ABSTRACT. We provide a positive analysis of effort allocation by a politician facing reelection when voters are uncertain about the politician’s preferences on a divisive issue. We then use this framework to derive normative conclusions on the desirability of transparency and other institutional design features. There is a pervasive incentive to “posture” by over-providing effort to pursue divisive policies, even if all voters would strictly prefer to have a consensus policy implemented. As such, the desire of politicians to convince voters that their preferences are aligned with the majority can lead them to choose strictly pareto dominated effort allocations. Transparency over the politicians’ effort choices can either mitigate or re-enforce the distortions depending on the strength of politicians’ office motivation and the efficiency of institutions. When re-election concerns are paramount, and executive institutions are strong, transparency about effort choices can be bad for both incentivizing politicians and for sorting.

Keywords: Posturing, Transparency, Effort Allocation, Strength of Executive Institutions.

JEL Classification numbers: D72, D78, D82.
1. Introduction

In any principal-agent relationship – voter-politician or owner-manager – the key dimensions of the contract (implicit or explicit) are the scope of the agent’s delegation, the structure of contingent rewards, and the amount of monitoring of actions. These three dimensions, of course, take different forms depending on the application. When considering the incentives of an elected policymaker, the scope of the agent’s authority depends, among other things, on the independence of executive power; the contingent rewards of holding office can be affected by the number of terms of potential reelection; and monitoring of a politician’s action is an issue of transparency in policymaking. Will the policymaker necessarily have the incentive to allocate her efforts across different tasks in a socially efficient manner? If not, is a transparency reform always “good” or does it depend on the degree of independence of the executive and on whether there are term limits? We develop a new theoretical framework to study these institutional design issues.

In any democracy that gives policymakers the possibility of re-election, incumbent politicians make policy choices considering the current relative importance of the potential choices as well as their impact on re-election prospects, with weights that obviously vary for different politicians and across institutional settings. Sometimes the difficult choices are between different policies on the same issue (e.g. fiscal discipline versus fiscal stimulus for the economy) and in other contexts the most difficult choices are about what issues to focus on the most during the term in office (e.g. whether to focus on reform of social security or the healthcare system, pursue legislation on social issues, or focus on stimulating the economy and on job creation). Campaign advisors may play an important role as well, and often the choice of which issues to focus on, and which actions to advertise or campaign about, do not align with our perception of importance ranking.\(^1\) Gay marriage, gun control, and other divisive issues like NPR or PBS funding, for example, have reached a level of importance in the political debate in the United States that could be viewed as excessive with respect to other large problems concerning our economic, financial, and social welfare system.

\(^1\)See Fiorina et al. (2006, 202): “Most citizens want a secure country, a healthy economy, safe neighborhoods, good schools, affordable health care, and good roads, parks, and other infrastructure. These issues do get discussed, of course, but a disproportionate amount of attention goes to issues like abortion, gun control, the Pledge of Allegiance, medical marijuana, and other narrow issues that simply do not motivate the great majority of Americans.”
Our objective is to understand what drives an incumbent politician’s choice about the allocation of effort across various policy issues – issues that may differ in terms of importance as well as in terms of how divisive they are. In focusing on the choice of which issues to address, our analysis differs from the standard political economy models that focus on the choice of which policy to implement in a one dimensional policy space.\(^2\) In considering the allocation of effort across different issues, our analysis relates to the classic multi-task problem introduced by Holmstrom and Milgrom (1991). Our framework differs from theirs in two important dimensions. First, the principal (voters) cannot commit to a contract with politicians, but rather decide whether to retain the her based on their beliefs about the incumbent. In addition, we study an environment in which heterogeneity is over ideology rather than competence, and so politicians seek to signal their preferences to the voters. This distinguishes our paper from other multi-task applications in the political economy literature (e.g., Ashworth and Bueno de Mesquita 2006, 2012) in which politicians seek to signal their competence in addressing common-value issues.

Clearly, from a welfare perspective, it would be socially optimal for policymakers to focus first on the most important issues and on issues on which there is widespread agreement about which action is desirable. While there is often disagreement on the most important issues, and the most logical solutions, there are many important issues on which there is little ideological disagreement: preparing for and managing the aftermath from disasters, ensuring the structural integrity of roads and bridges, providing care for veterans, preventing crime and arresting murderers, stabilizing the economy, preserving the U.S. credit rating and avoiding a default. However, we often see politicians focus less attention on these goals than they do on more narrow, divisive, and often less important issues. For example: In the 2004 election, while the country was mired in the war in Iraq, U.S. President George W. Bush made opposition to same-sex marriage a cornerstone of his re-election campaign. Similarly, in 2012, with public opinion shifting on the issue, Obama began talking about his support of same-sex marriage in the run-up to his own re-election. We have also seen much effort on relatively trivial issues such as the House of Representatives taking the time to introduce and pass a bill reaffirming “In God We Trust” as the national motto of the

\(^2\)There is a small literature that studies politicians’ choice of which issues to campaign over: Colomer and Llavador 2011, and Aragones et al. 2012 focus on politicians’ attempts to add salience to common values issues on which their party has a competence advantage; Negri (2012) focuses on issues of differing importance being also of differing difficulty. Our focus is on determining the characteristics of an issue in terms of distribution of preferences and state dependence that incentivize incumbent politicians to prioritize it.
United States on November 1, 2011, despite ongoing the concerns about the economy and un-
employment, and the extensive debate about Big Bird and PBS funding in the 2012 campaign.\textsuperscript{3}
Another example comes from the large amount of resources spent arresting illegal immigrants
and individuals in possession of illegal drugs—the desirability of which is questioned by many
Americans—while police forces across the country are often stretched for resources.

In each of the above cases, politicians prioritize issues which, though arguably not the most
important issues facing the country, provide an opportunity to signal to the voters, or to their
core supporters, that they hold desirable preferences. In this paper we study the incentives for
politicians to focus effort on different issues and consider why excessive effort may be applied
to issues that are considered divisive. We show that, when voters are uncertain of politicians’
preferences, and politicians with different policy preferences might pursue different policies in the
future, politicians have an incentive to over-provide effort on divisive issues, at the expense of
common values ones, in order to signal that they hold the majority preferences. As common val-
ues issues typically have an importance that varies over time and states of nature (like economic
reforms or national security issues), it makes sense to think that in some periods the common
value issues could be of dominant importance, while in some other periods the absence of pressing
common value reform or action needs can bring up the relevance of decisions on divisive issues.
We show that even when there exist very important common values issues that everybody agrees
should be solved first, incumbent politicians over-provide effort on divisive issues in order to sig-
nal their type. The uncertainty that voters have about the preferences of politicians on divisive
issues, coupled with the possibility that there will be disagreement about which issue should be
addressed in a future period, is what causes politicians to “posture” by focusing effort on divisive
issues, rather than common values ones.\textsuperscript{4} This incentive to posture still exists even if all voters

\textsuperscript{3}The U.S. government spends $445 million dollars per year, or 0.0012\% of its budget, on PBS.
\textsuperscript{4}We refer to focusing effort on actions with the maximal electoral benefit, rather than the greatest policy
benefit, as posturing (e.g., Fox 2007). Closely related is the literature on pandering (e.g., Canes-Wrone et al.
2001, Maskin and Tirole 2004), which examines the incentives for politicians to take actions which voters think are
in their interest, possibly at the expense of actions which actually are, in order to signal competence or congruence
with the voters. As shown in Morelli and Van Weelden (2013), the incentives to pander to public opinion vary with
the divisiveness of issues and to the informational advantage of politicians. Posturing distortions follow a different
logic: in contrast with the pandering situations, voters are perfectly aware that the politician’s action does not
maximize their first period welfare, but may re-elect her anyhow if it signals she is more likely to share the voters’
preferences. The incentive to focus attention on divisive issues in our model is similar to the incentive to choose
agree that the common value issue is more important and would receive a higher payoff if that issue was dealt with; hence, posturing may involve first period effort allocations that are strictly pareto dominated. As has been discussed in the previous literature (e.g., Canes-Wrone et al. 2001, Maskin and Tirole 2004), electoral pressures can have both positive and negative effects on politician behavior, and there is often a friction between incentivizing politicians to implement desirable policies today and selecting candidates who will implement desirable policies in the future (e.g., Fearon 1999). We focus on the distortions induced by electoral incentives, and show that politicians may secure re-election by focusing their effort in a pareto dominated way. The cost of these posturing incentives vary with the efficiency or independence of executive institutions: when the over-provision of effort on one issue comes at the expense of another important issue, the cost of posturing is obviously higher than when it is feasible to do both.

In the first part of the paper we assume that voters can observe the effort allocation choices by the incumbent politician. Using standard refinements from the signaling literature, we show that when the incumbent’s primary concern is with the policy outcome in the current period, the unique equilibrium is a separating one in which majority type politicians separate by focusing some effort on the divisive issue and minority types forgo re-election to focus on the common value issue. On the other hand, with a sufficiently strong re-election motive, we get a pooling equilibrium in which both types posture by focusing on the divisive issue. In the second part of the paper we ask what happens when voters cannot observe politicians’ effort allocation, but only the policy consequences that result. In some cases it is more difficult than in others to observe the effort exerted, and only results are observable, perhaps even with delay. Further, the degree of transparency in policymaking can be influenced by institutional and legal factors: from the level of detail publicly released in budgets about how resources are allocated across departments, to the amount of access cameras and cable news organizations have to congressional, committee and cabinet deliberations, to the ongoing legal battles about the disclosure of the emails of white house staffers.

The effect of transparency on policymaking has recently attracted much attention (e.g., Prat 2005, Fox and Van Weelden 2012). Although the degree of transparency can influence the political process in many ways, in our analysis we focus on how transparency influences the allocation of “anti-minoritarian” policies in a one-dimensional spatial model (e.g., Acemoglu et al. 2013, Fox and Stephenson 2013), and the prediction that elections result in policies more extreme than voter preferences has been supported empirically (e.g., Fiorina et al. 2006, Bafumi and Herron 2010, Lax and Phillips 2011) in American politics.
of effort between common values and divisive issues. We show that, far from guaranteeing that politicians will focus their effort on socially efficient policies, increased transparency can increase the electoral benefit politicians receive from engaging in socially inefficient posturing.\footnote{Don Rostenkowski, the longtime chairman of the House Ways and Means committee, shared this concern, arguing that “as much as people criticize the back room, the dark room, or the cigar or smoke-filled room, you get things done when you’re not acting.” (Koeneman 2013)} In particular, transparency can be harmful when executive institutions are powerful and the rewards from office are very high – perhaps due to the absence of term limits or high politician salaries. The intuition for this is that, as posturing is more advantageous when effort choices are more transparent, greater transparency increases the likelihood that the equilibrium involves pooling with maximal posturing. So, with strong executive institutions and office motivated politicians, transparency can be harmful both in terms of discipline and in terms of sorting, while for the other parameter combinations there is a trade off.

