The Safety Trap

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Abstract

In this paper we provide a model of the macroeconomic consequences of a shortage of safe assets. In particular, we discuss the emergence of a deflationary safety trap equilibrium which is an acute form of a liquidity trap. Issuing public debt, swapping private risky assets for public debt, or increasing the inflation target, stimulate aggregate demand and output in a safety trap. Instead, forward guidance is ineffective. The safety trap can be arbitrarily persistent, as in the secular stagnation hypothesis, despite the existence of infinitely lived assets.

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1 Introduction

One of the main structural features of the global economy in recent years is the apparent shortage of safe assets. The unmistakable signature of the growing shortage of safe assets at any given safe real interest rate, is the secular downward trend in equilibrium real interest rates for more than two decades.

In this paper we provide a simple model of the macroeconomic implications of such a shortage. In particular, we discuss the emergence of a deflationary safety trap equilibrium. It is an acute form of liquidity trap, in which the shortage of a specific form of assets (safe assets), as opposed to a general shortage of assets, is the fundamental driving force. The safety trap can be arbitrarily persistent, or even permanent, consistent with secular stagnation hypothesis (Hansen 1939, Summers 2013), and despite the existence of infinitely lived assets.

The model is a perpetual youth OLG model with heterogeneous agents: Neutrals (risk neutral) and Knightians (infinitely risk averse). Neutrals own risky Lucas trees (aggregate risk) and issue safe assets to Knightians. This securitization process is hampered by a financial friction. As the supply of safe assets shrinks relative to demand (at a given safe interest rate), the safe interest rate drops and the spread between risky and safe rates rises. This mechanism transfers resources from Knightians to Neutrals, reduces the demand for safe assets, and restores equilibrium in the safe asset market.

Once the safe rate hits the zero lower bound, the transfer mechanism breaks down, and instead equilibrium in the safe asset market is restored through a drop in output, which reduces the demand for safe assets. In fact, if deflationary forces emerge, then the transfer flow is actually reversed as safe real interest rates rise, resulting in an even larger drop in output.

This leads to a dual view of safe asset shortages. As long as safe interest rates are positive, safe asset shortages are essentially benign. But when safe interest rates reach the zero lower bound, they become malign. This is because at the zero lower bound tipping point, the virtuous equilibration mechanism through a reduction in safe interest rates is replaced by a perverse equilibrating mechanism through a reduction in output.

The equilibrium of our model admits an aggregate supply-aggregate demand representation, where the stock of safe assets plays the role of an aggregate demand shifter. The equilibrium impact on output is magnified by a Keynesian multiplier. The multiplier is
higher, the more responsive is inflation to output (the steeper is the Phillips curve), or equivalently, the more flexible prices are.

In this context, policies that increase the stock of safe assets, such as public debt issuances and some versions of QE, stimulate aggregate demand and output in a safety trap.\(^1\) Instead, policies that seek (and fail) to stimulate aggregate demand by directly increasing the value of risky assets, such as forward guidance, are largely ineffective because they are mostly dissipated in higher risk spreads. Finally, policies that seek to directly reduce the safe real interest rate (transferring wealth from Knightians to Neutrals and reducing the demand for safe assets), such as a large enough increase in the inflation target, lead to the emergence of a good equilibrium with no recession, inflation and negative safe real interest rates.

The capacity of the government to increase the supply of safe assets depends on two factors: fiscal capacity and crowding out of private safe assets by public safe assets. In a safe asset shortage situation, the relevant form of fiscal capacity is the government’s ability to raise taxes in the bad events feared by Knightians. Crowding out, on the other hand, depends on how much these taxes reduce the private sector’s capacity to issue safe claims backed by the risky dividends of Lucas trees. In our model, there is less crowding out when the securitization capacity of the economy is impaired (when the financial friction is severe). In a safety trap, issuing public debt, and possibly purchasing private risky assets, increases the supply of safe assets and stimulates the economy.

The low rates of a safety trap environment create a fertile ground for the emergence of bubbles. However, we show that risky bubbles do not alleviate the safety trap situation, as they do not expand the stock of safe assets. This formalizes some observations in Summers (2013) that in secular stagnation environments, even large financial bubbles only seem to create moderate economic expansions. Conversely, safe bubbles do alleviate the problem. We associate the latter concept to that of public debt, and show that the existence of a bubbly region expands the fiscal capacity of the government and reduces the crowding out effect, as bubble-debt does not require future taxation if real rates remain secularly low.

\(^1\)In a previous version of this paper, we argued that the benefit of QE1 type policies are unlikely to extend to the swapping of short-run public debt for long-run public debt (which we refer to as Operation Twist (OT), and which encompass the recent QE2 and QE3 in the U.S). In fact, OT can be counterproductive since long term public debt, by being a “bearish” asset that can be used to hedge risky private assets, has a safe asset multiplier effect that short term public debt lacks. That is, long term public debt is not only a safe asset in itself, but also makes risky private assets safer through portfolio effects.

Of course, part of the benefit of OT policies is to support the bearish nature of long term public debt, and in this sense it is the commitment to future support of these assets, should conditions deteriorate, that generates the benefit, for reasons similar to those we highlight in QE type policies.

We refer the reader to a previous version of this paper (Caballero and Farhi 2013) for a detailed exposition.
**Related literature.** Our paper is related to several strands of literature. First and most closely related is the literature that identifies the shortage of safe assets as key macroeconomic fact (see e.g. Caballero 2006, Caballero et al. 2008a and 2008b, Caballero and Krishnamurthy 2009, Caballero 2010, Bernanke et al. 2011, Barclay’s 2012). Our paper provides a model that captures many of the key insights in that literature and that allows us to study the main macroeconomic policy implications of this environment more precisely. Like us, Barro and Mollerus (2014) considers an environment with heterogenous risk aversion. They show that such a model can quantitatively match the value of safe assets to GDP as well as a number of asset pricing facts, and also study the crowding out of private safe assets by public safe assets.

Second, there is the literature on liquidity traps (see e.g. Keynes 1936, Krugman 1998, Eggertsson and Woodford 2003, Christiano, Eichenbaum and Rebelo 2011, Correia, Farhi, Nicolini and Teles 2012, Werning 2012). This literature emphasizes that the binding zero lower bound on nominal interest rates presents a challenge for macroeconomic stabilization. In most models of the liquidity trap, the corresponding asset shortage arises from an exogenous increase in the propensity to save (a discount factor shock). Some recent models (see e.g. Guerrieri and Lorenzoni 2011, and Eggertsson and Krugman 2012) provide deeper microfoundations and emphasize the role of tightened borrowing constraints in economies with heterogeneous agents (borrowers and savers). Similarly, our model can be seen as providing deeper microfoundations, introducing a distinction between safe and risky assets (a distinction which is irrelevant in most liquidity trap analyses which are either deterministic or linearized), and exploring the specific role of safe assets shortages. This distinction has important policy implications.

Third, there is an emerging literature on secular stagnation: the possibility of a permanent zero lower bound situation (see e.g. Kocherlakota 2013 and especially Eggertsson and Mehrota 2014). Like us, they use an OLG structure with a zero lower bound. Unlike us, they do not consider risk and risk premia. As we show, this difference has important consequences for the relative effectiveness of different policy options. An additional difference has to do with the theoretical possibility of permanent zero lower bound equilibrium.

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2 Caballero, Farhi, and Gourinchas (2008a,b) developed the idea that global imbalances originated in the superior development of financial markets in developed economies, and in particular the U.S. Global imbalances resulted from an asset imbalance. Although we do not develop the open economy version of our model here, our model could capture a specific channel that lies behind global imbalances: The latter were caused by the funding countries’ demand for financial assets in excess of their ability to produce them, but this gap is particularly acute for safe assets since emerging markets have very limited institutional capability to produce them.
in the presence of infinitely-lived assets, such as land. Indeed, infinitely-lived assets would rule out a permanent zero lower bound in Kocherlakota (2013) and Eggertsson and Mehrota (2014). In our model, trees are indeed infinitely lived. But their value remains finite even when the safe interest rate is permanently at zero. This is only possible because our model features risk and risk premia, which are ignored in most liquidity trap analyzes. Finally, our modelling of inflation in Section 5 borrows heavily from Eggertsson and Mehrota (2014).

Fourth, our paper is related to the literature on aggregate liquidity (see e.g. Woodford 1990 and Holmström and Tirole 1998), which analyzes the role of governments in providing (possibly contingent) stores of value that cannot be created by the private sector. Our paper shares the idea that liquidity shortages are important macroeconomic phenomena, and that the government has a special role in alleviating them. However, it shifts the focus to a very specific form of liquidity—safe assets—and works out its distinct consequences.

Fifth, there is a literature that documents significant deviations from the predictions of standard asset-pricing models—patterns which can be thought of as reflecting money-like convenience services—in the pricing of Treasury securities generally, and in the pricing of short-term T-bills more specifically (Krishnamurthy and Vissing-Jorgensen 2011, 2012, Greenwood and Vayanos 2010, Duffee 1996, Gurkaynak, Sack and Wright 2006). Our model offers an interpretation of these stylized facts, where the “specialness” of public debt is its safety during bad aggregate states.

Sixth, there is a literature which emphasizes how the aforementioned premium creates incentives for private agents to rely heavily on short-term debt, even when this creates systemic instabilities (Gorton 2010, Stein 2012, Woodford 2012, Gennaioli, Shleifer and Vishny 2012). Greenwood, Hanson and Stein (2012) consider the role of the government in increasing the supply of short-term debt and affecting the premium. Gorton and Ordonez (2013) also consider this question but in the context of a model with (asymmetric) information acquisition about collateral where the key characteristic of public debt that drives its premium is its information insensitivity.

Finally, our paper relates to an extensive literature, both policy and academic, on fiscal sustainability and the consequences of current and future fiscal adjustments (see, e.g., Giavazzi and Pagano 1990, 1996, Alesina and Ardagna 1998, IMF 1996, Guihard et al 2007). Our paper revisits some of the policy questions in this literature but highlights the government’s capacity to create safe assets at the margin, as the key concept to determine the potential effectiveness of further fiscal expansions as well as the benefits of future fiscal
The paper is organized as follows. Section 2 describes our basic model and introduces the key mechanism of a safety trap. Section 3 introduces public debt and considers the effects of QE policies. Section 4 analyzes the role of forward guidance (monetary policy commitments). Section 5 introduces inflation. Section 6 explores the possibility and consequences of rational bubbles. Section 7 concludes. Appendix A.4 develops a version of a liquidity trap in the context of our model but with no risk premia, where the distinction between safe and risky assets is irrelevant, and explains the similarities and differences with a safety trap.

2 A Model

In this section we introduce our basic model. We use a simple stochastic overlapping generations model. We start by developing a real model of an endowment economy. We then extend it to a production economy with nominal rigidities and money. In the model, when a shortage of safe assets arises, safe interest rates need to drop to reduce the return on these assets and hence their demand. However, if there is a zero lower bound for safe interest rates, then a safety trap emerges and asset markets are cleared through a recession instead.

2.1 Real Model

The horizon is infinite and time is continuous.

Demographics. Population is constant and normalized to one. Agents are born and die at hazard rate $\theta$, independent across agents. Each dying agent is instantaneously replaced by a newborn. Therefore, in an interval $dt$, $\theta dt$ agents die and $\theta dt$ agents are born.

Output and aggregate risk. We model aggregate risk as follows. There are two aggregate Poisson processes: a good Poisson process with intensity $\lambda^+$ and a bad Poisson process with intensity $\lambda^-$. We denote by $\sigma^+$ and $\sigma^-$ the stopping times for the realizations of the good and bad Poisson processes.

We define a Poisson event to be the first realization of either the good or the bad Poisson process, with corresponding stopping time $\sigma = \min\{\sigma^+, \sigma^\}$. We say that the Poisson event is good if $\sigma = \sigma^+$ and bad if $\sigma = \sigma^-$. Before the Poisson event, for $t < \sigma$, output $X_t$ per unit of time (output for short) is equal
to $X$. After the Poisson event, for $t \geq \sigma$, output $X_t$ is equal to $\mu^+ X > X$ if the Poisson event is good, and to $\mu^- X < X$ if the Poisson event is bad.

Preferences. We assume that agents only have an opportunity to consume when they die. We define preferences over stochastic consumption processes $\{c_t\}$ which depends only on age and the realization of aggregate shocks.

