (typically unique)*
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credible, strategy-proof ↔ ascending

Theorem 2

Assume (G, S_N) is optimal and orderly. (G, S_N) is credible and strategy-proof if and only if (G, S_N) is an ascending auction.

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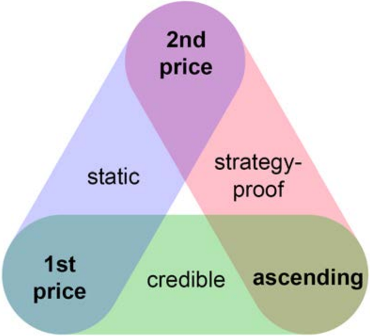
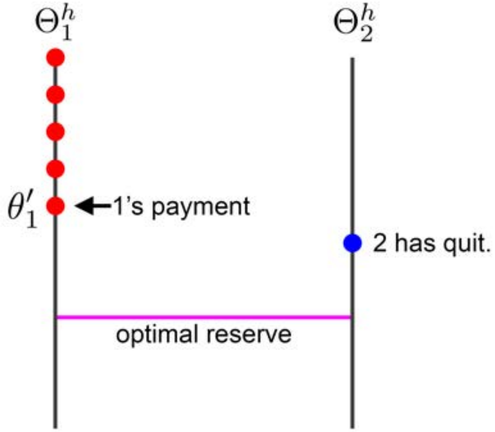
Green-Laffont-Holmström, Theorem 1, and Theorem 2 →



Why is the ascending auction credible?



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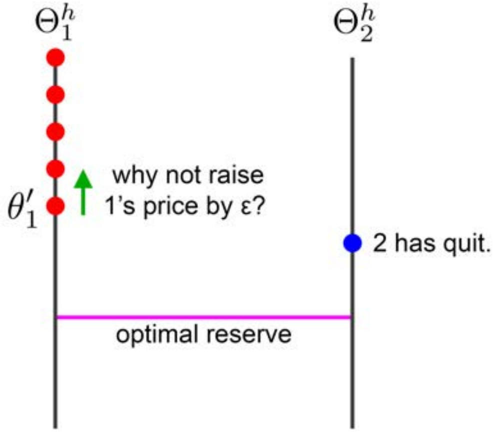

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A: 1's virtual value is positive.

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Ceci n'est pas une proof.

Proof: ascending $\rightarrow$ credible

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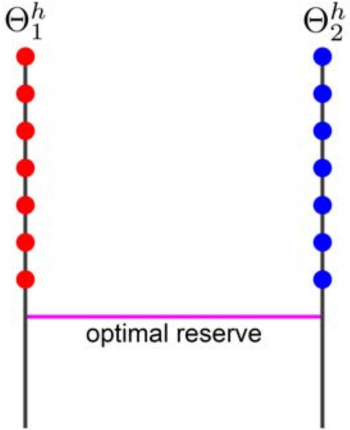
4. For all i, S_i remains a best response to $(S_0', S_{N\setminus i})$.
5. (G', S_N) is also BIC, yields more revenue than (G, S_N). Contradiction, QED.

Proof sketch: credible, SP $\rightarrow$ ascending

A key feature of ascending auctions:
All the types who might still win **pool on the same action**.

Suppose (G, S_N) SP. not pooling $\rightarrow$ not credible.



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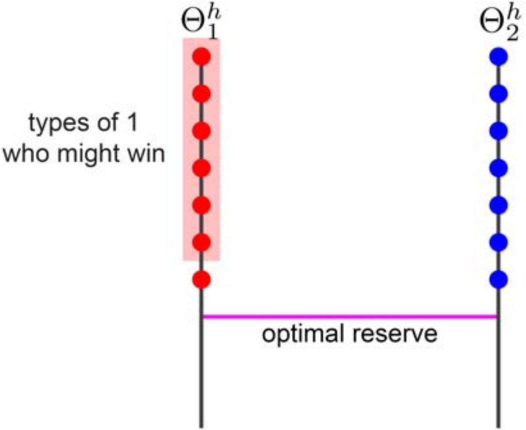