As known at least since Holmstrom (1979), welfare is increasing in transparency when complete contingent contracts can be written. However, in our setting long-term contracting is likely impossible and, as in other career concern models (e.g., Holmstrom 1999, Dewatripont et al. 1999) this makes it possible for transparency to be undesirable. In career concern models such as those, the typical result is that transparency is bad for discipline but good for sorting. Our results point out that the effects of transparency on discipline and sorting depend crucially on preference and institutional parameters, and that it is possible for transparency to be good for discipline and bad for sorting, or for transparency to be worse in both dimensions. The closest paper to ours on the issue of transparency is Prat (2005), who also considers the potentially negative effects of transparency of actions in a model of career concerns, and also finds that transparency can be harmful for both agent behavior and principal learning. The reason why transparency of actions is bad in Prat (2005) is that it creates the incentive for “conformism”, whereby the agent is reluctant to take actions that do not conform to the principal’s prior. In our setting, in contrast, it is because greater transparency of actions gives the incumbent politician a greater incentive to focus on divisive issues rather than common values issues in order to signal their type. This “posturing” by elected officials differs from conformism and is equally important.

The paper is organized as follows: in section 2 we present the model, section 3 analyzes the equilibrium when politicians’ effort is observable, and section 4 describes the equilibrium when only the outcomes are. In section 5 we offer some discussion and section 6 concludes.
2. Model

We consider a two-period model in which a politician chooses a policy on behalf of the public. In each period there are two dimensions, $A$ and $B$, and the politician has to decide how to allocate effort, or other scarce resources such as money or personnel, between the two issues. That is, the politician allocates effort $w^A \in [0,1]$ to issue $A$ and $w^B \in [0,1]$ to issue $B$, and faces the constraint $w^A + w^B \leq W$, where $W \in (0,2)$. We normalize the status quo policy to be 0 in each dimension, and assume that if effort $w^A$ is exerted on issue $A$ then the policy will be $p^A = 1$ with probability $w^A$ and 0 with probability $1 - w^A$. Similarly devoting effort $w^B$ to issue $B$ results in policy $p^B = 1$ with probability $w^B$ and $p^B = 0$ with probability $1 - w^B$. We interpret $p = 0$ as the status quo policy on each issue that obtains if the politician does not address it, $p = 1$ as a new policy that takes effort to implement, and $W$ as the authority of a politician holding the office in question to pursue her agenda.\(^6\)

When $W$ is small, the politician knows that no matter which policy she pursues it is unlikely to take effect; when $W \approx 2$, she is able to get both policies implemented with high probability if she so chooses; for intermediate values of $W$ the politician faces a tradeoff where she can implement one policy but will find it difficult to get everything she wants implemented. Thus $W$ is related to the efficiency of decision making institutions and/or the power of the office in question. For example, the Prime Minister in a parliamentary system, as the head of both the executive and legislative branch is likely to have a higher $W$ than the U.S. President, particularly when the House and Senate have majorities from the other party. Similarly, within the same institutional system, a Congressman, Senator, or Member of Parliament would no doubt have a lower $W$ than the President or Prime Minister.

In addition to caring about policy, voters receive some additional payoff from having a politician who is high quality or who they like personally. We assume that the distribution of valence among politicians is normally distributed with mean 0 and variance $\varepsilon^2 > 0$. The politician’s valence is unknown to both the politician and voters initially, but is revealed to everyone when the politician is in office and constant across periods.\(^7\) As the incumbent does not know her own valence when

\(^6\)While we assume that the mapping between effort and policy change is the same for both issues this is not necessary. We could allow this to be asymmetric – for example, assuming the probabilities of policy change are $\alpha^A w^A$ and $\alpha^B w^B$ respectively – and the results would still hold just with additional parameters and algebra.

\(^7\)Similar assumptions about valence, and the time at which it is revealed, are made in Bernhardt et al. (2011) and Morelli and Van Weelden (2013).
choosing how to allocate effort in the initial period, and the voters will learn the politician's 
(time invariant) valence regardless of her action, the addition of the valence component serves 
only to ensure that voters are (generically) not indifferent between re-electing the incumbent 
and not. This ensures that the probability of re-election will vary continuously with the voters’ 
beliefs about the politician’s type. We focus on the case where $\varepsilon$ is small so the primary concern 
of the voters is with how politicians allocate their effort.

In each period, $t \in \{1, 2\}$, the stage game utility of voter $i$ is

$$-\gamma(\theta_t - p^A_t)^2 - (1 - \gamma)(x^B_i - p^B_t)^2 + v^j_t,$$

where $p^A_t$ and $p^B_t$ are the policies implemented in period $t$, $v^j_t$ is the valence of politician $j$ who 
is in office in period $t$, and $\theta_t$ and $x^B_i$ are the preferred policies in each dimension for voter $i$. So 
$\theta_t \in \{0, 1\}$ reflects whether all voters prefer policy $p^A = 1$ or $p^A = 0$ in period $t$. Conversely, the 
voters may be type $x^B = 1$ or $x^B = 0$ reflecting whether their preferred policy in dimension $B$ 
is 0 or 1. To keep the analysis as simple as possible we assume that the preferences in the $B$ 
dimension are deterministic, although this is not necessary for our analysis. We assume that a 
majority of voters, $m \in (\frac{1}{2}, 1)$, are type $x^B_i = 1$ and so prefer policy 1 in dimension $B$.

We assume that $\theta_1 = 1$ so that, in the current period, it is in the interest of all voters to 
have the $A$ issue addressed, and the probability that $\theta_2 = 1$ is $q \in (0, 1)$. This means that, 
with some probability, the voters will be content with the status quo policy in the next period.

As our analysis focuses on the behavior of politicians in the first period we assume that $\theta_1 = 1$ 
so a majority of voters would benefit from effort being exerted on two different tasks, making 
the politician’s multi-task problem non-trivial. In the second period, it is important that the 
politician’s type matters for voters’ payoffs. We ensure it is payoff relevant by assuming that 
$q < 1$ and so different types will prefer different effort allocations with positive probability in the 
second period. Finally, we assume that $\gamma \in (\frac{1}{2}, 1)$ so that all voters care more about the issue $A$ 
than issue $B$. The assumption that $\gamma > \frac{1}{2}$ is not necessary for our results to hold, but corresponds 
to the case where all players prefer $A$ to be done first, and so biases against politicians choosing 
$B$. When $\gamma < 1/2$, politicians still focus first on the $B$ issue, and this effort allocation is optimal 
for a majority of voters. We focus on the case in which $\gamma > \frac{1}{2}$ in order to provide a theory of 
why politicians may not address common values issues even if they are more important.

We assume that politicians are drawn from a (possibly proper) subset of the voters themselves 
(e.g., Besley and Coate 1997, Osborne and Slivinski 1996), and so, like the voters, the preferences
of the politicians are homogenous on the A dimension and heterogenous on the B dimension. We assume that fraction $m^P \in (\frac{1}{2}, 1)$ of the politicians have ideal point $x^B = 1$ and that $1 - m^P$ have ideal point $x^B = 0$. We refer to a politician of type $x^B = 1$ as a majority-type politician, since her policy preferences are aligned with the majority of voters, and a politician of type $x^B = 0$ as a minority type. If $m^P = m$ then the distribution of politician preferences is the same as that of the voters, and, although we allow for this possibility, we do not require it in our analysis.\(^8\)

In addition to having preferences over policy, we allow the politician to receive positive benefit $\phi$ from being in office. So the stage game utility of politician $j$ if $(p^A_t, p^B_t)$ is implemented is

\[
\phi - \gamma(\theta_t - p^A_t)^2 - (1 - \gamma)(x^B_j - p^B_t)^2,
\]

if they are in office, and, if not in office,

\[-\gamma(\theta_t - p^A_t)^2 - (1 - \gamma)(x^B_j - p^B_t)^2.\]

The parameter $\phi$ could include monetary and non monetary rewards from being elected, or could be a reduced form of the continuation value of remaining in office, and so would be affected by institutional factors such as the salary in office and whether there are term limits.

Voters form beliefs about the type of the politician. As there are only two types we can define

\[
\mu(w^A, w^B) = Pr(x^B_j = 1|w^A, w^B),
\]

to be the probability the politician is the majority type given allocation $w^A$ and $w^B$. The game is repeated with discount factor $\delta \in (0, 1)$. The timing is as follows.

1. In period 1 a politician is randomly selected to be in office for that period.
2. The politician decides how to allocate effort ($w^A$ and $w^B$). Two subcases:
   a. The voters observe the effort decision – transparency case;
   b. Voters do not observe the effort decision – no transparency case.
3. The incumbent’s valence $v^j$ is realized and publicly observed. The politician’s valence is constant across periods.
4. The policy is determined with all players receiving their utilities for period 1.

\(^8\)We do assume that a majority of politicians hold the same policy preferences as the majority of voters. This plays no role in the mechanism we consider, but, if $m^P < 1/2$, then, because majority type politicians would have more to lose from not securing re-election, it is possible, for some parameters, to support other equilibria in which there is additional costly signaling to convince the voters that the re-election motive is strong.
(5) Voters observe outcomes and update beliefs about the politician, then vote whether to re-elect her or not. If the politician is not re-elected a random replacement is drawn.

(6) \( \theta_2 \) is realized, and the politician decides how to allocate effort in period \( t = 2 \).

(7) The policy is realized with all players receiving their payoff for period 2.

Notice that we specify the game so that the policy in each dimension in period 2 is the status quo policy (\( p = 0 \)) regardless of the outcome in period 1 – perhaps because the policy will revert if the politician does not exert effort defending the new policy from legal challenges. This assumption does not drive our results: any second period stage game in which politician’s actions potentially affect the policy in the \( B \) dimension (or another divisive issue on which preferences are correlated with those about \( B \)) would generate the same first-period behavior. The focus of our analysis will be on politician behavior in the first period.