There are two types of agents in constant fractions in the population: a fraction $\alpha$ of Knightians and $1 - \alpha$ of Neutrals, with identical demographics. These agents have different preferences over risk: Knightians are infinitely risk averse over short time intervals, while Neutrals are risk neutrals over short time intervals. More precisely, for a given stochastic consumption process $\{c_t\}$, we define the utility $U^N_t$ of a Neutral alive at date $t$, and $U^K_t$ of a Knightian alive at date $t$, with the following stochastic differential equations

$$U^N_t = \theta dt c_t + (1 - \theta dt) \mathbb{E}_t[U^N_{t+dt}]$$

and

$$U^K_t = \theta dt c_t + (1 - \theta dt) \min_t\{U^K_{t+dt}\}$$

where we use the notation $\mathbb{E}_t[U^N_{t+dt}]$ to denote the expectation of $U^N_{t+dt}$ conditional on the information available at date $t$ and $\min_t\{U^K_{t+dt}\}$ to denote the minimum possible realization of $U^K_{t+dt}$ given the information available at date $t$.

Endowments, assets, and limited pledgeability. Between $t$ and $t + dt$, output $X_t dt$ is divided into an endowment $(1 - \delta)X_t dt$ distributed equally to agents who are born during that interval of time, and the dividend $\delta X_t dt$ of a unit measure of identical infinitely-lived Lucas trees.

Only Neutrals can own and operate Lucas trees (a Lucas tree owned and operated by a Knightian yields no dividends). A Neutral can then securitize (borrow against) a tree that he owns by issuing arbitrary state-contingent securities to outside investors (other Neutrals or Knightians) against the cash flows of each tree that he owns and operates.

Assumption 1 (Financial friction): The securitization process is hampered by an agency problem: only a fraction $\rho$ of the cash flows of each tree can be pledged to outside investors.

This could be motivated in various ways. One popular microfoundation in the financial constraints literature (see e.g. Holmstrom and Tirole 1998, Kiyotaki and Moore 1997 and a
vast literature since then) is the existence of a moral hazard problem whereby the owner of a tree can abscond with a fraction \(1 - \rho\) of the cash flows.

**Equilibrium.** Newborns trade their endowments for assets. They keep reinvesting and rebalancing their portfolio until they die, at which point they sell their assets for goods and consume them. We focus on the period before the (aggregate) Poisson event and, for simplicity, we study the limit \(\lambda^- \to 0\) and \(\lambda^+ \to 0\). Agents choose their portfolios of assets to maximize their utility. Crucially, Knightians and Neutrals choose different portfolios.

The main features of the portfolios can be understood intuitively. The formal derivations are in Appendix A.1. In order to maximize his utility at date \(t\), a Knightian agent chooses to invest his wealth between \(t\) and \(t + dt\) in safe assets; that is assets whose value is independent of the realization of aggregate shocks between \(t\) and \(t + dt\). Similarly, in order to maximize his utility at date \(t\), a Neutral agent chooses to hold some Lucas trees and to issue some safe assets to Knightians against their pledgeable dividends. This is all we need to know about optimal portfolios in order to derive the equilibrium.

Because of the linearity of preferences and the i.i.d. (across agents and time) nature of death, the model aggregates cleanly. We denote by \(W^K_t\) the total wealth of Knightians and \(W^N_t\) the total wealth of Neutrals. We denote by \(V^S_t\) the total value of safe assets that can be issued against the Lucas trees, and by \(V^R_t\) the total value of risky assets, by which we mean the value of the Lucas trees net of the value of the safe assets that can be issued against them. Note that we have defined \(V^S_t\) as the total value of safe assets that can be issued, not the value of safe assets that are actually issued by Neutrals to Knightians. It is therefore possible that some safe assets are held by Neutrals. We denote total wealth by \(W_t\) with \(W_t = W^K_t + W^N_t\), and the total value of assets by \(V_t = V^R_t + V^S_t\). Note that \(V_t\) is the total value of Lucas trees, the only assets in positive net supply.

Between \(t\) and \(t + dt\) a fraction \(\theta\) of agents die and consume. Because dying agents are a representative sample of the population, consumption between \(t\) and \(t + dt\) is \(\theta W_t dt\). Given that output between \(t\) and \(t + dt\) is \(X dt\), market clearing in the goods market pins down the equilibrium level of wealth:

\[
W_t = W = \frac{X}{\theta}.
\]

Asset market clearing then determines the value of existing assets:

\[
V_t = V = W = \frac{X}{\theta}.
\]
We now can find $V_t^S$ by solving backwards. After the bad Poisson shock, the total value of Lucas trees can be found by applying a similar logic to that prior the shock, so that:

$$V^{\mu^-} = \mu^- \frac{X}{\theta}.$$

Given that only a fraction $\rho$ of the cash flows can be pledged, the maximal value of safe assets is $\rho V^{\mu^-}$. We will verify below that the equilibrium value of safe assets before (and after) the Poisson is indeed

$$V_t^S = V^S = \rho \mu^- \frac{X}{\theta}.$$

For now, we proceed as if it were the case. Risky assets (before the Poisson event) are worth the residual

$$V_t^R = V^R = (1 - \rho \mu^-) \frac{X}{\theta}.$$

Let $r_t$, $r^K_t$, and $\delta^S_t$ denote the rate of return on risky assets, the rate of return on safe assets, and the dividend paid by safe assets, respectively. Then equilibrium is characterized by the following equations:\footnote{Note that in the limit that we consider ($\lambda^+ \to 0$ and $\lambda^- \to 0$), $\mu^+$ does not appear in the equilibrium equations before the Poisson event. Only $\mu^-$ does, because it determines the supply of safe assets.}

$$r^K_t V^S = \delta^S_t X,$$

$$r_t V^R = (\delta - \delta^S_t) X,$$

$$\dot{W}^K_t = -\theta W^K_t + \alpha (1 - \delta) X + r^K_t W^K_t,$$

$$\dot{W}^N_t = -\theta W^N_t + (1 - \alpha) (1 - \delta) X + r_t W^N_t,$$

$$W^K_t + W^N_t = V^S + V^R.$$
The asset pricing equations equate the rates of returns to dividend yields and capital gains for both safe and risky assets, taking into account that capital gains are zero because the value of safe and risky assets are constant over time. Focusing on the evolution equation for Knightian wealth (the intuition for the evolution equation of Neutral wealth is similar), the wealth equations can be understood as follows: First, between $t$ and $t + dt$, a fraction $\theta dt$ of Knightians die, sell their assets, and consume. Because the dying Knightians are a representative sample of Knightians, this depletes the stock of Knightian wealth by $\theta W_t^K dt$. Second, between $t$ and $t + dt$, $\theta dt$ new Knightians are born with a total endowment $(1 - \delta) X dt$, which they sell to acquire assets. This increases Knightian wealth by $\alpha (1 - \delta) X dt$. Third, between $t$ and $t + dt$, Knightians collect interest rates $r^K W_t^K dt$. Overall, the increase in Knightian wealth is therefore $W_{t+dt}^K - W_t^K = -\theta W_t^K dt + \alpha (1 - \delta) X dt + r^K W_t^K dt$. Taking the limit $dt \to 0$ yields the stated equation.

Taken together, these equations constitute an equilibrium if and only if two conditions are satisfied: $W_t^K \leq V^S$ and $\delta S \leq \delta \rho$. These conditions are necessary and sufficient for the pledgeability constraints to be verified; i.e. that no Neutral pledges more than a fraction $\rho$ of the cash flows of his Lucas trees to Knightians in the form of safe assets. We shall see below that in equilibrium we can either have $W_t^K < V^S$ or $W_t^K = V^S$, but that we always have $\delta S < \delta \rho$. This validates our earlier claim that the value of safe assets is given by $V^S = \rho V^\mu^-$. 

**Two regimes.** We focus on steady states and drop $t$ subscripts. There are two regimes, depending on whether the constraint $W_t^K \leq V^S$ is slack (unconstrained regime) or binding (constrained regime).

In the unconstrained regime, since Neutrals are the marginal holders of safe assets, safe and risky rates must be equal. A few steps of algebra show that in this case:

$$\delta S = \delta \rho \mu^- < \delta \rho,$$

$$r = r^K = \delta \theta.$$

The interesting case for us is the constrained regime, which captures the safe asset shortage environment. In it, Knightians gobble up all safe assets and wish they had more, so that:

$$W^K = V^S = \rho \mu^- \frac{X}{\theta}.$$

It is easy to verify that this regime holds (after possibly a transitional period) as long as the following safe asset shortage condition holds (which we shall assume holds henceforth)
Assumption 2 \textit{(Safe asset shortage):} $\alpha > \rho \mu^{-}$.

With this assumption we have:

$$\delta^S = \delta \rho \mu^{-} - (\alpha - \rho \mu^{-})(1 - \delta) < \delta \rho \mu^{-} < \delta \rho,$$

$$r^K = \delta \theta - (1 - \delta) \theta \frac{\alpha - \rho \mu^{-}}{\rho \mu^{-}} < \delta \theta,$$

$$r = \delta \theta + (1 - \delta) \theta \frac{\alpha - \rho \mu^{-}}{1 - \rho \mu^{-}} > \delta \theta.$$  

It follows that in this region there is a safety premium

$$r - r^K = (1 - \delta) \theta \frac{\alpha - \rho \mu^{-}}{\rho \mu^{-} (1 - \rho \mu^{-})} > 0.$$

The supply of safe assets is determined by the severity of the potential bad shock ($\mu^{-}$) and the ability of the economy to create safe assets ($\rho$). In fact $\rho$ and $\mu^{-}$ enter the equilibrium equations only through the sufficient statistic $\rho \mu^{-}$. Similarly, the demand for safe assets is summarized by the fraction of Knightians ($\alpha$). Together, these sufficient statistics determine whether we are in the unconstrained regime ($\alpha \leq \rho \mu^{-}$) or in the constrained regime ($\alpha > \rho \mu^{-}$).

Remark 1 \textit{Our demographics and preferences allow us to capture a stylized version of a life-cycle model with a portfolio choice, abstracting from intertemporal substitution in consumption which is not central to the questions we want to analyze. The only decision that agents are making is how to invest their wealth at every point in time, and because Knightians and Neutrals have different attitudes towards risk, they choose different portfolios.}

Remark 2 \textit{Our model features two forms of market incompleteness. The first one is tied to our overlapping generations structure, which makes future endowment (“wages”) non-pledgeable. The second market incompleteness is the pledgeability of dividends constraint ($\rho$) which limits the ability to securitize (tranche) Lucas trees. Tranching is desirable because it decomposes an asset into a safe tranche which can be sold to Knightian agents and a risky tranche. In the policy discussion we will not exploit the first form of incompleteness (we use the OLG structure mostly because it yields a stationary distribution of wealth between}
Neutrals and Knightians) and focus instead on the second form.\footnote{An alternative would have been to use a model where all agents are alive from the initial period onwards. The problem is that differences in risk aversion lead to explosive wealth distribution dynamics To induce stationarity, Barro and Mollerus (2014), who follow this route, must introduce mean reversion in risk aversion in the form of a Poisson shock that transforms a Neutral into a Knightian and vice versa.} The latter implies that in equilibrium risky assets held by Neutrals contain an unpledgable safe claim of size \((1-\rho)\mu^{-\frac{X}{\delta}}\) which is the starting point of our macroeconomic policy analysis later on. See Section 3 for a more detailed discussion.

2.2 Aggregate Demand and the Safety Trap

In this section, we extend the real model to a production economy with nominal rigidities and money. The flexible price equilibrium of this extended model is the same as that of the real model. We call the resulting safe interest rate and allocation the Wicksellian safe natural interest rate and the natural allocation. For natural output, we also sometimes use the term potential output.

With nominal rigidities, appropriate monetary policy can ensure that actual output is at potential as long as the safe natural interest rate is positive. But when the safe natural interest rate is negative, the economy reaches the zero lower bound and actual output drops below potential, a situation which we call a safety trap. The zero lower bound on safe nominal interest rates arises endogenously because money can be held as a safe store of value.

2.2.1 A New Keynesian Cash-In-Advance Economy

We extend the model in two steps. The first step consists of making output demand determined and to associate real to nominal safe rates by adding standard New Keynesian features. The second step adds money and captures its transaction role with a Cash-In-Advance constraint, which introduces a lower bound for safe rates.

Demand determined output. Let us incorporate the traditional ingredients of New Keynesian economics: imperfect competition, sticky prices, and a monetary authority.