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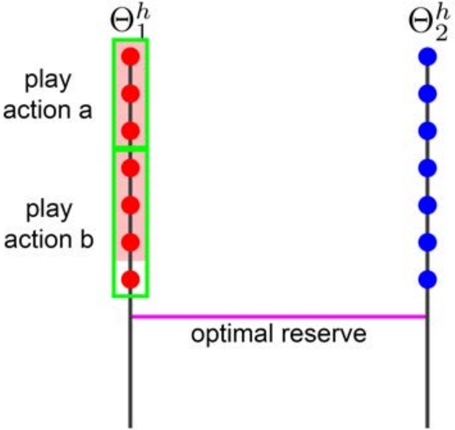

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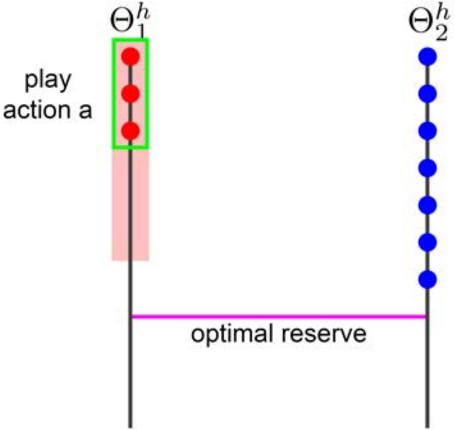

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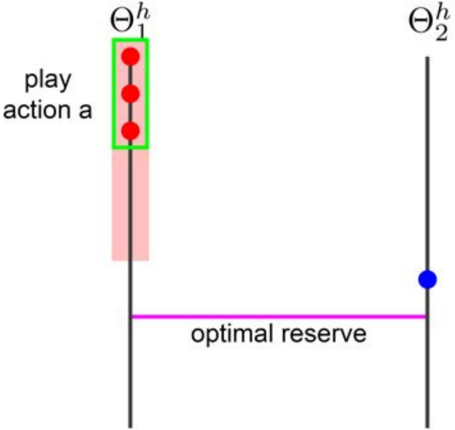

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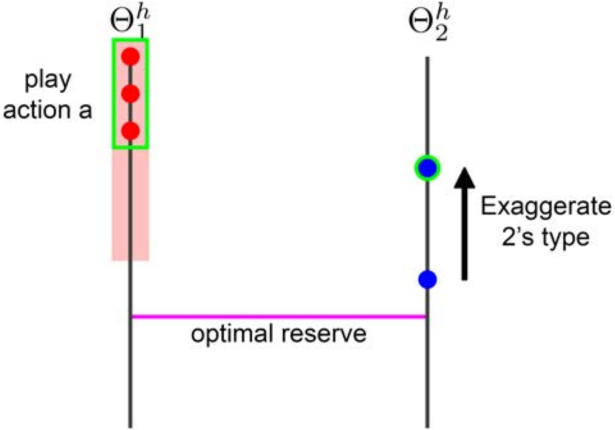

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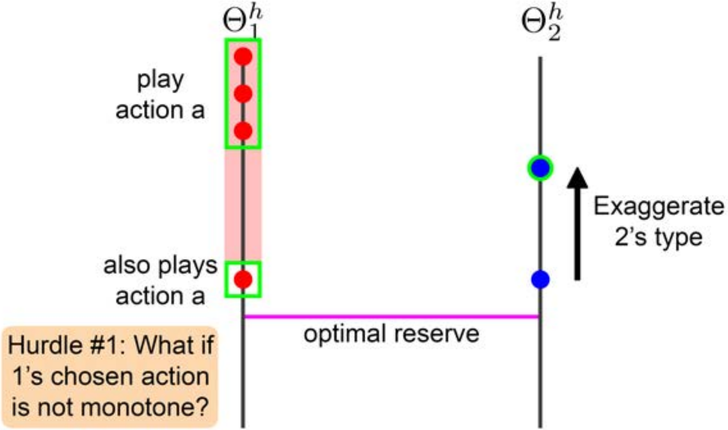