3. Equilibrium with Transparency

We look for Perfect Bayesian Equilibria, restricting attention to those in which all voters always hold the same beliefs about the politician’s type. We begin by solving for politician behavior in period \( t = 2 \). As \( \gamma > 1/2 \), all politicians, as well as all voters, care more about issue \( A \) than issue \( B \). Hence, when \( t = 2 \), the politician focuses first on addressing issue \( A \), if any change is desired on that issue (\( \theta_2 = 1 \)). If the politician is in the majority then she also prefers to act on issue \( B \) and will exert any left over effort after securing the preferred policy in dimension \( A \) on policy \( B \). The minority type will never exert effort on policy \( B \). We then have the following lemma.

**Lemma 1. Politician Action in the Second Period**

*In period \( t = 2 \),*

1. a politician of the majority type will choose \( w^A = \min\{W,1\} \) and \( w^B = 1 - w^A \) when \( \theta_2 = 1 \), and \( w^B = \min\{W,1\} \) and \( w^A = 0 \) when \( \theta_2 = 0 \).

2. a politician of the minority type will choose \( w^A = \min\{W,1\} \) when \( \theta_2 = 1 \) and \( w^A = 0 \) when \( \theta_2 = 0 \), and \( w^B = 0 \) for either \( \theta_2 \in \{0,1\} \).

Note that, as \( q < 1 \), the second period behavior of different politician types differs with positive probability regardless of \( W \). This makes the policy preferences of the politician relevant to the voters. Now that we have determined the behavior of the politician in the second period, we can consider the decision faced by the voters. Voters who are of the majority type will support
the incumbent if they believe she is sufficiently likely to be of the majority type relative to a random replacement; similarly, voters of the minority type will support the incumbent if they believe that she is sufficiently less likely to be the majority type than a random replacement.

How high a probability voters must place on the politician being their desired type to support her depends on her valence. We assume that all voters vote for the candidate they prefer, and that the politician is re-elected if and only if she receives at least half the votes. Note that this means that the politician will be re-elected if and only if majority type voters support her re-election.

We now look for the parameters under which separating and pooling equilibria exist. When $\varepsilon$ is small, in a separating equilibrium, the majority type is elected with probability close to 1, and the minority type with probability close to 0. In a pooling equilibrium the politician will be re-elected if and only if $v^j \geq 0$, which happens with probability 1/2.

**Lemma 2. Voter Behavior**

For all $\varepsilon > 0$, the probability the incumbent is re-elected is strictly increasing in $\mu(w^A, w^B)$, with

$$Pr(\text{re-elect}|\mu(w^A, w^B) = m^P) = 1/2,$$

and

$$Pr(\text{re-elect}|\mu(w^A, w^B) = 1) = X(\varepsilon, W),$$

$$Pr(\text{re-elect}|\mu(w^A, w^B) = 0) = Y(\varepsilon, W).$$

Moreover, $\lim_{\varepsilon \to 0} X(\varepsilon, W) = 1$ and $\lim_{\varepsilon \to 0} Y(\varepsilon, W) = 0$.

Now that we have determined the voter behavior, given the beliefs induced by the politician’s action, we turn to analyzing the first period behavior of the politician. As this is a signaling game it will admit many equilibria, especially when re-election concerns are paramount ($\phi$ is high). While it is possible to support many different first period actions by assigning unnatural off-path beliefs, applying criterion D1 from Cho and Kreps (1987) generates a unique equilibrium prediction. As a majority type has a greater willingness than a minority type to take action $B$, criterion D1 will require that voters believe that a politician who exerted greater than the equilibrium level of effort on $B$ to be of the majority type.

As this is not a standard sender-receiver game we must first define how this condition applies in our setting. While Cho and Kreps (1987) define these refinements in terms of Sequential Equilibrium, because there are a continuum of potential actions, we analyze the game using
Perfect Bayesian Equilibrium. For our purposes, the only relevant restriction on off-path beliefs from Sequential Equilibrium is that all voters hold the same beliefs at all information sets, and we restrict attention to equilibria with that property.

In order to facilitate the definition, we first define \( u^*(x^B_j) \) to be the expected utility of a type \( x^B_j \) politician in a given Perfect Bayesian Equilibrium. Further we define \( u(w^A, w^B, \mu|x^B_j) \) to be the expected utility, given the equilibrium strategies of the other players, of a type \( x^B_j \) politician from choosing allocation \( (w^A, w^B) \) in period 1 if the belief the voters form about her type from choosing that allocation is \( \mu \) and her behavior in the second period is unchanged.

**Definition 1. Criterion D1 (Cho and Kreps 1987)**

A Perfect Bayesian Equilibrium satisfies criterion D1 if,

1. at all information sets all voters hold the same beliefs, \( \mu \), about the politician’s type.
2. if for some off-path allocation \( (w^A, w^B) \), and \( x^B \in \{0, 1\} \),
   \[
   \{ \mu \in [0, 1] : |u(w^A, w^B, \mu|x^B) > u^*(x^B) \},
   \]
   is a proper superset of
   \[
   \{ \mu \in [0, 1] : |u(w^A, w^B, \mu|1-x^B) \geq u^*(1-x^B) \},
   \]
   then
   \[
   \mu(x^B|w^A, w^B) = 1.
   \]

Criterion D1 simply says that, if the voters see an out of equilibrium effort allocation, they should believe it was taken by the type of politician who would have an incentive to deviate for the least restrictive set of beliefs. As the majority type receives positive utility from implementing \( B \), while the minority type receives a negative payoff from doing so, the majority type has a greater incentive to take action \( B \) than the minority type does. There is one caveat to this however. As politicians care about the policy implemented after leaving office, a politician has a greater incentive to secure re-election if her replacement is less likely to be the same type. So, if \( \phi \) is very low, and \( m^P \) is close to one, a majority politician receives little benefit from re-election, and so has less incentive to posture even though it is comparatively less costly. Any equilibrium with D1 will involve signaling behavior, however, when \( \phi > \hat{\phi} \) where

\[
(1) \quad \hat{\phi}(W) \equiv \begin{cases} 
\max\{\frac{1}{2}q(2\gamma m^P - 1)W, 0\} & \text{if } W \leq 1, \\
\max\{\frac{1}{2}(W-2)q(2\gamma m^P - 1), 0\} & \text{if } W > 1. 
\end{cases}
\]
If $\phi < \hat{\phi}$, something that is only possible if $m^P$ is close to 1, minority-type politicians value re-election so much more than majority types that they would have a greater incentive to posture, and in the unique D1 equilibrium all politicians focus effort on issue $A$. However, when $\phi > \hat{\phi}$, the returns from holding office are similar enough across types that the differences in the costs of implementing $B$ dominate and the majority type always has a greater incentive to exert effort on $B$. Combining this with criterion D1 generates a unique equilibrium prediction.

Consider first the case in which $\phi$ is low (but greater than $\hat{\phi}$), so politicians are more concerned with the policy implemented in the current period than with securing re-election. Consequently, in equilibrium, both types focus the bulk of their energies on ensuring that $A$ is implemented. Notice, however, that the equilibrium must involve the majority type separating themselves by placing strictly positive effort on $B$: as the majority type has a strictly greater incentive to choose $B$ than the minority type, by placing slightly more weight on $B$ the majority type can reveal themselves to be the majority type and guarantee re-election with probability $X(\varepsilon, W) > \frac{1}{2}$. Hence, for $\phi$ low, we have a separating equilibrium with minority types focusing on $A$ and majority types exerting just enough effort on $B$ to reveal themselves to be the majority type. The requirement D1 on off-path beliefs guarantees that the separating equilibrium will be the one with the minimum effort diverted from $A$ to $B$.

Now consider the case in which $\phi$ is high, and so the primary concern of politicians is to secure re-election. In this setting, though the majority type still has an incentive to try to separate by putting additional emphasis on issue $B$, the minority type is no longer willing to forsake re-election by focusing effort on her preferred policy. As the minority type would always have an incentive to mimic the majority type, and the majority type would always have an incentive to try to separate by attaching more effort to $B$, the only possible equilibrium involves all politicians putting maximal effort $w^B = \min\{W, 1\}$ on issue $B$ in the initial period.

Finally note that emphasizing $B$ in a separating equilibrium results in re-election with a higher probability than by emphasizing $B$ in a pooling equilibrium. For intermediate levels of office-motivation, then, it would not be possible to have a D1 equilibrium that is either separating (the minority type would have an incentive to mimic the majority type and be elected with probability $X(\varepsilon, W)$) or pooling (the minority type would prefer to lose election than be re-elected with probability $1/2$ by exerting effort on $B$). For this range of parameters the equilibrium is partial-pooling, with the majority type emphasizing issue $B$, and the minority type randomizing...
between focusing all their effort on $A$ and losing election, and focusing on $B$ and being re-elected with probability greater than $1/2$ but less than $X(\varepsilon, W)$.

As the above discussion suggests, we have the following characterization of the unique equilibrium with off-path beliefs that satisfy $D1$.

**Proposition 1. Characterization of Equilibrium with $D1$ Beliefs**

If $\phi > \hat{\phi}(W)$ then there exists a unique Perfect Bayesian Equilibrium that satisfies criterion $D1$. Further, there exist $\tilde{\phi}(\varepsilon, W)$ and $\phi^* (\varepsilon, W)$ with $\hat{\phi}(\varepsilon, W) \leq \tilde{\phi}(\varepsilon, W) \leq \phi^*(\varepsilon, W)$ such that, in the first period in the unique $D1$ equilibrium,

1. if $\phi \in (\hat{\phi}, \tilde{\phi}]$ the majority type chooses $w^B = w^*_A(\phi) > 0$ and $w^A = W - w^B$ and the minority type chooses $w^A = \min\{W, 1\}, w^B = 0$.
2. if $\phi \in (\tilde{\phi}, \phi^*)$ the majority type chooses $w^B = \min\{W, 1\}, w^A = W - w^B$ and the minority type randomizes with a non-degenerate probability between $w^B = \min\{W, 1\}, w^A = W - w^B$ and $w^A = \min\{W, 1\}, w^B = 0$.
3. if $\phi \geq \phi^*$ all politicians choose $w^B = \min\{W, 1\}$ and $w^A = W - w^B$.

Moreover, there exists $\bar{W} \in (1, 2]$ such that $0 \leq \hat{\phi}(\varepsilon, W) < \tilde{\phi}(\varepsilon, W) < \phi^*(\varepsilon, W)$ for all $W \in (0, \bar{W})$. Finally, there exists $\bar{\gamma} > 1/2$ such that $\bar{W} = 2$ when $\gamma < \bar{\gamma}$.

So, when there are sufficiently low rewards from reelection, we get a separating equilibrium in which the majority type chooses $w^B > 0$ and allocates the rest of her effort to issue $A$, whereas the minority type allocates all effort to $A$.