In this setting, in every period, non-traded inputs are used to produce differentiated varieties of goods \(x_k\) indexed by \(k \in [0,1]\) where each variety is produced using a different variety of non-traded good also indexed by \(k \in [0,1]\). We index trees by \(i \in [0,\delta]\), where each
tree $i$ yields a dividend of $X$ non-traded goods. Similarly, we index newborns by $j \in [\delta, 1]$ where each newborn $j$ is endowed with $X$ non-traded goods. Goods with indices $k \in [0, \delta]$ are produced with the non-traded inputs from the dividends of trees indexed by $k$, and goods with indices $k \in [\delta, 1]$ are produced with the non-traded inputs from the endowments of the newborns indexed by $k$. Each variety is sold by a monopolistic firm. Firms post prices $p_k$ in units of the numeraire. These differentiated varieties of goods are valued by consumers according to a standard Dixit-Stiglitz aggregator $C = \left( \int_0^1 x_k^{\frac{\sigma - 1}{\sigma}} \, dk \right)^{\frac{1}{\sigma - 1}}$, and consumption expenditure is $PC = \int_0^1 p_k x_k \, dk$, where the price index is defined as $P = \left( \int_0^1 p_k^{1-\sigma} \, dk \right)^{\frac{1}{1-\sigma}}$. The resulting demand for each variety is given by $x_k = \left( \frac{p_k}{P} \right)^{-\sigma} C$.

The prices of different varieties are entirely fixed (an extreme form of sticky prices, which we shall relax later in Section 5) and equal to each other, $p_k = P$. Firms accommodate demand at the posted price, and their profits accrue to the agent owning and supplying the corresponding non-traded input. Without loss of generality, we use the normalization $P = 1$. Note that because the prices of all varieties are identical, the demand for all varieties is the same. Output is demand-determined, and as a result, $x_k = C = \xi X$ for all $k$ where the capacity utilization rate $\xi \leq 1$ is the same for all firms. Capacity utilization $\xi$ represents the wedge between actual output $\xi X$ and potential output $X$.

Finally, a monetary authority sets a safe nominal interest rate $i$. Because prices are rigid, this determines the real interest rates $r^K = i$.

**Money, the zero lower bound and the cashless limit.** To justify a zero lower bound, $r^K \geq 0$, we introduce money into the model. We then define and focus on the cashless limit (see e.g. Woodford 2003).

We represent the demand for real money balances for transactional services using a Cash-In-Advance constraint that stipulates that individuals with wealth $w_t$ and money holdings $m_t$ can only consume $\min(w_t, \frac{m_t}{\epsilon})$. When $i > 0$, money is held only for transaction services. When $i = 0$ money is also held as a safe store of value, which competes with its transaction services. This model has no equilibrium with $i < 0$, because then money would dominate other safe assets. Hence there is a zero lower bound $i \geq 0$.

The demand for real money balances for transactional services is $\varepsilon W_t^K$ and $\varepsilon W_t^N$ for Knightians and Neutrals respectively. We assume that the money supply is $\varepsilon M^\varepsilon$ with $M^\varepsilon = \frac{X}{\theta}$ before the Poisson event and that it adjusts to accommodate one for one the change in potential output after the Poisson shock ($M^{\varepsilon-} = \mu^- \frac{X}{\theta}$ and $M^{\varepsilon+} = \mu^+ \frac{X}{\theta}$ for bad and good
shocks, respectively).

After the (bad) Poisson shock, the value of the safe tranches of trees is a fraction $\rho$ of the total value of assets excluding money (government liability), and the total value of safe assets is

\[ V^S = \rho (\mu - \frac{X}{\theta} - \varepsilon M^\varepsilon) + \varepsilon M^\varepsilon. \]

\[ \quad = \rho \mu - \frac{(1 - \varepsilon) X}{\theta} + \varepsilon \mu - \frac{X}{\theta}. \]

and the equilibrium equations are now,

\[ r^K V^S = \delta^S \xi X, \]

\[ r V^R = (\delta - \delta^S) \xi X, \]

\[ \dot{W}^K_t = -\theta W^K_t + \alpha (1 - \delta) \xi X + r^K (1 - \varepsilon) W^K_t, \]

\[ \dot{W}^N_t = -\theta W^N_t + (1 - \alpha) (1 - \delta) \xi X + r (1 - \varepsilon) W^N_t, \]

\[ \varepsilon (W^K_t + W^N_t) \leq \varepsilon M^\varepsilon \quad \text{with equality if} \quad r^K > 0 \]

\[ W^K_t + \varepsilon W^N_t \leq V^S, \]

\[ W^K_t + W^N_t = V^S + V^R, \]

and the requirement that

\[ r^K = i \geq 0. \]

In the rest of the paper, we focus on the cashless limit $\varepsilon \to 0$. We also focus on steady states and drop $t$-subscripts.

### 2.2.2 The Zero Lower Bound and the Safety Trap

If the safe natural interest rate is positive, then by setting the nominal interest rate equal to the natural safe interest rate $i = r^K > 0$, the monetary authority can replicate the natural allocation, ensuring that output is at potential $\xi = 1$.

**The safety trap.** If the safe natural interest rate is negative, then there is a recession

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\^ We refer the reader to Appendix A.2 for an analysis of some interesting issues that arise away from the cashless limit.
(\xi < 1) even with \(i = r^K = 0\). To determine the severity of the recession, let us work backwards and recall that we have assumed that actual and potential (now lower) output coincide after the bad Poisson shock and therefore the value of safe assets (before the shock) is still given by

\[ V^S = \rho \mu^- \frac{X}{\theta}. \]

Mechanically, the expanded (to incorporate a zero lower bound) model is identical to the basic real model but with \(\rho \mu^-\) replaced by \(\frac{\rho \mu^-}{\xi}\) and \(X\) replaced by \(\xi X\). The requirement that \(r^K = 0\) determines the severity of the recession \(\xi\):

\[ 0 = \delta \theta - (1 - \delta) \theta \frac{\alpha - \frac{\rho \mu^-}{\xi}}{\rho \mu^-}, \]

yielding

\[ \xi = \frac{\rho \mu^-}{\rho \mu^-} < 1, \]

where \(\rho \mu^- = \alpha (1 - \delta)\) corresponds to the value of these combined parameters for which zero is the natural safe interest rate.\(^6\)

The mechanism is akin to that of a liquidity trap. We call it a “safety trap”. At full employment, there is an excess demand for safe assets. A recession lowers the absolute demand for safe assets while keeping the absolute supply of safe assets fixed and restores equilibrium. Figure 1 illustrates this mechanism, which we describe next.

The supply of safe assets is given by \(V^S = \rho \mu^- \frac{X}{\theta}\) and the demand is given by \(W^K = \frac{\alpha (1 - \delta) \xi X}{\theta - r^K}\). Equilibrium in the safe asset market requires that \(W^K = V^S\), i.e.

\[ \frac{\alpha (1 - \delta) \xi X}{\theta - r^K} = \rho \mu^- \frac{X}{\theta}. \]

Consider an unexpected (zero ex-ante probability) shock that lowers the supply of safe assets (a reduction in \(\rho \mu^-\)). The mechanism by which equilibrium in the safe asset market is restored has two parts. The first part immediately reduces Knightian wealth \(W^K\) to a lower level, consistent with the lower supply of safe assets \(\rho \mu^- \frac{X}{\theta}\). The second part maintains

\[ r = \delta \theta + (1 - \delta) \theta \frac{\alpha - \frac{\rho \mu^-}{\xi}}{1 - \frac{\rho \mu^-}{\xi}}. \]

\(^6\)Note that the risky interest rate \(r\) is increasing in \(\xi\), so that the deeper the recession, the lower is \(r\):
Knightian wealth $W^K$ at this lower level.

The first part of the mechanism is as follows. The economy undergoes an immediate wealth adjustment (the wealth of Knightians drops) through a round of trading between Knightians and Neutrals born in previous periods. At impact, Knightians hold assets that now carry some risk. They react by selling the risky part of their portfolio to Neutrals. This shedding of risky assets catalyzes an instantaneous fire sale whereby the price of risky assets collapses before immediately recovering once risky assets have changed hands. Needless to say, in reality this phase takes time, which we have removed to focus on the phase following the initial turmoil.

The second part of the equilibrating mechanism differs depending on whether the safe interest rate $r^K = i$ is above or at the zero lower bound. If $r^K = i > 0$, then a reduction in the safe interest rate $r^K = i$ takes place. This reduction in the safe interest rate effectively operates a transfer (in every period) from Knightians to Neutrals, which limits the growth of Knightian wealth. As a result, the safe asset market remains in equilibrium, and so does the goods market. If the safe interest rate is against the zero lower bound $r^K = i = 0$, then this reduction in the safe interest rate cannot take place and the associated transfer cannot occur. The only adjustment mechanism is with a decline in output (income), which also drags down Neutral’s wealth.
Figure 2: AS-AD representation and the Keynesian cross. Recession caused by a decrease in the supply of safe assets. Aggregate demand shifts down.

**AS-AD representation and the Keynesian cross.** With safe interest rates fixed at $r^K = i = 0$, output is determined as follows. Recall that $rW^N = rV^R = (\delta - \delta^S)X = \delta X - r^KV^S$, which replaced into the wealth accumulation equation of Neutrals yields the total value of Neutral wealth:

$$W^N = (1 - \alpha)(1 - \delta)\xi X_{\theta} + \delta\xi X_{\theta} - \frac{r^K}{\theta} V^S$$

while the total value of Knightian wealth is $W^K = V^S$. Aggregate demand for goods is therefore given by $\theta (W^N + W^K) = AD (\xi X)$ where

$$AD (\xi X) = (1 - \alpha)(1 - \delta)\xi X + \delta\xi X + (\theta - r^K) V^S,$$

with $r^K = 0$. Aggregate supply is simply the 45 degree line

$$AS (\xi X) = \xi X.$$

Aggregate demand and aggregate supply are two increasing functions of income $\xi X$. Aggregate demand is flatter than aggregate supply. Equilibrium is determined by a *Keynesian*
cross at the intersection of the aggregate demand and aggregate supply curves. Crucially, a reduction \( dV^S < 0 \) in the value of safe assets \( V^S \) represents an adverse (downward) shift to aggregate demand. This reduction in aggregate demand lowers output, which further reduces aggregate demand, etc. ad infinitum. This is a familiar Keynesian multiplier

\[
\frac{\xi X}{\theta V^S} > 1,
\]

which amplifies the effect of the initial reduction \( \theta dV^S \) in aggregate demand to a final effect of

\[
d (\xi X) = \frac{\xi X}{\theta V^S} \theta dV^S - a \text{ proportional increase in output.}^{7,8}
\]

**Secular stagnation.** Because we are studying the limit \( \lambda^- \to 0 \) and \( \lambda^+ \to 0 \), the safety trap is essentially permanent. The safety trap also can be seen as a model of secular stagnation, understood by most recent observers to be a situation where the economy is at the zero lower bound forever.

The theoretical possibility of permanent zero lower bound equilibrium is sometimes disputed on the grounds that this would imply an infinite value for infinitely-lived assets, such as land. In our model, trees are indeed infinitely lived. But their value remains finite even when the safe interest rate is \( r^K = i = 0 \) essentially permanently. This is only possible because our model features risk and endogenous risk premia, which are ignored in most liquidity trap analyses.

**Policy options: a roadmap.** In the next sections we analyze policy options in a safety trap. The only determinants of aggregate demand that the policies that we will consider will be able to affect is the value of safe assets \( V^S \) (public debt and QE in Section 3) and

\[
7\text{The fact that the final effect of a decrease in } V^S \text{ is a proportional decrease in output } \xi X \text{ is an immediate consequence of the fact that the Keynesian cross is a linear system of two equations in } \xi X \text{ and } V^S.
\]

\[
8\text{The analysis above goes through as well if we raise the share of Knightian agents } \alpha \text{ instead of reducing } \rho \mu^-, \text{ in which case the recession factor is}
\]

\[
\xi = \frac{\alpha}{\alpha} < 1,
\]

where \( \alpha = \frac{\rho \mu^-}{1-\delta} \) corresponds to the value of this parameter for which zero is the natural safe interest rate. The increase in \( \alpha \) acts as an adverse shift in aggregate demand. This interpretation resembles the Keynesian paradox of thrift. Combining both, asset supply and demand factors, we have that the severity of the recession is determined by the sufficient statistic \( \frac{\rho \mu^-}{\alpha} \) according to the simple equation:

\[
\xi = \frac{\alpha}{\rho \mu^-} \frac{\rho \mu^-}{\alpha},
\]

where \( \frac{\rho \mu^-}{\alpha} = 1 - \delta \) corresponds to the value of these combined parameters for which zero is the natural safe interest rate.
increasing the inflation target in Section 5). In particular, it is important to note that the risky rate $r$ does not appear either in aggregate demand or in aggregate supply. The equilibrium is block-recursive in $\xi$ and $(r, V^R)$. Even when $\lambda^+ > 0$, the future value of risky assets after the good Poisson shock does not affect the determination of output $\xi X$. This point will prove crucial later on when we discuss the ineffectiveness of forward guidance in a safety trap (Section 4).