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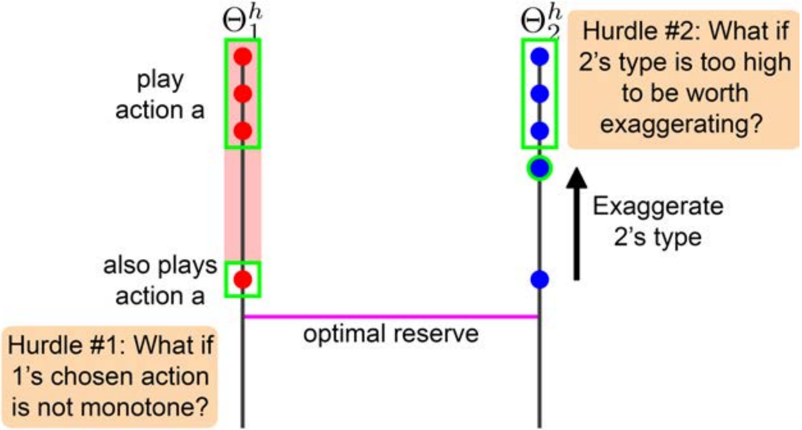

A deviating algorithm

(High-level description, omits fine details.)

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strategy-proof, not pooling → profitable safe deviation

credible, strategy-proof → pooling → ascending auction

bonus

What have the Romans ever done for us?

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Aqueducts, books, concrete, civil law. . .

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Aqueducts, books, concrete, civil law. . . and the ascending auction.

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Perspective #1

First-price and ascending auctions are used because of tradition/path-dependence.

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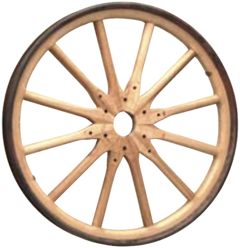
Aqueducts, books, concrete, civil law. . . and the ascending auction.

Perspective #1

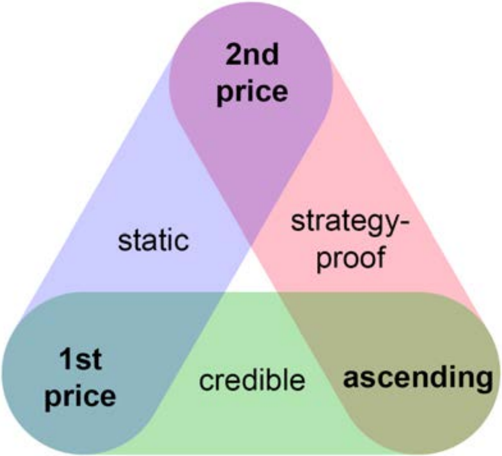
First-price and ascending auctions are used because of tradition/path-dependence.

Perspective #2

First-price and ascending auctions are good solutions to a well-defined commitment problem.

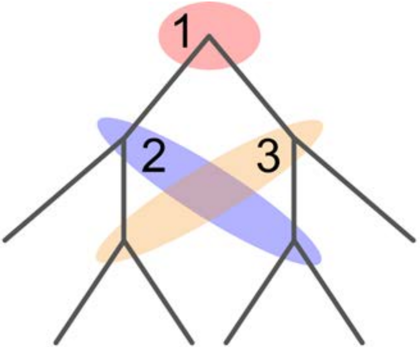


An Auction Trilemma



Pick any two of three.