With high enough office holding rewards, on the other hand, all politicians focus first on issue $B$ in order to maximize their probability of being re-elected. We refer to such an equilibrium as a “posturing” equilibrium. Finally, for an intermediate level of office-motivation, the minority type randomizes between posturing and revealing themselves to be in the minority by pursuing issue $A$.

While a posturing equilibrium always exists if the office motivation is strong enough, when $W \approx 2$ and $\gamma \approx 1$ a separating equilibrium may not exist for any $\phi$. For such parameters, posturing is not very costly, since issue $A$ will be addressed with probability close to 1, and exerting effort on $B$ provides little disutility to even the minority type politician, so to have a separating equilibrium requires lower office motivation than necessary to induce signaling. A

\footnote{Interestingly, this means that all voters, including majority type voters, may get higher first period payoff from a politician of the minority type.}
separating equilibrium always exists when $W$ is not too close to 2 ($W \leq \bar{W} \in (1, 2]$), or $\gamma$ is not close to 1 ($\gamma < \bar{\gamma}$).

We consider now how the ranges of $\phi$ for which equilibria of each form exist depend on the parameters of the model. The most important parameter for reflecting the institutional authority of the politician is $W$. We now consider how $\bar{\phi}$ and $\phi^*$ vary with this authority parameter. As it is only possible to support separating equilibria when $\phi \leq \bar{\phi}$, and only possible to support an equilibrium that does not involve everyone pooling on $B$ when $\phi < \phi^*$, $\bar{\phi}$ and $\phi^*$ are indices of how likely it would be (in a world of randomized parameter values) to live in an equilibrium without pervasive posturing incentives.

**Proposition 2. Existence of a Separating and Posturing Equilibrium**

Define $\bar{\phi}_0(W) = \lim_{\varepsilon \to 0} \bar{\phi}(\varepsilon, W)$ and $\phi^*_0(W) = \lim_{\varepsilon \to 0} \phi^*(\varepsilon, W)$. Both $\bar{\phi}_0(W)$ and $\phi^*_0(W)$ are strictly increasing on $(0, 1)$ and strictly decreasing on $(1, \bar{W})$.

We find that, when valence shocks are small, $\bar{\phi}$ and $\phi^*$ are non-monotonic in $W$. If $W$ is small, it is difficult to support a separating equilibrium. Since politicians know that their effort allocation is unlikely to affect the resulting policy, they have a greater incentive to choose the allocation most likely to get them re-elected – in this case, that means pooling on $B$.\(^{10}\) As $W$ increases, effort is more likely to lead to the policy being implemented, so the incentive for the politician to allocate effort to her preferred policy increases. However, as $W$ gets larger than 1, further increases in $W$ makes it more difficult to support a separating equilibrium. This is because, when $W$ is large, politicians are capable of getting both the $A$ and the $B$ policy implemented with a high probability. As the greatest cost of effort on the $B$ policy, when $W \leq 1$, is that it comes directly at the expense of effort which could be allocated to the $A$ policy, the costs of choosing the $B$ policy are lower when $W$ is large. As the policy consequences are the starkest when $W = 1$, this is when it is possible to support a separating equilibrium for the widest range of parameters.

4. **Equilibrium with Unobservable Effort Choices**

We now consider the incentives when the effort allocation is not transparent. That is, we assume that the voters can observe only the outcome but not $w^A$ or $w^B$. As the incentive for

\(^{10}\)Fox and Stephenson (2011) identify a similar effect. They present a model in which judicial review, by insulating politicians from their policy choices, can increase electoral induced distortions.
the politician to take each action depends on the beliefs the voters form after observing each outcome, the beliefs at off-path information sets can play a key role even here in determining the politician’s incentives. Further, since, in our model, a non-status-quo policy can never result if a politician does not exert positive effort on the issue, off-path information sets are produced by many politician strategies— if the politicians’ equilibrium strategies involve $w^j = 0$ for some $j \in \{A, B\}$ then observing $p^j = 1$ is off the equilibrium path. In our analysis, we focus on the case in which $\phi$ is large, so the dominant concern is to secure re-election. Then, if the politician’s effort allocation were transparent, the result would be the posturing equilibrium in which both types of politicians focus effort first on issue $B$. As the setting with unobservable efforts is farther removed from the original sender-receiver setting of Cho and Kreps (1987), rather than adapt the refinements notion further, we simply focus attention on the equilibria in which the majority type politician’s action corresponds to the transparency case.\footnote{This can be justified by adapting criterion D1 to an environment in which the sender’s message is not observed, by assuming that whenever there is one type that would be willing to choose every effort allocation which would result in $(p^A, p^B)$ being observed with positive probability for a wider range of beliefs than the other type, voters must believe the politician is of that type after observing $(p^A, p^B)$. The details are available upon request.} We then consider the minority type’s behavior.

Recall that, with transparency, if the voters observe any effort allocation other than that chosen by the majority type, they know with certainty that the politician deviated, and so is the minority type. With non-transparency if the minority type deviates this (may) not be observed with certainty. This means that parameter values that admit a posturing equilibrium with transparent effort will not necessarily result in the same behavior when the effort choices are non-transparent. In particular, when $W < 1$, we have the following result.

**Proposition 3.** For any $W < 1$, there exists a $\phi^A_{00}(\varepsilon, W) \geq \phi^*(\varepsilon, W)$ such that, for all $\phi > \phi^A_{00}$, there is a unique pure-strategy equilibrium in which the majority type chooses $w^B = W$. In this equilibrium the minority type’s strategy involves $w^A = 0$ and $w^B < W$ so first period welfare is lower than when effort is observable.

Proposition 3 characterizes the unique pure-strategy equilibrium in which the majority type focuses on $B$ when $W < 1$ and the re-election motive is strong. There are also mixed strategy equilibria: because the effort choice is not transparent, any two strategies leading to the same
probability of \(p^B = 1\) are equivalent for voter updating and the politician’s payoff. For high values of \(\phi\), when \(W < 1\), the lack of transparency creates even further welfare loss in the first period beyond that already determined by posturing incentives: not only will no politician exert effort on \(A\), with some probability a minority type politician will exert less than full effort on \(B\). The reason for this is simple. With non-transparency there cannot be a pooling equilibrium, for then the voters would not update based on the observed outcome, and the minority type would have no incentive to put any effort on \(B\). However, if the majority type is focusing entirely on \(B\), then policy \(A\) would never be obtained, and \(p^A = 1\) would reveal that the politician is the minority type with certainty. As the politician would not be willing to guarantee her own defeat by implementing \(A\), the only possible pure-strategy equilibrium involves the minority type choosing \(w^A = 0\) and \(w^B < W\). As such, when \(W < 1\), transparency over the effort allocation is beneficial for first period welfare. However, transparency over the effort choices will impede sorting as we have a pooling equilibrium only when effort is transparent, and, when effort is non-transparent, the voters update their beliefs based on the policy outcome. That transparency is good for incentives but bad for sorting is the reverse of what has often been found in the literature (e.g. Dewatripont et al. 1999, Fox 2007) on transparency and policymaking.

We now consider the case in which \(W > 1\). As noted above, in order to support a posturing equilibrium with transparency, we need only check that the politician does not have an incentive to deviate to her most preferred policy and lose re-election. Hence we can support a posturing equilibrium if and only if the policy gain from deviating is not enough to justify taking an action which drops the re-election probability from \(1/2\) to \(Y(\epsilon, W)\),

\[
\gamma(2 - W) + (1 - \gamma) \leq \left(\frac{1}{2} - Y(\epsilon, W)\right) \delta(\phi + m^P(1 - \gamma)[1 + q(W - 2)]),
\]

or equivalently

\[
\phi \geq \phi^* = \frac{2(1 - \gamma(W - 1))}{\delta(1 - 2Y(\epsilon, W))} - m^P(1 - \gamma)[1 + q(W - 2)].
\]

As in the transparency case, when the effort allocation is non-transparent, the minority type has the option of deviating to her preferred first-period policy \(w^A = 1, w^B = 0\) and revealing herself to be the minority type: she will prefer to do this if and only if \(\phi < \phi^*\). Note, however, that she could also deviate to choose \(A\) with higher probability putting the rest of the effort on \(B\), and get re-elected with probability greater than \(Y(\epsilon, W)\), so there is an additional deviation we need to check to verify that a posturing equilibrium can be supported. As, in the posturing equilibrium,
the politician will be re-elected with probability $\frac{1}{2}$ if $B$ is obtained and $Y(\varepsilon, W)$ if it is not, and in the proposed deviation $B$ occurs with probability $2 - W$, she will not have an incentive to shift effort away from $B$ if and only if
\[
(2 - W) \leq (2 - W)(\frac{1}{2} - Y(\varepsilon, W))\delta(\phi + mP(1 - \gamma)[1 + q(W - 2)]),
\]
or equivalently
\[
\phi \geq \phi_{NA}^* \equiv \frac{2}{\delta(1 - 2Y(\varepsilon, W))} - mP(1 - \gamma)[1 + q(W - 2)] > \phi^*.
\]
As such, it is more difficult to support a posturing equilibrium when the action is non-transparent.

We then get the following proposition.

**Proposition 4.** There exists an equilibrium in which both types always choose allocation $w^B = 1, w^A = W - 1$ if and only if $\phi \geq \phi_{NA}^* > \phi^*$. However, there exists $\phi^{**} < \phi_{NA}^*$ such that, when $\phi \in [\phi^{**}, \phi_{NA}^*)$, an equilibrium exists in which the majority type chooses $w^B = 1, w^A = W - 1$ and the minority types chooses allocation $w^A = 1, w^B = W - 1$ with probability $r \in (0, 1]$ and allocation $w^B = 1, w^A = W - 1$ otherwise. Hence, for $\phi \in [\phi^{**}, \phi_{NA}^*)$, non-transparency increases first period welfare and the probability of a majority type politician in office in the second period.

Note that, on the range $\phi \in [\phi^{**}, \phi_{NA}^*)$, for the equilibrium we described, the welfare comparison is unambiguous for the majority voters. The minority type places more effort on $A$, which gives higher payoff to everyone in the first period. Further, because $p^A = 1$ is more likely, and $p^B = 1$ is less likely, to occur when the politician is the minority type, the voters learn about the politician’s type, and a majority type is more likely to be retained. Hence, as in Prat (2005), non-transparency over actions is beneficial in this range, both in terms of the first period action, and in terms of selection for the future. As non-transparency decreases the reputational benefit from posturing, minority types have less incentive to posture. This breaks the equilibrium with pooling on maximal posturing, leading to more efficient policy choices by politicians, and more learning by voters.