3 Public Debt and Quantitative Easing

In this section we introduce public debt to our analysis and discuss situations where the government can use its debt capacity to alleviate the safety trap. We make three points: First, there is a region in which public debt crowds out one-for-one the private sector’s capacity to produce its own safe debt. Second, there is a region in which this crowding out does not occur, and instead public debt increases the total supply of safe assets one for one. And third, we show that in such contexts, QE type policies can be effective.

3.1 Public Debt and Fiscal Capacity

Public debt. We start by introducing public debt and discussing the role of public purchases and sales of such debt. The government taxes dividends, $\delta X$. The tax rate is $\tau^+$ after the good Poisson shock occurs, $\tau^−$ after the bad Poisson shock occurs, while the tax rate before the Poisson event is set to a value $\tau_t$ that satisfies the government flow budget constraint. The government issues a fixed amount of risk-free bonds that capitalize future tax revenues and pays a variable rate $r^K_t$. The proceeds of the sales of these bonds are rebated lump-sum to agents at date 0.

Let the value of public debt be given by $D$, then we have

$$D = \tau^− \mu \frac{X}{\theta}.$$

Assumption 3 (Regalian taxation power): Taxes backing government safe debt can be levied on the claim $(1 - \rho)\mu^- X \frac{X}{\theta}$ to privately unpledgable of future dividends.

It is the latter feature that makes this debt “short-term,” since its value remains constant over time as its coupons vary with the riskless rate. In the previous version of this paper (Caballero and Farhi 2012), we introduce long-term public debt and study Operation Twist (OT) policies that swap long-term debt for short-term debt.
That is, the government is essentially better than private investors at collecting dividend revenues from Neutrals once borrowers’ incentives are weak. This confers the government a comparative advantage in the production of safe assets.\footnote{This mechanism has some commonality with the idea in Holmström and Tirole (1998) that the government has a comparative advantage in providing liquidity. In their model like in ours, this result arises from the assumption that some agents (consumers in their model) lack commitment and hence cannot borrow because they cannot issue securities that pledge their future endowments. This can result in a scarcity of stores of value. The government can alleviate this scarcity by issuing public debt and repaying this debt by taxing consumers. The proceeds of the debt issuance can actually be rebated to consumers. At the aggregate level, this essentially relaxes the borrowing constraint of consumers: They borrow indirectly through the government. In their model like in ours, the comparative advantage of the government in providing liquidity arises because it is better than private lenders at collecting revenues from consumers.}

Note, however, that the scope for policy depends crucially on the degree of financial development of the economy (indexed by $\rho$). In a very developed market, $\rho$ is high, and soon public debt starts to crowd out privately produced safe assets. To see this, note that because the consumption of a Neutral cannot be negative (a form of limited liability), the fraction of dividends that it can pledge is now $\rho (\tau^-) = \min \{\rho, 1 - \tau^-\}$. Thus, as long as $\tau^- \leq 1 - \rho$ there is no crowding out, but above this threshold public safe assets crowd out private safe assets one for one (we return to this issue below).

The total (private and public) value of safe assets is then given by
\[
V^S = \rho (\tau^-) \mu^- \frac{X}{\theta} + D = [\rho (\tau^-) + \tau^-] \mu^- \frac{X}{\theta}.
\]

The model is isomorphic to the one described in Section 2, with $\rho$ replaced by $[\rho (\tau^-) + \tau^-]$ (and note that $\rho \leq \rho (\tau^-) + \tau^- \leq 1$). Thus we have
\[
r^K = \delta \theta - (1 - \delta) \theta \frac{\alpha - [\rho (\tau^-) + \tau^-] \mu^-}{[\rho (\tau^-) + \tau^-] \mu^-},
\]
\[
r = \delta \theta + (1 - \delta) \theta \frac{\alpha - [\rho (\tau^-) + \tau^-] \mu^-}{1 - [\rho (\tau^-) + \tau^-] \mu^-},
\]

and we can also use $r^K D = \tau \delta X$ to compute
\[
\tau = \tau^- \mu^- \frac{r^K}{\delta \theta}.
\]

The economy is in the constrained regime if and only if $\alpha > [\rho (\tau^-) + \tau^-] \mu^-$, which we
assume. The safety premium is then given by

\[ r - r^K = \theta (1 - \delta) \frac{\alpha - [\rho(\tau^{-}) + \tau^{-}][\mu^{-}]}{[\rho(\tau^{-}) + \tau^{-}]\mu^{-}\left[1 - [\rho(\tau^{-}) + \tau^{-}]\mu^{-}\right]} \geq 0. \]

**Crowding out.** As we mentioned earlier, in this model government debt acts exactly like tranching, with \( \tau^{-} \) playing the same role as \( \rho \), as long as public debt is low enough (\( \tau^{-} < 1 - \rho \)). In this non-Ricardian region, issuing public debt does not crowd out private safe assets, resulting in a one-for-one expansion of the supply of safe assets \( V^S \), an increase in the safe interest rate \( r^K \), a reduction in the risky interest rate \( r \), and a reduction in the safety premium \( r - r^K \). There is also a Ricardian region where public debt is high enough (\( \tau^{-} \geq 1 - \rho \)) so that issuing public debt crowds out private safe assets one for one, leaving unchanged the supply of safe assets \( V^S \), the safe and risky interest rates \( r^K \) and \( r \) as well as the safety premium \( r - r^K \). The economy is more likely to be in the Ricardian region than in the non-Ricardian region, the higher is the securitization capacity \( \rho \).

It will sometimes prove convenient to write

\[ V^S = \frac{\mu^{-}X}{\theta} v^S(D/X) \]

where the function \( v^S(D/X) \) is defined together with the function \( \tau^{-}(D/X) \) by the following equations

\[ v^S(D/X) = \rho(\tau^{-}(D/X)) + \frac{\theta}{\mu^{-}} \frac{D}{X} = [\rho(\tau^{-}(D/X)) + \tau^{-}(D/X)], \]

and

\[ \tau^{-}(D/X) = \frac{\theta}{\mu^{-}} \frac{D}{X}. \]

Crowding out of private safe assets \( \frac{\mu^{-}X}{\theta} v^S(D/X) - D \) by public debt \( D \) is

\[ \frac{\mu^{-}}{\theta} \left( 1 - \frac{dv^S}{d(D/X)} \right) = -\frac{d\rho}{d\tau^{-}} \]

is either 0 (in the non-Ricardian region) or 1 (in the Ricardian region).

**Intermediate crowding out.** We can easily extend the model to feature intermediate crowd out by assuming that the trees differ in their pledgeability \( \tilde{\rho} \) with a distribution \( dF(\tilde{\rho}) \). This extension can be exactly mapped into the model above. The only difference is that we now have
\[ \rho(\tau) = \int \min \{ \tilde{\rho}, 1 - \tau \} \, dF(\tilde{\rho}). \]

Crowd out is now given by
\[ -\frac{d\rho}{d\tau} = [1 - F(1 - \tau^{-})] \in [0, 1]. \]

**Crowding out and Ricardian equivalence.** The tight link between imperfect crowding out and the failure of Ricardian equivalence that exists in our model relies on the assumption that we have made that taxes are capitalized. It would not hold if, for example, taxes were levied on the endowment of newborns.\(^{11}\) We view this feature as desirable since it allows us to focus on on the market failure that is central to our analysis—the financial friction that hampers the securitization process—rather than on more conventional and perhaps debatable features of OLG models.

**Public debt and the safety trap.** Now imagine that the economy is in a safety trap where the safe interest rate is fixed at zero and output is below potential with \( \xi < 1 \). Then increasing public debt from \( D \) to \( \hat{D} > D \) increases the supply of safe assets \( \hat{V}^S > V^S \) where \( \hat{V}^S = \frac{\mu^- X}{\theta} v^S(\hat{D} X) \) and \( V^S = \frac{\mu^- X}{\theta} v^S(D X) \) as long as there is less than full crowding out. This then stimulates output, increasing \( \xi \) to \( \hat{\xi} \) where
\[ \hat{\xi} = \frac{\hat{V}^S}{V^S} \xi > \xi. \]

The stimulative effects of public debt can be understood most clearly by going back to our AS-AD equilibrium representation. Because issuing public debt increases the supply of safe assets \( V^S \), it produces and upward shift in aggregate demand, which in turn results in a proportional increase in output through the Keynesian multiplier.

In the model, we necessarily have \( \tau^+ \mu^+ = \tau^- \mu^- \), which implies that \( \tau^+ < \tau^- \). For this reason, it is natural to expect fiscal constraints to be more binding after the bad Poisson shock than after the good Poisson shock. This is why we adopt \( \tau^- \), a measure of the ability of the government to raise tax revenues after the bad Poisson shock, as our measure of future fiscal capacity. In particular, in a safety trap, increasing the supply of public debt to \( \hat{D} \)

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\(^{11}\) More generally, the distribution of taxes matters for this result. We refer the reader to an earlier version of this paper, Caballero and Farhi (2013), for an exploration of this idea. In a recent paper, Barro and Mollerus (2014) consider a model with heterogeneous risk aversion and assume that taxes are distributed independently of risk aversion. They generate a crowding out of 0.5 despite the fact that Ricardian equivalence holds in their model.
requires the government to have spare future fiscal capacity, that is to have the ability to raise more taxes after the bad Poisson shock

\[ \hat{\tau}^− = \frac{\hat{D}}{D} \tau^− > \tau^−. \]

Note however that taxes do not have to be raised while the economy is in the safety trap before the Poisson event. Indeed \( \hat{\tau} = \tau \) (and both are equal to 0) since safe interest rates on debt are \( r^K = 0 \).

**Remark 3** Issuing money while at the zero bound is equivalent to issuing short-term bonds, and both are constrained by the long-term fiscal capacity of the government. Indeed, after the bad Poisson shock, the government must raise taxes to retire the additional money that it has issued before the Poisson event. See Appendix A.2 for a detailed exposition of these arguments.

### 3.2 Quantitative Easing

Here we use the term QE loosely to encompass policies that swap risky assets for safe assets such as QE1, LTRO, and many other lender of last resort central bank interventions. We model QE as follows. The government issues additional short term debt and purchases private risky assets (the value of which drops to zero after the bad Poisson shock). Let \( \hat{\beta}^g \) be the fraction of the value of risky assets purchased by the government where\(^{12}\)

\[ \hat{\beta}^g[1 - [\rho(\hat{\tau}^−) + \hat{\tau}^−]μ^−] \frac{X}{\theta} = \hat{D} - D. \]

The key difference betweenQE and simply issuing more public debt is what the government does with the proceeds from the debt issuance. In QE, the government uses the proceeds to purchase private risky assets instead of simply rebating them lump sum to private agents. The revenues from the debt issuance taxes \( \hat{\tau} \) before the bad Poisson shock can be lowered to

\[ \hat{\tau} = \tau - \frac{r}{\delta\theta} \hat{\beta}^g[1 - [\rho(\hat{\tau}^−) + \hat{\tau}^−]μ^−]. \]

\(^{12}\)As long as \( \hat{\beta}^g \) is low enough, this can be done without violating the pledgeability condition. The precise condition is

\[ \hat{\beta}^g < \frac{\delta\rho - \{\delta[\hat{\tau}^− + \rho(\hat{\tau}^−)]μ^− - [\alpha - [\hat{\tau}^− + \rho(\hat{\tau}^−)]μ^−](1 - \delta)\}}{\delta - \{\delta[\hat{\tau}^− + \rho(\hat{\tau}^−)]μ^− - [\alpha - [\hat{\tau}^− + \rho(\hat{\tau}^−)]μ^−](1 - \delta)\}}. \]
because the government can now avail itself of additional investment revenues from its holdings of private risky assets. If $\tau = 0$, as would be the case in a safety trap, then we can get $\hat{\tau} < 0$. This should then be interpreted as a possibility to reduce taxes if there were some other reasons for which taxes had to be raised.

As long as there is less than full crowding out, the safe asset shortage is alleviated by this policy: $r^K$ increases, $r$ decreases, and the safety premium shrinks. Here QE works not so much by removing risky private assets from private balance sheets, but rather by injecting public assets into private balance sheets. In other words, QE works by increasing the supply of safe assets. The proceeds of the extra debt issuance can either be rebated lump sum or reinvested in a portfolio of private assets as long as these assets are risky and not safe. More formally, what matters is that the value of the acquired asset drop to zero after the bad Poisson shock.

If the economy is in a safety trap where the safe interest rate is fixed at zero and output is below potential with $\xi < 1$, QE acts by stimulating output, increasing the value of $\xi$ to $\hat{\xi}$ where

$$\hat{\xi} = \frac{\hat{V}^S}{V^S}\xi > \xi,$$

with as before $\hat{V}^S > V^S$ where $\hat{V}^S = \frac{\mu - X}{\theta}v^S(DX)$ and $V^S = \frac{\mu - X}{\theta}v^S(DX)$ as long as there is less than full crowding out.