Calendar time isn't 'built into' extensive forms



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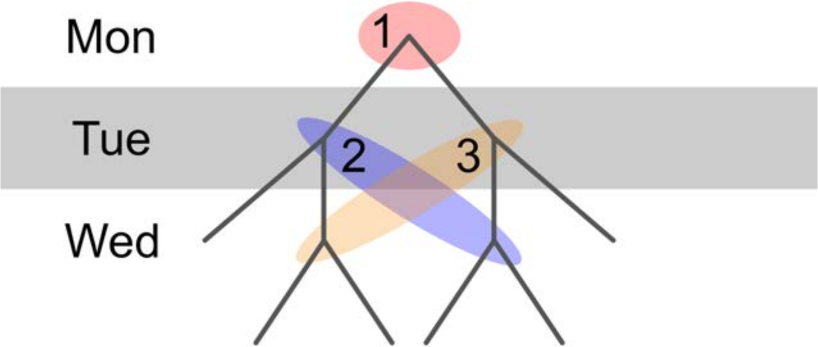

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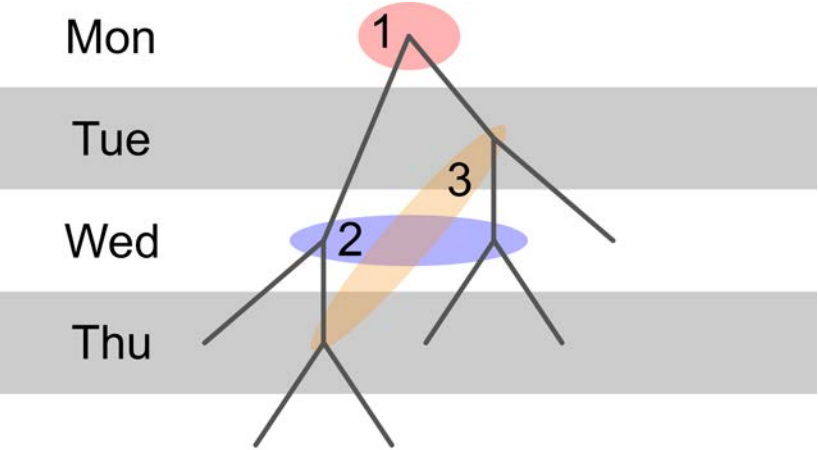

What about asymmetric distributions?

First-price auction (static, credible)

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There exist asymmetric distributions such that no credible static (G, S_N) is ϵ-optimal.

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First-price auction (static, credible)

'Robustly' credible. May not be optimal.
Sometimes impossible to restore optimality.

Proposition

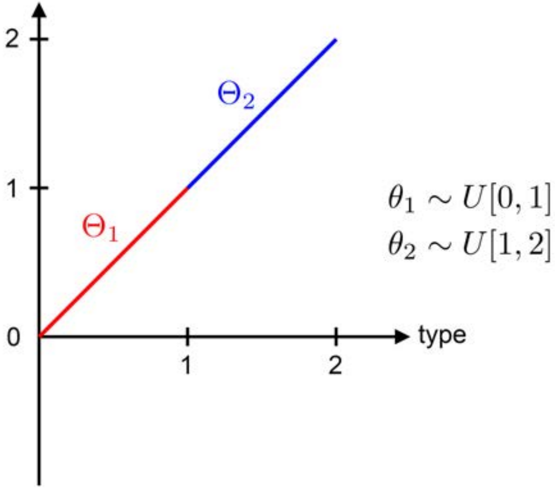
There exist asymmetric distributions such that no credible static (G, S_N) *is* ϵ*-optimal.*

Ascending auction (strategy-proof, credible)

May not be credible or optimal.
Easy to restore both.
The **virtual values** ascending auction.

return

Example: No ϵ-optimal credible static auction



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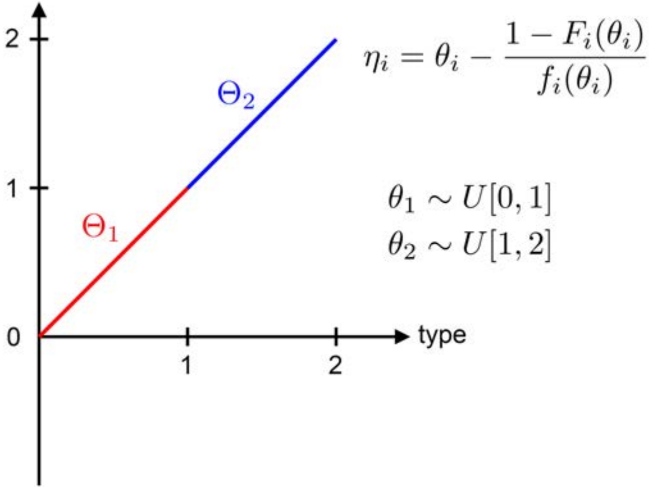