This does not mean that we cannot have a posturing equilibrium with non-transparent action and $W > 1$. When $\phi > \phi_{NA}^*$ the benefits from holding office are great enough that no politician would want to risk $p^B = 0$ and likely electoral defeat. Hence, even with non-transparency, the equilibrium must involve maximal posturing by all politicians. As, regardless of the transparency
regime, voters cannot update about the politicians in a pooling equilibrium, when $\phi > \phi_{NA}^*$ the equilibrium outcome is unaffected by the transparency of effort.

So we have shown that, on the range $\phi \in [\phi^{**}, \phi_{NA}^*)$, greater transparency increases posturing by elected officials and decreases the amount voters can learn from this behavior. For example, it is likely that advent of cable news causes politicians to focus more time on trivialities and polarizing debates; similarly, we may worry that if cabinet meetings were televised, or the minutes were publicly released, that concern about signaling popular preferences would distract members from working to advance the most important goals. Further, if greater transparency leads all politicians to focus on posturing, in equilibrium, though politicians spend their time engaging in socially harmful signaling, voters don’t actually learn anything about the politician’s preferences. These results show that in models in which career-minded agents distort their messages in order to protect their reputation (e.g. Morris 2001), if the messages the agent sends are (partially) obscured, because there is less incentive to send an “anti-minoritarian” or “politically correct” message, not only may the messages be more informative, leading to more desirable policy outcomes, but more could be learned about the agent in the process.

Of course, it is a long way from these results to the conclusion that transparency is “bad”. In our analysis we have considered only one of many reasons why politicians may pursue socially suboptimal policies. When evaluating the desirability of a given transparency reform, the risks of increased posturing must be balanced against the potential benefits from making the system more transparent – for example, decreasing the opportunities for rent-seeking behavior or to target transfers or promises to special interests – that are not considered in our model.

5. Discussion: Posturing and Polarization

In our model, we studied the incentive for politicians to focus energy on divisive issues rather than common values issues that may be more important. Such an incentive appears to be a central feature of American politics: much has been written on how American politics has become polarized (e.g., Fiorina et al. 2006, Abramowitz 2010), and how a disproportionate amount of time (Fiorina et al. 2006) and media attention (Prat and Stromberg 2011) is devoted to divisive issues at the expense of issues that voters are believed to consider more important. In our model, politicians focus undue attention on divisive issues in order to signal that they hold the majority position on that issue. Notice that this means that, although all voters agree on the
most important issue to be addressed, and share identical preferences on that issue, the voters are “polarized” over the issue politicians focus their attention on, with a majority of the voters feeling change on the divisive issue is for the better, but others feeling it is harmful.

While our model cannot be directly applied to study polarization of candidates and parties – there are no parties in this model, only a clearly defined majority that all politicians need to win over – one could imagine extending this framework to an environment with parties. In such a model, different politicians may be predominantly concerned with signaling to members of their own party, perhaps in order to prevent a primary challenge; for example, Republicans may feel the need to signal their commitment to preventing tax increases in order to ward off potential primary challenges from the “tea party” or the Club for Growth.\textsuperscript{12} There are many examples where politicians appear to focus excessive effort on divisive issues in order to signal their commitment to their core supporters. For example: In March of 2005 U.S. President George W. Bush returned from vacation early in order to sign a bill at 1:11 AM transferring the controversial Terri Schiavo case to federal courts; such a sense of urgency is particularly striking considering that six months later it took a day and a half for Bush to end his vacation and return to Washington after Hurricane Katrina. After taking office in January 2009 at the depths of the recession, a leading piece of the legislative agenda of the new Democratic majority was the Employee Free Choice Act seeking to bolster the power of unions, something which was popular only with the Democratic base. And, during the debt ceiling crisis in July of 2011, Republican insistence that no deal could involve any tax increases, no matter how small, was cited by Standard and Poor as a reason for downgrading the U.S. credit rating.

While there is a concern that posturing to a core constituency may lead politicians to take actions the majority opposes, our results suggest that the greater concern may be that it distracts

\textsuperscript{12}The Club for Growth has backed successful primary challenges in the past, notably against incumbent Senators Bob Bennett and Chuck Hagel in 2010. Further, Club for Growth president Chris Chocola, when discussing the Obama administration’s omnibus bill warned that “every Republican aye vote will likely face a serious primary challenge from the right in their next reelection campaign and should.” (Cantanese 2010) The threat of primary challenges also exists for Democratic incumbents: the head of the United Steelworkers Union, Leo Gerrard, warned that any Democrat who opposes Obama’s policies isn’t a “real Democrat” and that should “remember what happened to Blanche Lincoln”, who lost re-election after a difficult primary challenge in 2010. See Hummel (2010) and Padro-i-Miquel and Snowberg (2012) for models of primaries and their impact on policy choices.
politicians from more important common values issues. If different politicians are posturing to different constituencies, and Republicans posture in order to show that they are committed to not raising taxes by refusing to accept even mild tax increases, while Democrats posture to show they are committed to maintaining government spending on social issues, it may not be possible to reach an agreement on a common value goal (e.g. dealing with the debt crisis, maintaining the nation’s credit rating) even if all involved agree that is an important priority. Appearing on Meet the Press soon after Standard and Poor downgraded the U.S. debt rating, former chairman of the Council of Economic Advisors Austan Goolsbee wondered if, for the sake of the economy, “Can’t we wait on the things that we’re going to yell at each other about and start on the things that we agree on?” (Goolsbee 2011) Our analysis provides an explanation for why it is often difficult to get politicians to come together to address the “things we agree on.”

6. CONCLUSIONS AND DIRECTIONS FOR FUTURE RESEARCH

We have considered the incentives of politicians to “posture” by focusing their efforts on issues that, though perhaps not the most important ones, present the greatest opportunity for politicians to signal their preferences to voters. We have shown that this incentive can lead politicians to spend their time pursuing policies that are not only harmful to the minority, but also an inefficient use of time from the majority’s perspective. Further, we have shown that greater transparency about how politicians allocate their time can increase socially inefficient posturing, while at the same time impeding the selection of majority type politicians.

While we have focused on only one component of the policymaking process, our analysis raises important issues for the design of political institutions – and, perhaps, after re-interpreting the parameters, organizational design more generally. Given that our analysis has emphasized the difficulty of incentivizing electorally accountable politicians to focus attention on common values issues, our findings highlight the potential advantage of delegating such tasks to individuals who are politically insulated or whose authority is task specific. This can be accomplished by delegating to city managers that are, at least somewhat, politically insulated and who have clearly defined tasks (Vlaicu and Whalley 2012) or by leaving such issues in the hands of a competent bureaucracy. The design of such institutions, and a full analysis of the tradeoffs between the distortions induced by electoral pressures and transparency and the benefits that have previously been highlighted, is an important avenue for future research.
Another priority for future work is the empirical investigation of posturing incentives. From the preliminary results of a study on American senators that we have started, posturing behavior can be identified for the senators up for reelection, and in particular for the party that holds the majority in the Senate. Selecting a set of issues that have been addressed in American politics in the last 40 years or so, we can use public opinion data to rank such issues in terms of divisiveness, perceived importance, degree of common value and state dependence, whether the voters supporting a reform on each issue in each period are mostly democrats or republicans, controlling for whether the incumbent was democrat or republican. Text analysis of Congress and Senate speeches will be used to verify whether posturing is indeed strongest when comparing pairs of issues that resemble the assumptions of our model. In other words, for any pair of issues where one of them has a higher perceived importance and higher state dependence and the other is more divisive, we should expect an excessive focus on the latter, especially for the senators that are up for reelection, and especially if they belong to a party whose supporters are greatly in favor of action on such a divisive issue.

REFERENCES


Appendix: Proofs

We begin by characterizing the unique equilibrium consistent with D1 in the game with transparent effort. We then proceed to consider the non-transparency case.

Proof of Results with Transparent Effort. Proof of Lemma 1 Immediate. □

Proof of Lemma 2 As the election is determined by the majority voters, we consider the expected second period payoff to a majority voter from a majority type incumbent, a minority type incumbent, and a random replacement, for any \( W \). If the incumbent is a majority type with valence \( v^j \) then the expected payoff is

\[
  u_1 = \begin{cases} 
  -q[(1 - W)\gamma + (1 - \gamma)] - (1 - q)(1 - \gamma)(1 - W) + v^j & \text{if } W \leq 1, \\
  -q(1 - \gamma)(2 - W) + v^j & \text{if } W > 1. 
  \end{cases}
\]

Similarly, if the incumbent is a minority type with valence \( v^j \) the expected payoff is

\[
  u_0 = \begin{cases} 
  -q(1 - W) - (1 - \gamma) + v^j & \text{if } W \leq 1, \\
  -(1 - \gamma) + v^j & \text{if } W > 1. 
  \end{cases}
\]

So a majority voter’s payoff if the incumbent is the majority type with probability \( \mu(w^A, w^B) \) is

\[
  u(\mu, v^j) = u_1\mu + u_0(1 - \mu) + v^j.
\]

Combining these equations payoffs with the fact that a random replacement has expected valence of 0, the expected payoff from a random replacement is

\[
  u_r = \begin{cases} 
  -q(1 - W)\gamma - (1 - \gamma)[qm^P + (1 - q)(1 - W)m^P + (1 - m^P)] & \text{if } W \leq 1, \\
  -(1 - \gamma)[q(2 - W) + (1 - m^P)(q(2 - W) + (1 - q))] & \text{if } W > 1. 
  \end{cases}
\]

The incumbent will be re-elected if and only if \( u(\mu, v^j) \geq u_r \). As \( u(\mu, v^j) \) is increasing in \( \mu \) we can see immediately that the re-election probability is increasing in \( \mu \). Similarly, \( u(m^P, 0) = u_r \), so
the re-election probability if $\mu = m^P$ is $1/2$. And, evaluating at $\mu = 1$ and $\mu = 0$, an incumbent known to be the majority type is re-elected if and only if

$$v^j \geq \begin{cases} -(1 - \gamma)(1 - m^P)(1 - q)W & \text{if } W \leq 1, \\ -(1 - \gamma)(1 - m^P)(q(2 - W) + (1 - q)) & \text{if } W > 1, \end{cases}$$

and an incumbent known to be the minority type if and only if

$$v^j \geq \begin{cases} (1 - \gamma)m^P(1 - q)W & \text{if } W \leq 1, \\ (1 - \gamma)m^P(q(2 - W) + (1 - q)) & \text{if } W > 1. \end{cases}$$

So, in a separating equilibrium, the re-election probabilities of majority and minority types are

$$X(\varepsilon, W) = \begin{cases} F\left(\frac{(1 - \gamma)(1 - m^P)(1 - q)W}{\varepsilon^2}\right) & \text{if } W \leq 1, \\ F\left(\frac{(1 - \gamma)(1 - m^P)(q(2 - W) + (1 - q))}{\varepsilon^2}\right) & \text{if } W > 1, \end{cases}$$

$$Y(\varepsilon, W) = \begin{cases} F\left(\frac{-(1 - \gamma)m^P(1 - q)W}{\varepsilon^2}\right) & \text{if } W \leq 1, \\ F\left(\frac{-(1 - \gamma)m^P(q(2 - W) + (1 - q))}{\varepsilon^2}\right) & \text{if } W > 1, \end{cases}$$

where $F$ is the cdf of the standard normal. We can see immediately that

$$\lim_{\varepsilon \to 0} X(\varepsilon, W) = 1,$$

$$\lim_{\varepsilon \to 0} Y(\varepsilon, W) = 0.$$

□

We now turn to characterizing first period behavior in the unique D1 equilibrium. To do this, we begin with the following three supporting Lemmas. The proofs of Lemma 3–5 are included in the Supplemental Appendix. We then characterize the equilibrium for each range of parameters in Lemmas 6–8. We first show that, in any equilibrium satisfying criterion D1, the majority type must always choose $w^A + w^B = W$.