### 4 Forward Guidance

Another major policy tool advocated in the context of zero lower bound of interest rates is Forward Guidance (the commitment to low future interest rates once the economy recovers). However, in this section we show that when the reason for this low interest rate is a shortage of safe assets, the policy is ineffective.

The reason is that only policy commitments that support future bad states work in safety traps. This is a higher level of requirement than in the standard New-Keynesian liquidity-trap mechanism where any future wealth increase has the potential to stimulate the economy, including wealth created after the recovery is completed.

We illustrate this point with an example of forward guidance policy that would work in a standard liquidity trap environment but not in a safety trap. We refer the reader to Appendix A.4 for a version of the standard liquidity trap in the context of our model (assuming that
there are no Knightians) and the demonstration of forward guidance effectiveness in that context. In this section we focus on the safety trap case.

Since public debt is not key to our main concern here, we temporarily revert to our model in Section 2 where there are only private assets. We introduce two modifications to that model. First, we temporarily (only for this section) assume a non-zero intensity of the good Poisson shock $\lambda^+ > 0$. Second, we allow agents to produce $\zeta > 1$ units of output per unit of input. However, we imagine that there is a large utility loss from doing so.

This model functions as in Section 2 with $\lambda^+ = 0$. Indeed all the variables $V^S$, $V^R$, $W^K$, $W^N$, $r^K$, $\delta^S$ have exactly the same equilibrium values. The only difference is in the (risky) interest rate $r$. The interest rate $r$ is now determined by the following set of equations (and $\lambda^+$ only enters the last of these equations):

$$r = \delta \theta + (1 - \delta) \theta \frac{\alpha - \rho \mu^-}{1 - \rho \mu^-} + \lambda^+ \frac{\mu^+ - \rho \mu^-}{1 - \rho \mu^-}.$$

We can also look at safety traps where $r^K = i = 0$. Output $\xi X$ is determined by the exact same equation as in the model with $\lambda^+ = 0$. To see this most transparently, it is useful to go back to our AS-AD representation. The key is to note that both the aggregate supply $AS(\xi X) = \xi X$ and aggregate demand $AD(\xi X) = (1 - \alpha)(1 - \delta)\xi X + \delta \xi X + (\theta - r^K)V^S$ (with $r^K = 0$) relations are unchanged. The risky interest rate $r$ is then determined by the same equation as above but with $X$ replaced by $\xi X$, $\mu^+$ replaced by $\frac{\mu^+}{\xi}$ and $\mu^-$ replaced by $\frac{\mu^-}{\xi}$:

$$r = \delta \theta + (1 - \delta) \theta \frac{\alpha - \rho \mu^-}{1 - \rho \mu^-} + \lambda^+ \frac{\mu^+}{\xi} - \frac{\rho \mu^-}{\xi}.$$

In New-Keynesian models of the liquidity trap (see e.g. Krugman 1998, Eggertsson and Woodford 2003, and Werning 2012), committing to keep the interest rate low in the future once the economy recovers (after the good Poisson shock) stimulates the economy—a policy often referred to as forward guidance. The latter works by creating a boom in the future, which raises current demand through a combination of a wealth effect (higher income in the future) and substitution effect (lower real interest rates because of inflation). In Appendix A.4, we show how to model a standard liquidity trap in the context of our framework, by

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13 This occurs because the value of safe assets $V^S = \rho \mu^- \frac{\xi}{\theta}$ is unchanged. As a result, the dividend accruing to Neutrals $(\delta - \delta^S)X = \delta \xi X - r^K V^S$ is itself unchanged. Plugging it back into the unchanged wealth accumulation equation for Neutrals $W^N = (1 - \alpha)(1 - \delta)\xi \frac{X}{\theta} + \delta \xi \frac{X}{\theta} - r^K V^S$ yields the result.
considering a version of the model with no Knightians ($\alpha = 0$). There, we show that forward guidance does stimulate output in our version of the liquidity trap.\textsuperscript{14}

In a safety trap, we show that a commitment to low interest rates after the good Poisson shock translates into an increase in $r$ but fails to stimulate the economy, as it does not affect value of assets $V^S$, which is the key shifter of aggregate demand. The reconciliation of this ineffectiveness result with the obvious ex-post (after the good Poisson shock) increase in risky asset values and Neutral wealth of the policy is that the risky rate $r$ rises just enough to offset the present value effect of the policy. We show this result formally next.

Consider the following policy: Suppose that the good Poisson shock occurs at $\sigma^+$. After the good Poisson shock, the central bank stimulates the economy by setting the interest rate $i_t$ below the natural interest rate $\delta \theta$ until $\sigma^+ + T$, at which point it reverts to setting the nominal interest rate equal to the natural interest rate $\delta \theta$. For $t > \sigma^+ + T$, output is equal to potential so that $\zeta_t = 1$. For $\sigma^+ \leq t \leq \sigma^+ + T$, output is above potential, and capacity utilization satisfies a simple differential equation

$$\frac{\dot{\zeta}_t}{\zeta_t} = i_t - \delta \theta \leq 0,$$

with terminal condition

$$\zeta_{\sigma^+ + T} = 1.$$

The solution is

$$\zeta_t = e^{\int_{\sigma^+ + T}^{t} (\delta \theta - i_s) ds}.$$

By lowering interest rates, the central bank creates a temporary boom after the Poisson shock. This boom boosts the total value of assets immediately after the good Poisson shock from

$$\mu^+ \frac{X}{\delta \theta}$$

to

$$\mu^+ \zeta_{\sigma^+} \frac{X}{\delta \theta} > \mu^+ \frac{X}{\delta \theta}.$$

Let us now work backwards to understand the effects of this policy before the Poisson event, while the economy is in a safety trap. The only effect of this policy is to increase the interest

\textsuperscript{14}The model with constant prices de facto shuts down the effect through inflation and lower real interest rates. The expectation of a boom still produces a positive wealth effect which stimulates output. We can also introduce inflation as in Section 5. Then forward guidance gains extra kick by increasing inflation and reducing real interest rates.
rate $r$ during the safety trap to
\[
   r = \delta \theta + (1 - \delta) \theta \frac{\alpha}{\xi} \frac{\mu^- - \xi}{1 - \frac{\mu^-}{\xi}} + \lambda^+ \frac{\zeta_\sigma^+ \mu^+ \theta}{\xi} - \frac{\rho \mu^-}{\xi}. 
\]

This increase in the interest rate is such that the contemporaneous value of risky assets $V^R$ (and hence the wealth of Neutrals $W^N$) is unchanged, despite the fact that its future value after a good Poisson shock has increased to $\frac{\zeta_\sigma^+ \mu^+ X}{\theta} - V^S$. But there is no effect on output $\xi X$. This can be seen most clearly by going back to the AS-AD equilibrium representation. Neither $\zeta_\sigma^+$ nor $r$ affects aggregate demand. The future increase in risky asset values and the contemporaneous increase in risky interest rates are orthogonal to the safe-asset shortage problem. Since the policy leaves the supply of safe assets unchanged, it does not expand aggregate demand or output.\(^{15}\)

A safety trap is addressed more directly by committing to provide support during bad rather than good times, as would be the case of a commitment to lower interest $i_t$ rates after the bad Poisson shock.\(^{16}\) By setting the nominal interest rate $i_t$ below the natural interest rate $\delta \theta$ after the bad Poisson shock, monetary authorities stimulate the economy and inflate the value of safe assets to
\[
   \dot{V}^S = \rho \mu^- \frac{\zeta_\sigma^+}{\theta} X, 
\]
where
\[
   \zeta_\sigma^+ = e^{\int_{i_t}^{\theta + T} (\delta \theta - i_s) ds}. 
\]

This mitigates the recession in the safety trap by raising $\xi$ to $\xi \zeta_\sigma^+ > \xi$ (the analysis is almost identical to that of a monetary stimulus after the good Poisson shock explained above).\(^ {17,18}\)

\(^{15}\)There is one caveat to this conclusion. We have assumed that prices are entirely rigid. If prices could adjust gradually over time in a forward looking manner, then forward guidance could regain some kick: A commitment to lower interest rates after the good Poisson shock could increase inflation while the economy is in a safety trap. This would lower the safe interest rate $r^K$ and mitigate the recession. Note than when we model inflation in Section 5, we assume, motivated by a desire to capture downward wage rigidity, that inflation is determined by a myopic Philipps curve rather than an expectations-augmented Philipps curve, so that this effect does not arise.

A similar comment applies to the unconventional tax policies considered by Correia, Farhi, Nicolini and Teles (2012), which here could simply take the form of an increasing path of sales taxes—say through a sales tax holiday—which would create inflation in consumer prices and hence reduce $r^K$.

\(^{16}\)Another example is the OMT (outright monetary transactions) program established by the ECB in late 2012, which had an immediate impact on the Eurozone risk perception.

\(^{17}\)Note we could just as well have used the model with public debt. The central banker’s put works by increasing both the public and private sectors’ ability to provide safe assets.

\(^{18}\)Just like standard models of the liquidity trap, and to the extent that they are possible at all, these forms of policy commitments raise time-consistency issues: Their efficacy hinges on the ability of monetary
However, it is natural to question whether monetary authorities would have the ability to lower interest rates in that state. If indeed the bad state happens to coincide with yet another safety or liquidity trap, monetary authorities could find themselves unable to deliver a lower interest rate. Perhaps a more realistic policy option would be a commitment by the authorities to buy up safe assets at an inflated price after the Poisson shocks—a form of government (central bank?) put. A commitment to buy up safe private assets at an inflated value $\omega \rho \mu - \frac{\mu - \theta}{\rho} > \rho \mu - \frac{\mu - \theta}{\rho}$ would mitigate the recession and increase the value of $\xi$ to $\hat{\xi}$ where

$$\hat{\xi} = \omega \xi > \xi.$$ 

It could be carried out by monetary authorities but it does require spare fiscal capacity (in the form of taxes or seigniorage). This kind of public insurance policy can potentially play a crucial role in a safety trap.\(^{19}\)

5 Inflation

In this section we relax the extreme sticky price rigidity assumption and show how safety traps trigger deflationary forces, which exacerbate the output drop. We also show that the policy multiplier (of increasing safe assets) is enhanced by its positive feedback with inflation. Finally, as always, inflation also opens the door for expectations and targets to alleviate the trap by lowering real rates.

5.1 Safety Trap, Deflation, and Inflation Targets

We assume that prices cannot fall faster than at a certain pace.\(^{20}\)

$$\pi_t \geq -(\gamma + \beta(1 - \xi_t)).$$

The more slack there is in the economy, the more prices can fall.

\(^{19}\)See, e.g., Caballero and Kurlat (2010) for a proposal to increase the resilience of the financial system in a shortage of safe assets environment. Also, see Brunnermeir et al (2012) for a related proposal in the context of the current Euro crisis.

\(^{20}\)Our motivation is to capture downward wage rigidities. This requires reinterpreting our varieties as varieties of labor which can then be transformed into final goods by competitive firms with a one-to-one technology. Our modeling strategy in this section borrows heavily from Eggertsson and Mehrota (2014).
We impose that if there is slack in the economy, prices fall as fast as they can: \( \xi_t < 1 \) implies \( \pi_t = -(\gamma + \beta(1-\xi_t)) \). We capture this requirement with the complementary slackness condition

\[
[\pi_t + (\gamma + \beta(1 - \xi_t))](1 - \xi_t) = 0.
\]

This is a Phillips curve which we plug in our model in conjunction with a (truncated) Taylor rule

\[
i_t = \min\{0, r^K,n_t + \pi^* + \phi(\pi_t - \pi^*)\},
\]

with \( \phi > 1, \pi^* \geq 0 \) where \( r^K,n_t \) is the safe natural interest rate at \( t \).

Now imagine that the real model is such that the safe interest rate is negative \( r^K,n < 0 \). The equilibrium equations are then

\[
\min\{0, r^K,n_t + \pi^* + \phi(\pi_t - \pi^*)\} - \pi = \delta \theta - (1 - \delta) \theta \frac{\alpha - \rho \mu^-}{\rho \mu^- - \xi},
\]

\[
[\pi + (\gamma + \beta(1 - \xi))](1 - \xi) = 0,
\]

where

\[
r^K,n = \delta \theta - (1 - \delta) \theta \frac{\alpha - \rho \mu^-}{\rho \mu^-} < 0.
\]

The first equation is simply the requirement that \( i - \pi = r^K \), where we have used the Taylor rule to replace \( i \) and the equilibrium equations to replace \( r^K \). The second equation is the Phillips curve. These two correspondences link inflation \( \pi \) and output \( \xi X \) and can be interpreted as aggregate demand and aggregate supply respectively. We denote them by \( \pi = AD^\pi(\xi X) \) and \( \pi = AS^\pi(\xi X) \) to distinguish them from the aggregate supply and demand functions \( AS(\xi X) \) and \( AD(\xi X) \) that we introduced in the Keynesian cross equilibrium representation in Section 2. We discuss the link between these two representations in detail below.