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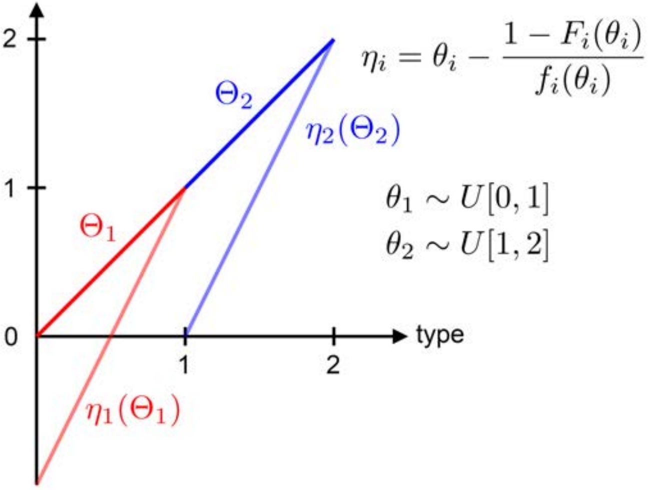

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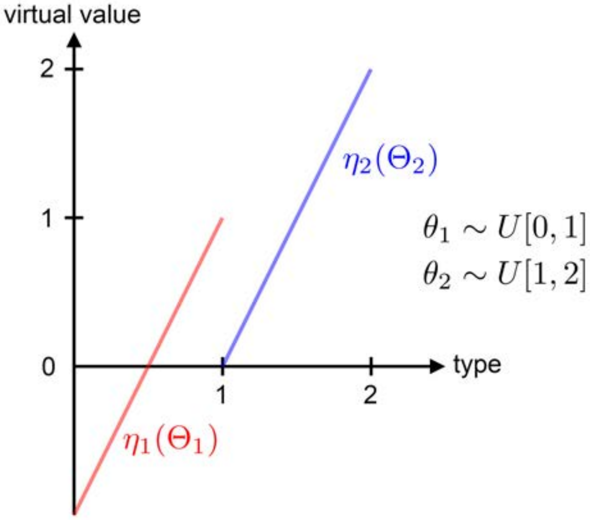

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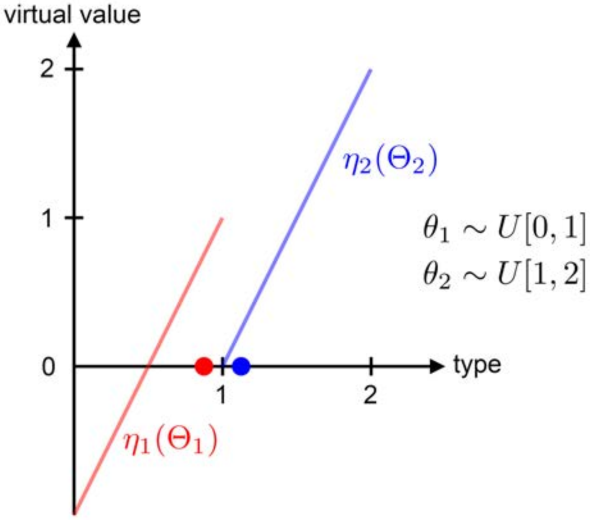