**Lemma 3.** There cannot exist a Perfect Bayesian Equilibrium satisfying criterion D1 in which the majority type chooses any allocation $w^A, w^B$ satisfying $w^A + w^B < W$ with positive probability in period 1.

Next we show that, as choosing $B$ instead of $A$ is less costly for the majority type than the minority type, a deviation to exerting less effort on $B$ is beneficial for a larger set of beliefs for the minority type than the majority type. Recall the definition of $\hat{\phi}$ from equation (1).
Lemma 4. Consider an allocation $w^B > 0$ and $w^A = W - w^B$, and suppose the probability of being re-elected after that allocation is $\pi$. Then, if $\phi > \hat{\phi}$, at any allocation $(w', w'')$ with $w'' < w^B$, one of the following must hold:

1. both types would prefer $(W - w^B, w^B)$ to allocation $(w', w'')$ for all beliefs.
2. both types would prefer $(w', w'')$ to $(W - w^B, w^B)$ for all beliefs.
3. the set of beliefs for which a minority type strictly prefers $(w', w'')$ to $(W - w^B, w^B)$ is a proper superset of those for which a majority type weakly prefers $(w', w'')$ to $(W - w^B, w^B)$.

The above lemma will be useful for determining the beliefs after observing that the politician chose a level of $w^B$ lower than what is on the equilibrium path. Combining the above lemma with criterion D1, except for deviations which no politicians would make regardless of the induced beliefs, the voters would have to believe such a deviation was made by the minority type.

We now consider the beliefs after observing the politician allocate more weight to $B$ than expected. As the majority politician pays a smaller policy cost than a minority politician for increasing $w^B$, they would be willing to take such an action for a less-restrictive set of beliefs. The following lemma then shows that, in any equilibrium consistent with D1, if the highest equilibrium level of effort does not reveal the politician to be the majority type with certainty, then the equilibrium must involve the majority type allocating maximal effort to $B$.

Lemma 5. If $\phi > \hat{\phi}(W)$ then, in any equilibrium satisfying criterion D1 in which the majority type is re-elected with probability $\pi < X(\varepsilon, W)$, the majority type’s effort allocation is $w^B = \min\{W, 1\}$ and $w^A = W - w^B$ in period 1.

With these the lemmas we can determine when a separating, pooling, and partial-pooling equilibrium consistent with D1 exist. We begin by considering separating equilibria, and show that only the minimally separating equilibrium is consistent with criterion D1 on off-path beliefs.
Lemma 6. If \( \phi > \hat{\phi}(\varepsilon, W) \), then there exists a Separating Equilibrium that satisfies Criterion D1 if and only if \( \phi < \hat{\phi} \equiv \max\{\phi_1, \hat{\phi}\} \), where

\[
\phi_1 = \begin{cases} 
\frac{\delta(X(\varepsilon, W) - Y(\varepsilon, W))}{\delta(X(\varepsilon, W) - Y(\varepsilon, W)) - (1 - \gamma)(1 - q)m^P W} - (1 - \gamma)(1 - q)m^P W & \text{if } W \leq 1, \\
\frac{1 - (W - 1)\gamma}{\delta(X(\varepsilon, W) - Y(\varepsilon, W)) - m^P (1 - \gamma)(1 + (W - 2)q)} - m^P (1 - \gamma)(1 + (W - 2)q) & \text{if } W > 1.
\end{cases}
\]

In this equilibrium, the minority chooses \( w^A = \min\{W, 1\} \), \( w^B = 0 \) and the majority chooses

\[
w^B = w_*(\delta, \phi) = \begin{cases} 
\delta[X(\varepsilon, W) - Y(\varepsilon, W)](\phi + (1 - \gamma)(1 - q)m^P W) & \text{if } W \leq 1, \\
(W - 1)\gamma + \delta[X(\varepsilon, W) - Y(\varepsilon, W)](\phi + m^P (1 - \gamma)(1 + (W - 2)q)) & \text{if } W > 1.
\end{cases}
\]

and \( w^A = W - w^B \). Moreover, there exists \( \bar{W} \in (1, 2] \) such that \( \phi_1(\varepsilon, W) > \hat{\phi}(\varepsilon, W) \) for all \( W \in (0, \bar{W}) \). Finally, there exists \( \bar{\gamma} > 1/2 \) such that, if \( \gamma < \bar{\gamma} \) then \( \bar{W} = 2 \).

Proof. We begin by showing that, if \( \phi \in (\hat{\phi}, \tilde{\phi}) \), the behavior described can be supported in an equilibrium with off-path beliefs that satisfy Criterion D1. First note that, since the politician is revealed to be the majority type with certainty when \( w^B = w_*(\delta) \), \( w^A = W - w_*(\delta, \phi) \), and all politicians strictly prefer to implement \( w^B = w_*(\delta, \phi) \), \( w^A = W - w_*(\delta) \) to any allocation with \( w^B > w_*(\delta, \phi) \), any allocation with \( w^B > w_* \) are equilibrium dominated. The beliefs after such allocations then are not relevant for the equilibrium behavior. Next note that, under the specified strategies a minority type that chooses \( (W - w_*(\delta, \phi), w_*(\delta, \phi)) \) would be re-elected with probability \( X(\varepsilon, W) \), and by following her prescribed strategy of \( (\min\{W, 1\}, 0) \) she is re-elected with probability \( Y(\varepsilon, W) \). The value to a minority type politician of being re-elected rather than being replaced by a random replacement is

\[
\begin{cases} 
\delta[X(\varepsilon, W) - Y(\varepsilon, W)](\phi + (1 - \gamma)(1 - q)m^P W) & \text{if } W \leq 1, \\
\delta[X(\varepsilon, W) - Y(\varepsilon, W)](\phi + m^P (1 - \gamma)(1 + (W - 2)q)) & \text{if } W > 1.
\end{cases}
\]

However the cost of implementing \( (W - w_*(\delta, \phi), w_*(\delta, \phi)) \) instead of \( (\min\{W, 1\}, 0) \) is

\[
\gamma[\min\{W, 1\} + w_*(\delta, \phi) - W] + (1 - \gamma)w_*(\delta, \phi)
\]

which equals

\[
\begin{cases} 
\delta[X(\varepsilon, W) - Y(\varepsilon, W)](\phi + (1 - \gamma)(1 - q)m^P W) & \text{if } W \leq 1, \\
\delta[X(\varepsilon, W) - Y(\varepsilon, W)](\phi + m^P (1 - \gamma)(1 + (W - 2)q)) & \text{if } W > 1.
\end{cases}
\]
As the costs and benefits of deviating are equal the minority type has no incentive to deviate. Moreover, since \( \phi > \hat{\phi} \) this implies that the majority type strictly prefers \((W - w_*(\delta, \phi))\) to \((\min\{W, 1\}, 0)\).

Now consider the beliefs after \( w^B = w' \in (0, w_*(\delta, \phi)) \). Note that by construction the minority type is indifferent between \( w^B = 0 \) and \( w^B = w_*(\delta, \phi) \) in the initial period. Hence, by Lemma 4, the set of beliefs for which the majority type would have a weak incentive to deviate to \( w^B = w' \) are a proper subset of those for which the minority type would have a strict incentive to deviate, and so the voters must infer that a politician who chose any \( w' \in (0, w_*(\delta, \phi)) \) is the minority type with certainty. As the minority type would then prefer to implement \((\min\{W, 1\}, 0)\) to any allocation with \( w^B > 0 \) the minority type, and hence also the majority type, would have a strict incentive not to choose \( w' \in (0, w_*(\delta, \phi)) \). As such, the above strategies constitute a Perfect Bayesian Equilibrium which satisfies criterion D1.

Having now established that the above strategies constitute a D1 equilibrium we now turn to showing that no other separating equilibrium satisfies Criterion D1. Consider a Perfect Bayesian Equilibrium in which \( w^B = \hat{w} > w_*(\delta, \phi) \). Now consider the effort allocation \( w^B = w' \in (w_*(\delta, \phi), \hat{w}), w^A = W - w' \). We show that such an allocation is equilibrium dominated for the minority type, but not the majority type. Consider first the minority type. We have shown that a minority type politician is indifferent between choosing \( w^B = w_*, w^A = W - w_* \) and being re-elected with probability \( X(\varepsilon, W) \) and \((\min\{W, 1\}, 0)\) with probability \( Y(\varepsilon, W) \). Further, as the minority type strictly prefers the allocation \( w^B = w_*, w^A = W - w_* \) to \( w^B = w', w^A = W - w' \), she would then have a strict incentive not to choose \( w^B = w', w^A = W - w' \) for any voter beliefs. So \( w^B = w', w^A = W - w' \) is equilibrium dominated for the minority type. Now consider the majority type. Note first that the politician prefers allocation \( w^B = w', w^A = W - w' \) to \( w^B = \hat{w}, w^A = W - \hat{w} \) in period 1, so if the beliefs were such that she would be re-elected with probability \( X(\varepsilon, W) \) by choosing \( w^B = w', w^A = W - w' \) she would have an incentive to choose that allocation. Therefore, \( w^B = w', w^A = W - w' \) is equilibrium dominated for the minority type, but not the majority type, and so the voters must believe that any politician who took that action was the majority type with certainty. Hence, after observing allocation \( w^B = w' \in (w_*(\delta, \phi), \hat{w}), w^A = W - w' \) voters must believe the incumbent is the majority type with certainty so the probability of re-election is the same as from choosing \( w^B = \hat{w} \) and \( w^A = W - \hat{w} \). But, as the politician receives greater utility in the first period by increasing \( w^A \).
and decreasing $w^B$, she would not be optimizing by choosing $w^B = w'$. We can then conclude that it is not possible to support a separating equilibrium with $w^B > w_*(\delta, \phi)$ that is consistent with D1.