We make the further assumptions that there is “enough” price rigidity so that the deflationary spiral (the feedback loop between inflation and output) remains bounded:

\[
\beta < (1 - \delta) \theta \frac{\alpha}{\rho \mu^-},
\]

\[
\gamma + \beta < \theta.
\]

In this context, there is always a deflationary safety trap equilibrium (with \( \pi \leq 0 \) and
\[ \xi < 1 \). It is determined by the following equations

\[ \pi = (1 - \delta) \theta \frac{\alpha - \mu \xi}{\mu \xi} - \delta \theta, \]

\[ \pi = -(\gamma + \beta (1 - \xi)). \]

Both equations express \( \pi \) as increasing linear functions of \( \xi \). The first one is aggregate demand \( \pi = AD^\pi (\xi X) \), while the second is aggregate supply \( \pi = AS^\pi (\xi X) \). With our assumptions above, aggregate demand is steeper than aggregate supply and both curves intersect exactly once on \([0, 1]\) at some value \( \xi \). This is the "bad" safety trap equilibrium, analog to the one discussed in the previous sections.

But this model may also feature inflationary full employment equilibria (i.e., \( \pi > 0 \) and \( \xi = 1 \)). To see this, note that the supply curve becomes vertical at \( \xi = 1 \) and the demand curve has a kink at the value of \( \pi = \bar{\pi} \) that solves \( r^{K:n} + \pi^* + \phi (\pi - \pi^*) = 0 \): the demand curve is an upward sloping function \( \xi (\pi) \) for \( \pi \leq \bar{\pi} \) and downward sloping for \( \pi > \bar{\pi} \). As a result, there are either one or three intersections between the supply and demand curves.
We have already seen the bad equilibrium; the other equilibria (if they exist) feature $\pi > 0$ and $\xi = 1$.

In an equilibrium with $\pi > 0$ and $\xi = 1$, inflation is determined by

$$\pi = \frac{1}{\phi - 1} [\phi \pi^* + r^{K,n}],$$

if $r^{K,n} + \pi^* + \phi(\pi - \pi^*) > 0$ (in which case $i = r^{K,n} + \pi^* + \phi(\pi - \pi^*)$) or by

$$\pi = -r^{K,n},$$

if $r^{K,n} + \pi^* + \phi(\pi - \pi^*) \leq 0$ (in which case $i = 0$). The condition for existence of both equilibria is the same and is given by

$$0 \leq r^{K,n} + \pi^*.$$  

Hence the requirement is (roughly speaking) that the inflation target $\pi^*$ be high enough.

To summarize, when $r^{K,n} < 0$, there is always a safety trap equilibrium with $\xi < 1$ and $\pi < 0$. If the inflation target is high enough ($0 \leq r^{K,n} + \pi^*$), then there are also two other equilibria with $\xi = 1$ (and two different nominal interest rates $i = 0$ and $i > 0$ and two different inflation rates $\pi = -r^{K,n}$ and $\pi = \frac{1}{\phi - 1} [\phi \pi^* + r^{K,n}] > -r^{K,n}$). In other words, the increase in the inflation target needs to be large enough to even have a chance to work. Note however that even when there is a "good" equilibrium with $\xi = 1$ and $i > 0$, the "bad" equilibrium with $\xi < 1$ and $i = 0$ still exists. The best a high inflation target can achieve is the possibility of a good equilibrium, not the elimination of the possibility of the bad equilibrium.

Let us now connect our discussion with the Keynesian cross equilibrium representation developed in Section 2. The difference here is that output $\xi X$ and inflation $\pi$ are jointly determined by the new aggregate demand and aggregate supply relations $\pi = AD^\pi (\xi X)$ and $\pi = AS^\pi (\xi X)$. We can still use the Keynesian cross equilibrium representation to determine output once inflation has been solved out. Indeed we now have

$$AD (\xi X) = (1 - \alpha)(1 - \delta)\xi X + \delta \xi X + (\theta - r^K) V^S,$$

$$AS (\xi X) = \xi X,$$
with \( r^K = i - \pi \) and \( i = 0 \). Higher inflation therefore helps to reduce safe interest rates, causing an upward shift in aggregate demand and resulting in an increase in output. In the bad safety trap equilibrium, this logic works in reverse, with the recession causing deflation, increasing safe real interest rates, reducing aggregate demand, further reducing output etc. In fact, adding an inflation channel to the model increases the value of the Keynesian multiplier to

\[
\frac{1}{1 - \frac{\gamma + \beta}{\theta}}
\]

so that \( d(\xi X) = \frac{1}{1 - \frac{\gamma + \beta}{\theta}} \xi X \theta dV^S \). This is because increases in the value of safe assets increase aggregate demand, which increases output, increasing inflation, reducing the safe real interest rate, further increasing aggregate demand and output etc. ad infinitum. The more responsive is inflation to capacity utilization \( \xi \) (the larger is \( \beta \)), the larger is the Keynesian multiplier. In other words, increased price flexibility is destabilizing as in DeLong and Summers (1986).

### 5.2 Public Debt, QE and Forward Guidance with Inflation

We now turn to the effects of the policies considered in Sections 3 and 4 in this extended model with inflation. We focus on the safety trap equilibrium and show that the main conclusions remain qualitatively unchanged but the power of QE is enhanced.

We start with public debt and QE. In a safety trap, \( \xi < 1 \) and \( \pi < 0 \) are determined by the intersection of the demand and supply curves:

\[
\pi = (1 - \delta) \theta \frac{\alpha - \frac{v^S(D_X)\mu}{\xi}}{\frac{v^S(D_X)\mu}{\xi}} - \delta \theta, \\
\pi = -(\gamma + \beta(1 - \xi)).
\]

Clearly, increasing \( D \) to \( \hat{D} > D \) (and either rebating the proceeds to consumers or purchasing private risky assets) shifts the demand curve down which, given that it is steeper than the supply curve, results in an increase in \( \xi \) to \( \hat{\xi} > \xi \). Note that the stimulus is stronger in this extended setup with endogenous inflation

\[
\hat{\xi} > \frac{\hat{V}^S}{V^S} \xi,
\]

with \( V^S = v^S(\frac{D}{X}) \) and \( \hat{V}^S = v^S(\frac{\hat{D}}{X}) \). This is because of a virtuous circle whereby additional
safe assets increase output, which increases inflation (reduces deflation), which lowers the real interest, further stimulating output etc. ad infinitum, resulting in a larger value of the Keynesian multiplier as explained above.

We now turn to forward guidance. In order for monetary policy to be able to generate a boom after the good Poisson shock, we introduce the following modification of our setup. We assume that the Phillips curve only becomes vertical at a value $\bar{\xi} > 1$ so that we now have

$$[\pi_t + (\gamma + \beta(1 - \xi_t))](\bar{\xi} - \xi_t) = 0.$$  

Further more, we assume that $\gamma = -\pi^*$ so that inflation is $\pi^*$ if the economy is at capacity.

To capture forward guidance, we continue to assume that monetary policy follows the truncated Taylor rule

$$i_t = \min\{0, r_t^{K,n} + \pi^* + \phi(\pi_t - \pi^*)\}$$

before and after the bad Poisson shock, but we allow monetary policy to depart from this rule after the good Poisson shock and follow instead (recall that $r_t^{K,n} = \delta\theta$ after the good Poisson shock)

$$i_t = (\hat{i}_t - \delta\theta) + \delta\theta + \pi^* + \phi(\pi_t - \pi^*)$$

where $\hat{i}_t < \delta\theta$ for $\sigma^+ \leq t \leq \sigma^+ + T$ and $\hat{i}_t = \delta\theta$ for $t > \sigma^+ + T$. Then output is above potential after the good Poisson shock. Capacity utilization satisfies the differential equation

$$\dot{\zeta}_t = (\hat{i}_t - \delta\theta) + (\phi - 1)(\pi_t - \pi^*) \leq 0,$$

$$\pi_t - \pi^* + \beta(1 - \zeta_t) = 0$$

with terminal condition

$$\zeta_{\sigma^+ + T} = 1.$$  

Just as in Section 4, the solution features $\zeta_t > 1$ and $\pi_t \geq \pi^*$ for $\sigma^+ \leq t < \sigma^+ + T$. The rest of the analysis is identical and the conclusion is identical. Forward guidance stimulates the economy after the good Poisson shock, resulting in a boom and inflation above target. But this fails to stimulate the economy before the Poisson even when the economy is in a safety trap.
6 Bubbles and Fiscal Capacity

The very low interest rates that characterize a safety trap raises the issue of whether speculative bubbles may emerge, and whether these can play a useful role through their wealth effect. We show that bubbles can indeed arise in safety traps, but that only the emergence of safe bubbles (as opposed to risky bubbles) can stimulate economic activity. This is because only safe bubbles alleviate the shortage of safe assets. We associate the latter to public debt, and in fact the existence of a bubbly-region is equivalent to an expansion of the fiscal capacity of the government.

6.1 Growth

We extend the model to allow for bubbles. It is well understood in the rational bubbles literature that the growth rate of the economy is a key determinant of the possibility and size of bubbles.

We generalize our model by allowing for an arbitrary growth rate $g > 0$. At every point in time, there is a mass $X_t$ of trees. A mass $\dot{X}_t = gX_t$ of new trees are created, which are claims to a dividend of $\delta$ units of goods at every future date until a Poisson event occurs, at which point the dividend jumps permanently to $\delta\mu^+$ if the good Poisson shock takes place and to $\delta\mu^-$ if the bad Poisson shock takes place. For reasons that will appear clear below, we assume that new trees are initially endowed to Neutral newborns. A fraction $\rho$ of these new trees can be tranched into a safe and a risky component. Endowments also grow at the rate $g$.

We some abuse of notation, we suppress time indices throughout. Hence we write $X, V^S, V^R, V, W^K, W^N, W$ for $X_t, V^S_t, V^R_t, V_t, W^K_t, W^N_t, W_t$. All these variables grow at rate $g$ in equilibrium. We also write $r^K, r, \delta^S$ for $r^K_t, r_t, \delta^S_t$. All these variables are constant in equilibrium.

We focus on the constrained regime where $W^K = V^S = \frac{\rho\mu^- X}{g}$ and $r > r^K$. This occurs as long as

$$\frac{\rho\mu^-}{r^K} > \frac{g}{(1 - \delta) \theta}$$

If new trees are endowed in equal proportions to Knightians and Newborns, then bubbles do stimulate the economy in a safety trap because they reduce the value of the new trees endowed to Knightian newborns and hence reduce the growth rate of Knightian wealth. Endowing the new trees exclusively to Neutrals shuts down this somewhat artificial effect of bubble on safe asset demand.
The equilibrium equations in the constrained regime are

\[ r^K V^S = \delta^S X, \]

\[ rV^R = (\delta - \delta^S)X, \]

\[ gW^K = -\theta W^K + \alpha (1 - \delta) X + r^K W^K, \]

\[ gW^N = -\theta W^N + (1 - \alpha) (1 - \delta) X + g(V^S + V^R) + rW^N, \]

\[ W^K + W^N = V^S + V^R, \]

\[ W^K = V^S = \frac{\rho \mu^ - X}{\theta}. \]

We then have

\[ \delta^S = g \frac{\rho \mu^ -}{\theta} + \delta \rho \mu^ - (1 - \delta) \left( \alpha - \rho \mu^ - \right), \]

\[ r^K = g + \delta \theta - \theta (1 - \delta) \frac{\alpha - \rho \mu^ -}{\rho \mu^ -}, \]

\[ r = g + \delta \theta - \frac{g}{1 - \rho \mu^ -} + (1 - \delta) \theta \frac{\alpha - \rho \mu^ -}{1 - \rho \mu^ -}. \]

Now suppose that we are in a safety trap where

\[ g + \delta \theta - \theta (1 - \delta) \frac{\alpha - \rho \mu^ -}{\rho \mu^ -} < 0, \]

then as in Section 2, we have a recession determined by

\[ 0 = g + \delta \theta - (1 - \delta) \theta \frac{\alpha - \rho \mu^ -}{\rho \mu^ -}. \]

This formula shows that in the constrained regime with a scarcity of safe assets, the lower is the structural growth rate \( g \) of the economy, the lower is the natural safe interest rate, and if the latter is negative, the deeper is the recession (the lower is utilization capacity \( \xi \)). In this sense supply side secular stagnation can lead to or reinforce demand side secular stagnation—a powerful and perverse complementarity.
6.2 Bubbles

To ensure that bubbles are stationary, we allow for new bubbles to be created. Just like new trees, new bubbles are endowed to Neutral newborns. The total value of the bubble $B_t$ grows at rate $\dot{B}_t = gB_t$ until the bad Poisson shock occurs, at which point the bubble drops to $B_t^- < B_t$ and then keeps growing at rate $g$, or until the good Poisson shock occurs, at which point the bubble jumps to $B_t^+ \geq B_t$. The value of the bubble $B_t^+$ after the good Poisson shock is irrelevant for our analysis. The bubble can be separated into a safe bubble $B_{S_t} = B_t^- + \delta \mu \frac{X - r}{X}$ and a risky bubble $B_{R_t} = B_t - B_{S_t}$ with rate of return $r_t$.