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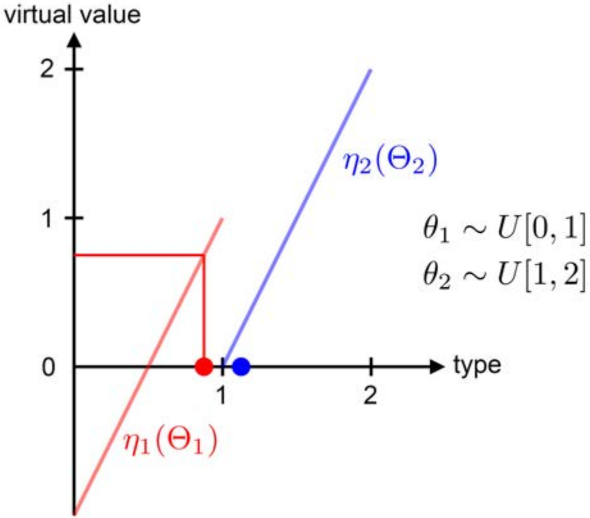

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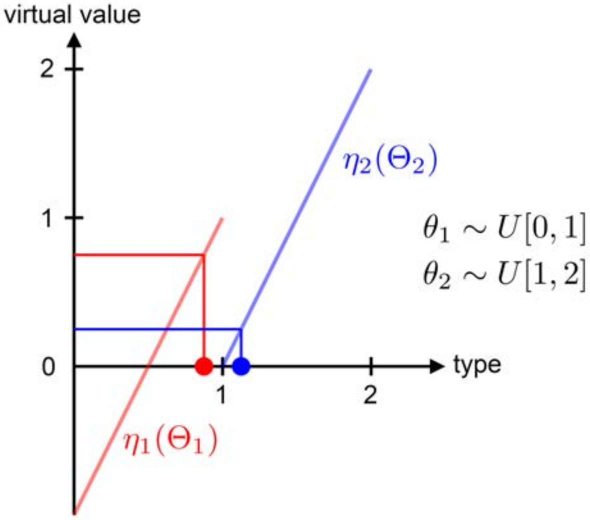

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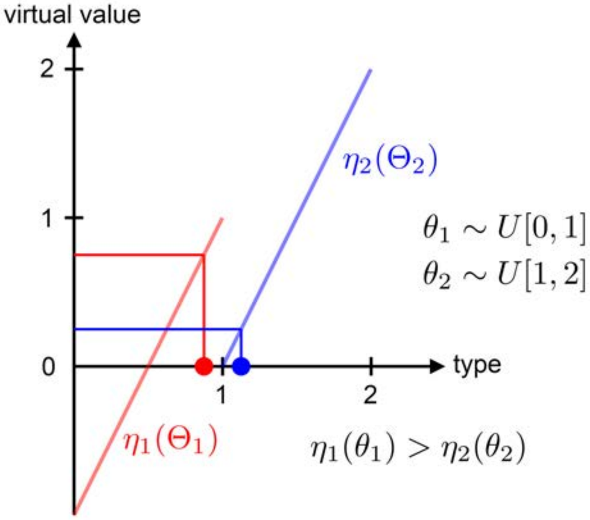

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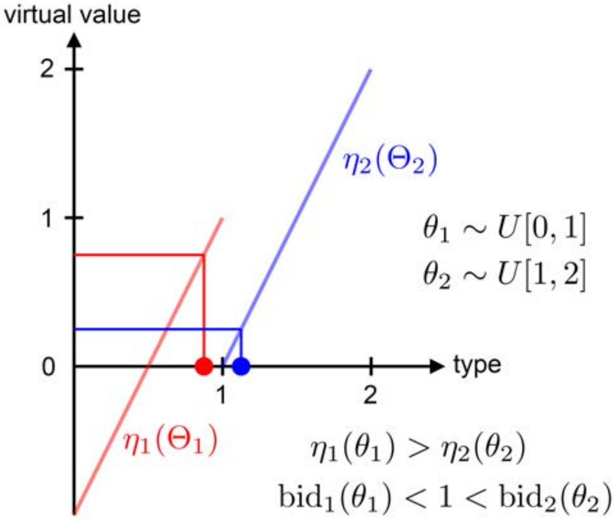