Finally, note that $w_* (\delta, \phi)$ is increasing in $\phi$, and, in order to have an equilibrium, we must have $w_* (\delta, \phi) \leq \min \{ W, 1 \}$. As $w_* (\delta, \phi_1) = \min \{ W, 1 \}$, we can conclude that there exists a separating equilibrium if and only if

$$\phi \leq \phi_1 \equiv \begin{cases} \frac{W}{\delta(x(\epsilon, W) - Y(\epsilon, W))} - (1 - \gamma)(1 - q)m^P W & \text{if } W \leq 1, \\ \frac{1 - (W - 1) \gamma}{\delta(x(\epsilon, W) - Y(\epsilon, W))} - m^P (1 - \gamma)(1 + (W - 2)q) & \text{if } W > 1. \end{cases}$$

Defining $\tilde{\phi} = \max \{ \phi_1, \hat{\phi} \}$ we have that a separating equilibrium exists if and only if $\phi \in (\hat{\phi}, \phi_1]$.

We now consider the conditions under which $\phi_1 > \hat{\phi}$. Recall from equation (1) that

$$\hat{\phi}(\epsilon, W) \equiv \begin{cases} \max \{ \frac{1 - q}{2} (2\gamma m^P - 1) W, 0 \} & \text{if } W \leq 1, \\ \max \{ \frac{1 + (W - 2)q}{2} (2\gamma m^P - 1), 0 \} & \text{if } W > 1, \end{cases}$$

When $W \leq 1$, notice that $\phi_1 > \hat{\phi}$ if and only if

$$1 - q(2\gamma m^P - 1) W < \gamma m^P W < W - (1 - \gamma)m^P W < \frac{2W}{\delta (1 - 2Y(\epsilon, W))} - (1 - \gamma)(1 - q)m^P W,$$

which follows immediately because

$$\frac{1 - q}{2} (2\gamma m^P - 1) W < \gamma m^P W < W - (1 - \gamma)m^P W < \frac{2W}{\delta (1 - 2Y(\epsilon, W))} - (1 - \gamma)(1 - q)m^P W.$$

Similarly, when $W > 1$ then $\phi_1 > \hat{\phi}$ if and only if

$$1 + (W - 2)q(2\gamma m^P - 1) < \frac{2(1 - (W - 1) \gamma)}{\delta(1 - 2Y(\epsilon, W))} - m^P (1 - \gamma)(1 + (W - 2)q),$$

or equivalently

$$(1 + (W - 2)q)(m^P - 1/2) < \frac{2(1 - (W - 1) \gamma)}{\delta(1 - 2Y(\epsilon, W))}.$$

As this inequality holds strictly when $W = 1$, by continuity there exists $\bar{W} \in (1, 2]$ such that this inequality is satisfied for all $W < \bar{W}$. Finally, since $W < 2$ and $2Y(\epsilon, W) > 0$ this inequality is satisfied for all $W \in (0, 2)$ if

$$\gamma < \bar{\gamma} \equiv 1 - \frac{(m^P - 1/2) \delta}{2} \in \left( \frac{1}{2}, 1 \right).$$

We can then conclude that there exists a $\bar{W} \in (1, 2]$ and $\bar{\gamma} \in (\frac{1}{2}, 1)$ such that $\phi_1 > \hat{\phi}$ for all $W \in (0, \bar{W})$ and $\bar{W} = 2$ if $\gamma < \bar{\gamma}$.
So the only separating equilibrium to satisfy criterion D1 is the minimally separating one, and such an equilibrium exists only if re-election pressures are not too strong. We now consider the possibility of a pooling equilibrium.

**Lemma 7.** Suppose $\phi > \hat{\phi}(\varepsilon, W)$. There exists a pooling equilibrium that satisfies Criterion D1 on off-path beliefs if and only if $\phi > \phi^* \equiv \max\{\phi_2, \hat{\phi}\}$ where

$$\phi_2 \equiv \begin{cases} 
\frac{2W}{\delta(1 - 2Y(\varepsilon, W))} - (1 - \gamma)(1 - q)m^P W & \text{if } W \leq 1, \\
\frac{2(1 - (W - 1)\gamma)}{\delta(1 - 2Y(\varepsilon, W))} - m^P (1 - \gamma)(1 + (W - 2)q) & \text{if } W > 1.
\end{cases}$$

In this equilibrium both types choose effort allocation $w^B = \min\{W, 1\}$ and $w^A = W - w^B$. Moreover, when $W \in (0, \bar{W})$, $\phi^* > \hat{\phi} > \phi$.

**Proof.** Since, in a pooling equilibrium, both politician types are re-elected with probability $1/2$, by Lemma 5 we cannot have a pooling equilibrium that satisfies D1 unless in that equilibrium all politicians choose $w^B = \min\{W, 1\}$ in period 1. We first determine the range of parameters for which a pooling equilibrium exists with $w^B = \min\{W, 1\}$ then verify that the beliefs satisfy criterion D1.

We show a pooling equilibrium with $w^B = \min\{W, 1\}$ can be supported if and only if $\phi \geq \phi^*$.

This follows because the maximum $w^B$ for which the minority type would rather choose allocation $(W - w^B, w^B)$ and be reelected with probability $1/2$ than be re-elected with probability $Y(\varepsilon, W)$ after choosing $(\min\{W, 1\}, 0)$ is

$$\bar{w}(\delta, \phi) = \begin{cases} 
\min\{\frac{\delta}{2}[\phi + (1 - q)m^P (1 - \gamma)W], W\} & \text{if } W \leq 1, \\
\min\{(W - 1)\gamma + \frac{\delta}{2}[\phi + m^P (1 - \gamma)(1 + (W - 2)q)], 1\} & \text{if } W > 1.
\end{cases}$$

Note that we have a pooling equilibrium if and only if $\bar{w}(\delta, \phi) \geq \min\{W, 1\}$, that $\bar{w}(\delta, \phi)$ is increasing in $\phi$, and that

$$\bar{w}(\delta, \phi_2) = \min\{W, 1\}.$$  

So a pooling equilibrium exists if and only if $\phi > \hat{\phi}$ and $\phi \geq \phi_2$. Hence, there exists a pooling equilibrium with $w^B = \min\{W, 1\}$ if and only if $\phi \geq \phi^*$.

Having now established that there exists a pooling equilibrium with $w^B = \min\{W, 1\}$ if and only if $\phi \geq \phi^*$, we now show that, when $\phi \geq \phi^*$, the beliefs supporting the equilibrium satisfy criterion D1. Now, by Lemma 4, the range of beliefs for which the minority type would have a strict incentive to choose any $w^B = w' < \min\{W, 1\}$ are a proper superset of those for which
the majority type would have a weak incentive to choose that allocation. Hence, in order to be consistent with criterion D1 the voters must believe any \( w^B < \min\{W,1\} \) was chosen by a minority type, and so the politician would be re-elected with probability \( Y(\varepsilon,W) \). As these are the most punitive beliefs the voters could hold after a deviation, the beliefs determined by criterion D1 are sufficient to support an equilibrium.

We can then conclude that, when \( \phi \geq \phi^* \), in the unique pooling equilibrium consistent with criterion D1 all politicians choose \( w^B = \min\{W,1\} \) in period 1 and, when \( \phi < \phi^* \), we cannot have a pooling equilibrium consistent with criterion D1. So a pooling equilibrium exists if and only if \( \phi \geq \phi^* = \max\{\phi_2,\hat{\phi}\} \). Finally, since \( \phi_2 > \phi_1 \), and \( \bar{\phi} = \phi_1 > \hat{\phi} \) when \( W \in (0,\bar{W}) \), it follows that \( \phi^* > \bar{\phi} > \hat{\phi} \) for all \( W \in (0,\bar{W}) \). □

So we have that when \( \phi \leq \bar{\phi} \) there exists a unique separating equilibrium but no pooling equilibrium that satisfies criterion D1 and when \( \phi \geq \phi^* \) there exists a unique pooling equilibrium, but no separating equilibrium. And, if \( \phi \in (\bar{\phi},\phi^*) \) neither a separating or pooling equilibrium consistent with D1 exists. We now explore the possibility of a semi-separating equilibrium. For this range, there exists a unique semi-separating equilibrium in which the minority-type randomizes so that the politician is re-elected with probability between 1/2 and \( X(\varepsilon,W) \) after choosing the posturing allocation.

**Lemma 8.** There exists a partial-pooling equilibrium that satisfies Criterion D1 if and only if \( \phi \in (\bar{\phi},\phi^*) \) and this equilibrium is unique. In this equilibrium, the majority type chooses \( w^B = \min\{W,1\}, w^A = W - w^B \) and the minority type randomizes with a non-degenerate probability between \( w^B = \min\{W,1\}, w^A = W - w^B \) and \( w^A = \min\{W,1\}, w^B = 0 \) in period 1.