New safe bubbles per unit of time are then given by $(g - r^K)B_{S_t}$, and new risky bubbles per unit of time by $(g - r_t)B_{R_t}$, which must both be positive for the equilibrium to exist. Again, we focus on balanced growth paths and suppress the dependence on time and write $B$, $B^-$, $B^+$, $B^S$ and $B^R$ for $B_t$, $B_t^-$, $B_t^+$, $B_{S_t}$ and $B_{R_t}$.

After the bad Poisson shock, goods market clearing requires that

$$\theta(B^- + \frac{\delta \mu X}{r^-}) = \mu X.$$

This implies that the interest rate $r^-$ is given by

$$r^- = \frac{\delta \mu X}{\mu X - B^-} = \frac{\delta \theta}{1 - \frac{\theta \mu X}{\mu X}}.$$  

The value of safe assets before the bad Poisson shock is therefore

$$V^S = B^- + \frac{\delta \rho \mu X}{r^-},$$

which can be rewritten as a function

$$V^S = \frac{\mu X^-}{\theta} v^{S,B}(\frac{B^-}{X})$$

with

$$v^{S,B}(\frac{B^-}{X}) = \rho + \frac{\theta}{\mu X^-} (1 - \rho).$$

This expression makes clear that safe bubbles increase the value of safe assets because of the implicit assumption that there is no agency problem involved in the tranching of bubbles into a safe and a risky part.

As above, we focus on the constrained regime where $W^K = V^S$ and $r > \delta \theta > r^K$. The
equilibrium equations in the constrained regime in the presence of bubbles are then

\[ r^K (V^S - B^S) = \delta^S X, \]
\[ r (V^R - B^R) = (\delta - \delta^S) X, \]
\[ gW^K = -\theta W^K + \alpha (1 - \delta) X + r^K W^K, \]
\[ gW^N = -\theta W^N + (1 - \alpha) (1 - \delta) X + \left[ g \left( V^S + V^R \right) - r^K B^S - rB^R \right] + rW^N, \]
\[ W^K + W^N = V^S + V^R, \]
\[ W^K = V^S = \frac{\mu^- X}{\theta} v^{S,B}(\frac{B^-}{X}). \]

We find the following expressions for the safe and risky interest rates:

\[ r^K = g + \delta\theta - \theta (1 - \delta) \frac{\alpha - \mu^- v^{S,B}(\frac{B^-}{X})}{\mu^- v^{S,B}(\frac{B^-}{X})}, \]
\[ r = \frac{(g + \theta) \left[ 1 - \mu^- v^{S,B}(\frac{B^-}{X}) \right] - (1 - \alpha) (1 - \delta) \theta - g + r^K \theta \frac{B^-}{X}}{1 - \mu^- v^{S,B}(\frac{B^-}{X}) - \theta \frac{B^R}{X}}. \]

The condition for the constrained regime is that \( r > r^K \) and the condition for bubbles to exist is that \( \max \{ r, r^-, r^+ \} \leq g \), where \( r^+ \) is the interest rate after the good Poisson shock

\[ r^+ = \frac{\delta \mu^+ X}{\nu^{S,B}(\frac{B^-}{X} - B^+)}. \]

Now suppose that we are in a safety trap. Then we have a recession with \( \xi < 1 \) determined exactly as in the bubbleless equilibrium analyzed in Section 6.1 above

\[ 0 = g + \delta\theta - \theta (1 - \delta) \frac{\alpha - \frac{\nu^{S,B}(\frac{B^-}{X})}{\xi} \mu^-}{\frac{\nu^{S,B}(\frac{B^-}{X})}{\xi}}. \]

Clearly, output \( \xi X \) is increasing in the size of the safe bubble \( \frac{B^-}{X} \). A larger safe bubble \( \frac{B^-}{X} > \frac{B^-}{X} \) stimulates output to \( \hat{\xi} X \) with

\[ \hat{\xi} = \frac{\hat{V}^S}{V^S} > \xi, \]

where \( \hat{V}^S = v^{S,B}(\frac{B^-}{X}) \) and \( V^S = v^{S,B}(\frac{B^-}{X}) \). However output is invariant to the size of the risky bubble \( \frac{B^R}{X} \). This is because the safe bubbles increase the supply of safe assets \( V^S \) while
the risky bubbles only increase the risky interest rate \( r \) (they fail to increase the supply of risky assets \( V^R \) since they perfectly crowd out other risky assets through an increase in the risky interest rate \( r \)). In other words, in terms of the AS-AD equilibrium representation, only safe bubbles increase aggregate demand, not risky bubbles.

The fact that risky bubbles have no effect on output in a safety trap formalizes some interesting observations in Summers (2013) that in secular stagnation environments, even large financial bubbles only seem to create moderate economic expansions.

6.3 Fiscal Capacity: Debt as a Safe Bubble

A natural interpretation of safe bubbles is that they are a form of government debt. To develop this idea, assume that the risky bubble is equal to zero, and interpret the safe bubble \( B^- \) as government debt \( B^- = D \). When \( r < g \), and by implication \( \delta \theta < g \), the government can sustain a stable debt to output ratio without ever having to levy any taxes \( (\tau = \tau^+ = \tau^- = 0) \). The government can then increase the supply of safe assets by leveraging on the bubble in \( V^S = \frac{\mu^- X}{\theta} v^{S,B}(\frac{B^-}{X}) \), with a crowding out (of private safe assets) by public debt of:

\[
\frac{\mu^-}{\theta} \left(1 - \frac{dv^{S,B}(\frac{B^-}{X})}{d(\frac{B^-}{X})}\right) = \rho < 1.
\]

Finally, note that as the government issues more public debt, \( r^- \) and \( r^+ \) increase, which puts a limit to how much public debt can be issued without ever having to tax, namely \( \frac{B^-}{X} \leq \frac{\mu^-}{\theta} \left(1 - \frac{\delta \theta}{\theta}\right) \). In a safety trap, this gives the government some fiscal space to increase debt and stimulate the economy.

7 Final Remarks

In this paper we provided a model that captures some of the most salient macroeconomic consequences and policy implications of a safety trap. Given the faster growth of safe-asset-consumer economies than that of safe-asset-producer economies as well as the aging of wealth-rich economies, absent major financial innovations, the shortage of safe assets is only likely to worsen over time, perhaps as a latent factor during booms but reemerging in full force during contractions. It is our conjecture that the shortage of safe assets will remain as a structural drag, lowering safe rates, increasing safety spreads, straining the financial system,
and weakening the effectiveness of conventional monetary policy during contractions.
References


A Appendix

A.1 Derivation of the optimal portfolios of Knightians and Neutrals

Recall that we focus on the period before the Poisson event and that we study the limit \( \lambda^- \to 0 \) and \( \lambda^+ \to 0 \). Between \( t \) and \( t + dt \), three events can occur. First there can be no Poisson event. Second, there can be a bad Poisson event. Third, there can be a good Poisson event.
We denote by \( q_{t,t+dt} \) the price at date \( t \) of an asset that pays one at date \( t+dt \) if and only if the first event occurs. We denote by \( q_{t,t+dt}^- \) the price at date \( t \) of an asset that pays one at date \( t+dt \) if the second event occurs. And we denote by \( q_{t,t+dt}^+ \) the price at date \( t \) of an asset that pays one at date \( t+dt \) if the third event occurs. Finally we denote by \( v_t \) the price of a tree, and we denote the price of the tree at date \( t+dt \) by \( v_{t+dt} \) if the first event occurs, \( v_{t+dt}^- \) if the second event occurs, and \( v_{t+dt}^+ \) if the third event occurs. In equilibrium \( v_t = v_{t+dt} = \frac{X}{\theta}, \quad v_{t+dt}^- = \mu^- \frac{X}{\theta} \) and \( v_{t+dt}^+ = \mu^+ \frac{X}{\theta} \). The equilibrium has \( q_{t,t+dt}^+ > 0, \quad q_{t,t+dt}^- > 0, \) and \( q_{t,t+dt}^+ = 0 \). We define
\[
1 + r^K_t dt = \frac{1}{q_{t,t+dt}^- + q_{t,t+dt}^+}
\]
and
\[
1 + r_t dt = \frac{1}{q_{t,t+dt}^-}
\]
so that to a first order in \( dt \), \( q_{t,t+dt}^- = (r_t - r^K_t) dt \). The case \( q_{t,t+dt}^- = 0 \) corresponds to the unconstrained regime where \( r_t = r^K_t \). The case \( q_{t,t+dt}^- > 0 \) corresponds to the constrained regime where \( r_t > r^K_t \).

Consider a Knightian agent with wealth \( w^K_t \). Let \( \beta^K_t, \beta^K_- \) and \( \beta^K_+ \), be the numbers of each assets in the agent’s portfolio with \( \beta^K_t q_{t,t+dt} + \beta^K_- q_{t,t+dt}^- = w^K_t \) and \( \beta^K_t \geq 0, \quad \beta^K_- \geq 0, \) and \( \beta^K_+ \geq 0 \). His utility is given by
\[
\theta w^K_t dt + (1 - \theta dt) w^K_t \min\{\beta^K_t, \beta^K_-, \beta^K_+\}.
\]
Clearly the solution is \( \beta^K_t = \beta^K_- = \beta^K_+ = \frac{w^K_t}{q_{t,t+dt}^- + q_{t,t+dt}^+} \), so that the Knightian agent chooses to invest his wealth between \( t \) and \( t+dt \) in a safe asset.

Consider now a Neutral agent with wealth \( w^N_t \). Let \( \beta^N_t, \beta^N_- \) and \( \beta^N_+ \) be the numbers of each assets in the agent’s portfolio, and let \( \beta^N_{v_t} \) be the number of trees owned by the agent, with \( \beta^N_t q_{t,t+dt} + \beta^N_- q_{t,t+dt}^- + \beta^N_{v_t} v_t = w^N_t \) and \( \beta^N_t \geq -\rho \beta^N_{v_t} v_{t+dt}, \quad \beta^N_- \geq -\rho \beta^N_{v_t} v_{t+dt}, \quad \beta^N_+ \geq -\rho \beta^N_{v_t} v_{t+dt}^+, \) and \( \beta^N_{v_t} \geq 0 \). His utility is given by
\[
\theta w^N_t dt + (1 - \theta dt) w^N_t [\beta^N_t + \beta^N_{v_t} (\delta X dt + v_{t+dt})].
\]
If we are in the unconstrained regime with \( q_{t,t+dt}^- = 0 \), a necessary condition for an interior solution in \( \beta^N_t \) and \( \beta^N_{v_t} \) (which must be the case in equilibrium) is \( \frac{1}{q_{t,t+dt}} = \frac{\delta X dt + v_{t}}{v_t} \) which we can rewrite as \( r_t = \delta \theta \), a condition which is verified in our equilibrium. If we are in the constrained regime with \( q_{t,t+dt}^- > 0 \), then we must have \( \beta^N_- = -\rho \beta^N_{v_t} v_{t+dt}^- \) and a necessary
condition for an interior solution in $\beta^N_t$ and $\beta^{Nv}_t$ (which must be the case in equilibrium) is

$$\frac{1}{q_{t:t+dt}} = \frac{\delta X dt + \mu_t dt}{-\rho v_{t+dt} q_{t:t+dt} + v_t + \mu_t dt}$$

which we can rewrite as

$$\frac{1}{q_{t:t+dt}} = \frac{\delta \theta dt + 1}{-\rho \mu^- q_{t:t+dt} + 1}$$

or to a first order in $dt$,

$$r_t = \delta \theta + \rho \mu^- (r_t - r^K_t),$$

a condition which is verified in our equilibrium.

### A.2 Helicopter Money and Fiscal Capacity

One may wonder why not directly addressing the shortage of safe assets directly by printing money. Here we show that this is entirely equivalent to issuing public debt and hence it is subject to the same fiscal constraints.