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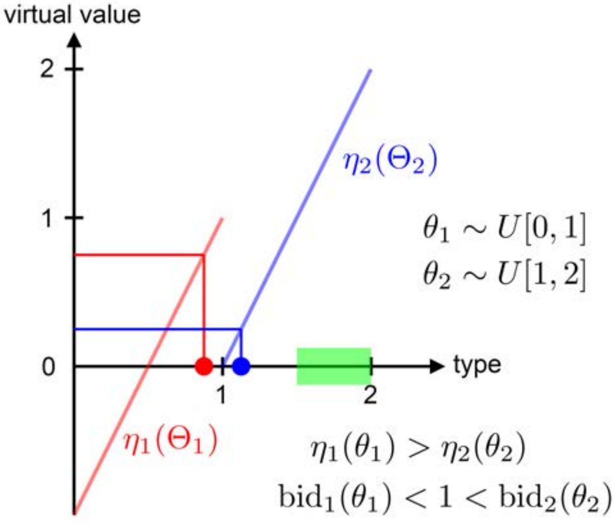

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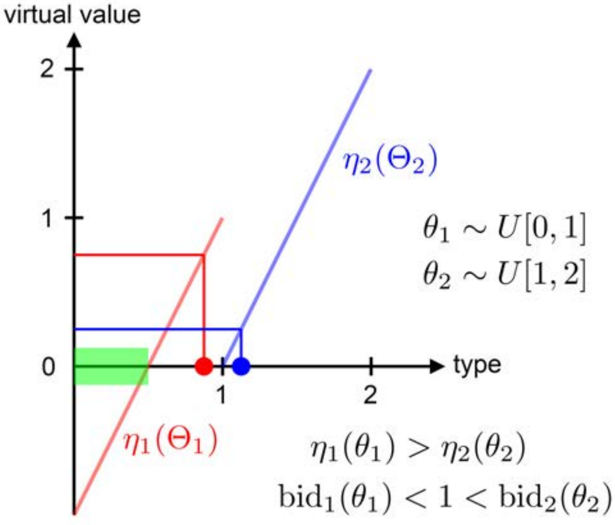

Bidders seldom display types on placards.

> *In the English system bids are . . . usually transmitted by signal. Such signals may be in the form of a wink, a nod, scratching an ear, lifting a pencil, tugging at the coat of the auctioneer or even staring into the auctioneer's eyes – all of them perfectly legal.*

Cassady 1967

Public communication affects aftermarkets and thus incentives.
Ausubel & Cramton 2004, Carroll & Segal 2016, Dworczak 2017.
(Outside the model today.)

A Menagerie

Table: ϵ-optimal auctions

	1P	2P	Asc
Strategy-proof		X	X
Static	X	X	
Credible	X		X
Ex Post IR	X	X	X
Non-winner 0 transfer	X	X	X

A Menagerie

Table: ϵ-optimal auctions

	1P	2P	Asc	Dutch
Strategy-proof		X	X	
Static	X	X		
Credible	X		X	X
Ex Post IR	X	X	X	X
Non-winner 0 transfer	X	X	X	X

A Menagerie

Table: ϵ-optimal auctions

	1P	2P	Asc	Dutch	All-Pay
Strategy-proof		X	X		
Static	X	X			X
Credible	X		X	X	X
Ex Post IR	X	X	X	X	
Non-winner 0 transfer	X	X	X	X	

A Menagerie

Table: ϵ-optimal auctions

	1P	2P	Asc	Dutch	All-Pay	Consol
Strategy-proof		X	X			
Static	X	X			X	X
Credible	X		X	X	X	X
Ex Post IR	X	X	X	X		X
Non-winner 0 transfer	X	X	X	X		

optimal $\cap$ first-price $= \emptyset$

$N = \{1, 2\}$
$\Theta_i = \{4, 5, 6\}$
Tie-breaking order: $1 \lhd 2$

Optimal reserve $= 4$.

Optimality requires:
$b_1(5) = 5$
$b_2(5) = 4.5$

When type profile is $(5, 5)$, tie-breaking rule requires to sell to bidder 2, even though he bid less. Not first-price auction!

return