Let us start backwards. In order to buy back the money stock after the bad Poisson shock, the government undertakes an open market operation immediately after the realization of the shock, swapping the extra supply of money $M^\varepsilon - M^{\varepsilon -}$ for debt $D$ where

$$D = M^\varepsilon - M^{\varepsilon -},$$

the interest on which it finances by a tax $\tau^-$ on the dividends of trees, where

$$D = \tau^- \mu^- \frac{X}{\theta}.$$

Consider what happens when the government issues additional money $\hat{M}^\varepsilon > M^\varepsilon = \frac{X}{\theta}$ in a safety trap, but maintains an adequate supply of money $M^{\varepsilon -} = \frac{\mu^- X}{\theta}$ after the bad Poisson shock. This stimulates output to

$$\hat{\xi} = \frac{\rho \mu^- + \frac{\varepsilon}{1-\varepsilon} \hat{M}^\varepsilon \theta}{\rho \mu^- + \frac{\varepsilon}{1-\varepsilon}} \xi > \xi.$$

This is exactly the same effect as that which would be achieved by issuing additional short-term debt in the amount $\varepsilon (\hat{M}^\varepsilon - M^\varepsilon)$, which is intuitive given that money and short-term debt are perfect substitutes at the zero lower bound. And exactly like this debt issuance policy, it requires that the government be able to increase taxes $\hat{\tau}^- > \tau^-$ after the bad Poisson shock where

$$(\hat{\tau}^- - \tau^-) \mu^- \frac{X}{\theta} = \varepsilon (\hat{M}^\varepsilon - M^\varepsilon).$$

Consider next what happens when the government issues additional money $\hat{M}^\varepsilon > M^\varepsilon = \frac{X}{\theta}$ in a safety trap, but keeps an excessive supply of money $\hat{M}^{\varepsilon -} > \frac{\mu^- X}{\theta}$ after the Poisson shock where
shock occurs (perhaps because it doesn’t have the fiscal capacity to retire the extra money), while maintaining an interest rate of $\delta \theta$. In this case output is above potential at $\zeta \mu - X$ where

$$\zeta = \hat{M}^\varepsilon \frac{\theta}{\mu - X}.$$  

Hence the value of private safe assets is increased to

$$\frac{\rho \mu^- \zeta X (1 - \varepsilon)}{\theta},$$

resulting in a mitigation of the recession before the Poisson shock when the economy is in a safety trap, increasing the value of $\xi$ to $\hat{\xi}$ where

$$\hat{\xi} = \frac{\rho \mu^- \hat{M}^\varepsilon - \theta}{\rho \mu^- + \varepsilon} \frac{\hat{M}^\varepsilon \theta}{X} \xi > \frac{\rho \mu^- + \varepsilon}{\rho \mu^- + \frac{\varepsilon}{1 - \varepsilon}} \xi > \xi.$$  

Thus issuing money while the economy is in a safety trap and not taking it away when the economy exits the safety trap further mitigates the recession associated with the safety trap. However, this extra effectiveness is no free lunch, as it comes with the important cost of excessively stimulating the economy when it exits the safety trap.

### A.3 Derivations for Section 6

To find the expression for $r$, we can write

$$(g + \theta - r) \frac{X}{\theta} [1 - \frac{\theta V^S(\frac{B^-}{\mu - X})}{X}] = (1 - \alpha) (1 - \delta) X + g \frac{X}{\theta} - r^K B^S - r B^R,$$

$$(g + \theta) \frac{X}{\theta} [1 - \frac{\theta V^S(\frac{B^-}{\mu - X})}{X}] - (1 - \alpha) (1 - \delta) X - g \frac{X}{\theta} + r^K B^S = r \frac{X}{\theta} \left[1 - \frac{V^S(\frac{B^-}{\mu - X})}{X}\right] - \frac{\theta B^R}{X},$$

$$r = \frac{(g + \theta) \left[1 - \frac{\theta V^S(\frac{B^-}{\mu - X})}{X}\right] - (1 - \alpha) (1 - \delta) \theta - g + r^K \frac{\theta B^-}{X}}{[1 - \frac{V^S(\frac{B^-}{\mu - X})}{X}] - \frac{\theta B^R}{X}}.$$
A.4 A Liquidity Trap without Risk Premia

Standard Keynesian liquidity trap analyses ignore the distinction between safe and risky assets, either because they are deterministic or because they rely on linearized models. In this section, we highlight the differences that arise in our model where this distinction is important. To do so, we develop a version of our model with no risk premia, where the economy can be in a liquidity trap but where there is no specific shortage of safe assets (but rather a shortage of assets in general which all carry the same zero interest rate).

We assume either that there are no Knightians ($\alpha = 0$) or that there are Knightians ($\alpha > 0$) but that the economy is in the unconstrained regime so that $r = r^K$. In both cases, the distinction between risky and safe assets is irrelevant, and the analysis is similar. We make one modification: The possibility of the bad shock is $\lambda^- > 0$ rather than studying the limit $\lambda^- \to 0$. This is necessary for the interest rate $r$ to reach zero (and it cannot go below zero because of the zero lower bound on nominal interest rates). We maintain our focus on the limit $\lambda^+ \to 0$ for now.\(^{22}\)

The equilibrium equations are

\[
    rV = \delta X + \lambda^- \left( \frac{\mu^- X}{\theta} - V \right),
\]

\[
    0 = -\theta W + (1 - \delta)X + rW - \lambda^- \left( \frac{\mu^- X}{\theta} - V \right) + gV,
\]

\[
    V = W.
\]

As long as the zero bound is not binding, we have $V = W = \frac{X}{\theta}$ and

\[
    r = \delta \theta - \lambda^- \left( 1 - \mu^- \right) > 0.
\]

When the zero bound $r = 0$ binds, the economy enters a recession ($\xi < 1$) where $\xi$ is determined by the requirement that $r = 0$:

\[
    0 = \delta \theta - \lambda^- \left( 1 - \frac{\mu^-}{\xi} \right),
\]

i.e.

\[
    \xi = \frac{\mu^-}{1 - \frac{\delta \theta}{\lambda^+}}.
\]

\(^{22}\)We relax this assumption later when we analyze forward guidance in Section ??.
The recession originates from a scarcity of assets (stores of value). It is more severe, the worse the expected bad shock (the lower is \( \mu^- \)), the more likely is the bad shock (the higher \( \lambda^- \)), the higher the propensity to save (the lower \( \theta \)), and the lower is the ability of the economy to create assets that capitalize future income (the lower is \( \delta \)).

We can use this model to examine the effects of the same policies that we have considered in the context of the safety trap: balance sheet policies (QE), fiscal policy (redistribution and government spending), and monetary policy commitments (forward guidance). We can also examine the possibility and the consequences of bubbles.

**Public Debt and QE.** We start with public debt and QE. We introduce public debt in the model exactly as in Section 3. The key point is that public debt issuances and QE have no effect at all on the recession \( \xi \). This precise irrelevance result relies on our assumption that dividends are taxed while the endowment of newborns (wages) is not. As a result, public debt issuances and QE simply reshuffle the fraction of dividends that accrues to private asset holders and the fraction of dividends that is absorbed by taxes to pay interest on debt of various maturities. This assumption essentially renders our framework Ricardian, despite the fact that we have overlapping generations of agents.\(^{23}\) These conclusions about the irrelevance of public debt issuances and QE in this standard liquidity trap environment must be contrasted with those reached in Section 3 for safety traps. The effects of public debt issuances and QE in safety traps rely entirely on the (assumed) superior ability of the government to address a form of market incompleteness—the difficulty to isolate safe from risky assets.

**Forward Guidance.** We now turn to monetary policy commitments. To do so, we introduce the possibility of a good shock as in Section 4. As in Section 4, we temporarily (only for this section) assume that \( \lambda^+ > 0 \). In a liquidity trap \( r = 0 \), the recession is now determined by

\[
0 = \delta \theta - \lambda^-(1 - \frac{\mu^-}{\xi}) - \lambda^+(1 - \frac{\mu^+}{\xi}),
\]

i.e.

\[
\xi = \frac{\lambda^- \mu^- + \frac{\lambda^+}{\lambda^- + \lambda^+} \mu^+}{1 - \frac{\delta \theta}{\lambda^- + \lambda^+}}.
\]

Consider the following policy: Suppose that the good Poisson shock occurs at \( \sigma^+ \). After

\(^{23}\)If we allowed the endowments of newborns to be taxed, then public debt issuances and QE could have some non-Ricardian effects, depending on exactly how these taxes are levied, and hence affect economic activity in a liquidity trap. For example, Kocherlakota (2013) studies a non-Ricardian environment where issuing public debt can stimulate the economy in a liquidity trap.
the good Poisson shock, the central bank stimulates the economy by setting the interest rate $i_t$ below the natural interest rate $\delta \theta$ until $\sigma^+ + T$, at which point it reverts to setting the nominal interest rate equal to the natural interest rate $i = \delta \theta$. For $t > \sigma^+ + T$, output is equal to potential so that $\zeta_t = 1$. For $\sigma^+ \leq t \leq \sigma^+ + T$, output is above potential, and capacity utilization satisfies a simple differential equation

$$\frac{\dot{\zeta}_t}{\zeta_t} = i_t - \delta \theta \leq 0,$$

with terminal condition

$$\zeta_{\sigma^+ + T} = 1.$$

The solution is

$$\zeta_t = e^{\int_t^{\sigma^+ + T} (\delta \theta - i_s) ds}.$$

By lowering interest rates, the central bank creates a temporary boom after the good Poisson shock. This boom boosts the value of risky assets immediately after the good Poisson shock from

$$\mu^+ \frac{X}{\theta}$$

to

$$\mu^+ \zeta_{\sigma^+} \frac{X}{\theta} > \mu^+ \frac{X}{\theta}.$$

This policy alleviates the recession while the economy is in a liquidity trap, pushing $\xi$ to $\hat{\xi}$ where

$$\hat{\xi} = \xi \frac{\lambda^-}{\lambda^- + \lambda^+} \mu^- + \frac{\lambda^+}{\lambda^- + \lambda^+} \zeta \mu^+ > \xi.$$

Basically, committing to low interest rates after the good Poisson shock increases the value of assets while the economy is in the liquidity trap. This wealth effect increases demand and mitigates the recession. Forward guidance works by alleviating the asset shortage that is at the root of the recession.

Forward guidance trades off a future boom against a mitigation of the current recession and hence raises time-consistency issues. Because of the utility loss that comes with the boom, monetary authorities might be tempted to renege on their commitment to keep interest rates low when the time comes to deliver on this promise. Nevertheless, our main point here is that the effectiveness of forward guidance in this standard liquidity trap is to be contrasted with its relative ineffectiveness in the safety trap model developed in this paper.
**Inflation.** We could also introduce inflation just like in Section 5, with a Philipps curve

\[ \pi_t + (\gamma + \beta(1 - \xi_t))(\bar{\xi} - \xi_t) = 0, \]

and a truncated Taylor rule

\[ i_t = \min\{0, r^n_t + \pi^* + \phi(\pi_t - \pi^*)\}. \]

This does not change our results for public debt and QE in a safety trap. Forward guidance gains an extra kick by increasing inflation, reducing real interest rates, further stimulating output and inflation, etc.

**Bubbles.** To consider the possibility and consequences of bubbles, we could generalize the environment to allow for growth as in Section 6 and introduce safe and risky bubbles \( B_t = B^R_t + B^S_t \). Such bubbles are possible as long as

\[ \delta \theta < g. \]

Now suppose that we are in a liquidity trap. We have

\[ r(V - B) = \delta \xi X + \lambda^{-[\frac{\mu^- X}{\theta} - B^- - (V - B)]}, \]

\[ \theta V = \xi X, \]

\[ r = 0, \]

which yields

\[ 0 = r\left(\frac{\xi X}{\theta} - B\right) = \delta \xi X + \lambda^{-[\frac{\mu^- X}{\theta} - B^- - \left(\frac{\xi X}{\theta} - B\right)]}, \]

or

\[ \xi = \frac{\mu^- + \frac{\theta}{X}(B - B^-)}{1 - \frac{\delta \theta}{\lambda}} = \frac{\mu^- + \frac{\theta}{X}B^R}{1 - \frac{\delta \theta}{\lambda}}. \]

Hence in contrast to a safety trap environment, in a standard liquidity trap environment, it is only risky bubbles (and not safe bubbles) that stimulate output. This is because safe bubbles, in contrast with risky bubbles, entirely crowd out the future value of other assets, and hence do not result in an increase the present value of assets at a given interest rate \( r \) \((r = 0)\).