**Proof.** By Lemma 5 we know that the equilibrium must either involve all majority types choosing \( w^B = \min\{W,1\} \) or have the majority type re-elected with probability \( X(\varepsilon,W) \). Since we cannot have a separating equilibrium, the majority type must choose \( w^B = \min\{W,1\} \) in period 1. Since the majority type always chooses \( w^B = \min\{W,1\}, w^A = W - w^B \), any other effort allocation would reveal the politician to be the minority type with certainty. Hence the equilibrium must involve the minority type randomizing between \( w^B = \min\{W,1\}, w^A = W - w^B \) and \( w^A = \min\{W,1\}, w^B = 0 \). Let \( \rho \in [0,1] \) be the probability with which the minority type takes action \( w^B = \min\{W,1\}, w^A = W - w^B \) and let \( \pi(\rho) \) be the associated probability of being re-elected
after the voter observes \( w^B = \min\{W, 1\}, w^A = W - w^B \). The voters’ updated beliefs are

\[
\mu(1|w^B = \min\{W, 1\}, w^A = W - w^B) = \frac{m^P}{m^P + (1 - m^P)\rho}
\]

As \( \mu(1|w^B = \min\{W, 1\}, w^A = W - w^B) \) is decreasing in \( \rho \) and equal to 1 when \( \rho = 0 \) and \( m^P \) when \( \rho = 1 \), the probability of re-election, \( \pi(\rho) \), is decreasing in \( \rho \) with \( \pi(0) = X(\varepsilon, W) \) and \( \pi(1) = 1/2 \). We now show that we have a solution with \( \rho \in (0, 1) \) if and only if \( \phi \in (\bar{\phi}, \phi^*) \), and that the probability of randomization is unique. In order for the minority type to be willing to randomize we must have that

\[
\pi(\rho) - Y(\varepsilon, W) = \begin{cases} 
\delta \phi + (1 - \gamma)(1 - \gamma)m^P W & \text{if } W \leq 1, \\
\delta \phi + m^P (1 - \gamma)(1 + (W - 2)\gamma) & \text{if } W > 1.
\end{cases}
\]

Notice that the left hand side of this expression is decreasing in \( \rho \) and the right hand side is constant. Note also that, when \( \rho = 0 \), \( \pi(\rho) = X(\varepsilon, W) \), and so when \( \phi > \bar{\phi} \),

\[
\pi(0) - Y(\varepsilon, W) > \begin{cases} 
\delta \phi + (1 - \gamma)(1 - \gamma)m^P W & \text{if } W \leq 1, \\
\delta \phi + m^P (1 - \gamma)(1 + (W - 2)\gamma) & \text{if } W > 1,
\end{cases}
\]

and, when \( \rho = 1 \), \( \pi(\rho) = 1/2 \), and so when \( \phi < \phi^* \),

\[
\pi(1) - Y(\varepsilon, W) < \begin{cases} 
\delta \phi + (1 - \gamma)(1 - \gamma)m^P W & \text{if } W \leq 1, \\
\delta \phi + m^P (1 - \gamma)(1 + (W - 2)\gamma) & \text{if } W > 1.
\end{cases}
\]

Hence, there exists a unique solution with \( \rho \in (0, 1) \) when \( \phi \in (\bar{\phi}, \phi^*) \) and no solution otherwise. Hence we can conclude that there exists a unique partial pooling equilibrium if \( \phi \in (\bar{\phi}, \phi^*) \), and there does not exist a partial pooling equilibrium otherwise.

We have now established that, for \( \phi \in (\bar{\phi}, \bar{\phi}) \) the only equilibrium to satisfy criterion D1 is the minimally separating equilibrium. When \( \phi \geq \phi^* \) the unique equilibrium to satisfy D1 is the pooling equilibrium. And when \( \phi \in (\bar{\phi}, \phi^*) \) the unique equilibrium is partial-pooling. Combining these lemmas then proves Proposition 1.

**Proof of Proposition 1** Follows immediately by combining Lemmas 6 – 8. □

**Proof of Proposition 2** Recall first that \( \lim_{\varepsilon \to 0} X(\varepsilon, W) = 1 \) and \( \lim_{\varepsilon \to 0} Y(\varepsilon, W) = 0 \) and that, when \( W \in (0, W) \), we have \( \bar{\phi}(\varepsilon, W) = \phi_1(\varepsilon, W) \) and \( \phi^*(\varepsilon, W) = \phi_2(\varepsilon, W) \), where \( \phi_1 \) and \( \phi_2 \) are
defined in equations (5) and (6). So we can see immediately that
\[
\bar{\varphi}_0(W) = \lim_{\varepsilon \to 0} \varphi_1(\varepsilon, W) = \begin{cases} 
\frac{W}{\delta} - (1 - \gamma)(1 - q)mPW & \text{if } W \leq 1, \\
\frac{1}{1 - (W - 1)\gamma} - mP(1 - \gamma)(1 + (W - 2)q) & \text{if } W > 1,
\end{cases}
\]
and
\[
\phi_0^*(W) = \lim_{\varepsilon \to 0} \varphi_2(\varepsilon, W) = \begin{cases} 
\frac{2W}{\delta} - (1 - \gamma)(1 - q)mPW & \text{if } W \leq 1, \\
\frac{2(1 - (W - 1)\gamma)}{\delta} - mP(1 - \gamma)(1 + (W - 2)q) & \text{if } W > 1.
\end{cases}
\]
Differentiating with respect to \( W \),
\[
\frac{\partial \bar{\varphi}_0(W)}{\partial W} = \begin{cases} 
\frac{1}{\delta} - (1 - \gamma)(1 - q)mP & \text{if } W \leq 1, \\
-\frac{1}{\delta} - mP(1 - \gamma)q & \text{if } W > 1,
\end{cases}
\]
and
\[
\frac{\partial \phi^*(W)}{\partial W} = \begin{cases} 
\frac{2}{\delta} - (1 - \gamma)(1 - q)mP & \text{if } W \leq 1, \\
-\frac{2}{\delta} - mP(1 - \gamma)q & \text{if } W > 1.
\end{cases}
\]
It then follows by inspection that \( \bar{\varphi}_0(W) \) and \( \phi_0^*(W) \) are increasing in \( W \) on \((0, 1]\) and decreasing on \((1, \bar{W})\). □

**Proof of Results with Non-Transparency.** Proof of Proposition 3: We begin by defining
\[
\phi_0^{A0}(\varepsilon, W) = \frac{2}{\delta(1 - 2Y(\varepsilon, W))} - (1 - q)(1 - \gamma)mPW \geq \phi^*(\varepsilon, W).
\]
For there to be an equilibrium of the form described, the minority type must be indifferent between \( p^B = 1 \) and \( p^A = 0 \) and prefer either alternative to \( p^A = 1 \). In order to have the minority type indifferent between \( B \) and 0 it must be that
\[
1 - \gamma = (\pi_B - \pi_0)\delta[\phi + (1 - q)(1 - \gamma)mPW]
\]
where \( \pi_i \) is the probability of being reelected after outcome \( i \). Assume that after seeing \( p^A = 1 \), the incumbent is re-elected with probability \( \pi_A = Y(\varepsilon, W) \) regardless of \( p^B \); after seeing nothing, reelected with prob \( \pi_0 \); after \( B \), reelected with prob \( \pi_B \). The above indifference condition is equivalent to
\[
\pi_B - \pi_0 = \frac{1 - \gamma}{\delta[\phi + (1 - q)(1 - \gamma)mPW]}.
\]
Note that in order to have an equilibrium, in addition to the above indifference condition, we must have that neither type wants to exert effort on \( A \), and that the majority type prefers \( B \).
to doing nothing. Note however that this implies that $\pi_B > \pi_0$, and since $\phi > \phi_0^{A0}$ implies that $\phi > \hat{\phi}$, if the minority type is optimizing it means that the majority type is as well.

We first show that there exists a unique $w^B$ such that the minority type is indifferent between $B$ and $0$. To see this, note that the right hand side is constant in $w^B$ but

$$
\mu(p^A = 0, p^B = 1) = \frac{m^P W}{m^P W + (1 - m^P)w^B}
$$

is decreasing and

$$
\mu(p^A = 0, p^B = 0) = \frac{m^P (1 - W)}{m^P (1 - W) + (1 - m^P)(1 - w^B)}
$$

is increasing in $w^B$. As the probability of re-election is increasing in the probability perceived to be the majority type, $\pi_B - \pi_0$ is decreasing in $w^B$. Furthermore, evaluating at $w^B = 0$ and $w^B = W$, we see that $\pi_B - \pi_0$ is greater than $X(\varepsilon, W) - 1/2$ when $w^B = 0$ and equal to 0 when $w^B = W$. Moreover, by equations (3) and (4), we know that when $\phi > \phi^{A0}$,

$$
X(\varepsilon, W) - 1/2 \geq 1/2 - Y(\varepsilon, W) > \frac{1 - \gamma}{\delta[\phi + (1 - q)(1 - \gamma)m^P W]} > 0.
$$

So by the intermediate value theorem we have a solution $w^B \in (0, W)$, and since $\pi_B - \pi_0$ is decreasing this solution is unique.

As the probability of re-election after $p^A = 1$ is $\pi_A = Y(\varepsilon, W)$, to show that the minority type would not want to deviate to $A$ it is sufficient to show that

$$
\pi_B - Y(\varepsilon, W) \geq \frac{1}{\delta[\phi + (1 - q)(1 - \gamma)m^P W]}.
$$

But, as $\pi_B > 1/2$ and

$$
\phi > \phi^{A0} \equiv \frac{2}{\delta(1 - 2Y(\varepsilon, W))} - (1 - q)(1 - \gamma)m^P W,
$$

this follows immediately. We have then established that there exists an equilibrium of the described form.

We now turn to showing that this is the unique pure strategy equilibrium of the specified form. We rule out all other equilibria by contradiction. There are three possibilities to rule out: the minority type chooses $w^B = W$, the minority type chooses $w^A = w^B = 0$, and the minority type chooses $w^A > 0$. Note, however, that we have already seen that $w^B = W$ and $w^A = 0$ cannot be
an equilibrium. If \( w^B = W \) then
\[
\pi_B - \pi_0 = 0 < \frac{1 - \gamma}{\delta[\phi + (1 - q)(1 - \gamma)mPW]},
\]
so the minority type prefers \( p^B = 0 \) to \( p^B = 1 \) and so would benefit from reducing effort on \( B \). Similarly, if \( w^A = w^B = 0 \) then
\[
\pi_B - \pi_0 > X(\varepsilon, W) - \frac{1}{2} > \frac{1 - \gamma}{\delta[\phi + (1 - q)(1 - \gamma)mPW]}
\]
and the minority type would benefit from increasing effort on \( B \).

We conclude by showing that we cannot have a pure-strategy equilibrium in which the minority type chooses \( w^A > 0 \). Suppose there is an equilibrium in which the minority type chooses \( w^A > 0 \) on issue \( A \) and \( w^B \in [0, W - w^A] \) on issue \( B \) and the majority type’s effort allocation is \((0, W)\). As \( p^A = 1 \) never happens when the incumbent is the majority type we have that the probability of re-election after \( p^A = 1 \), regardless of \( p^B \) is \( \pi_A = Y(\varepsilon, W) \). Next, note that by Bayes’s rule,
\[
\mu(p^A = 0, p^B = 0) = \frac{m^P(1 - W)}{m^P(1 - W) + (1 - m^P)(1 - w^A)(1 - w^B)} < m^P,
\]
\[
\mu(p^A = 0, p^B = 1) = \frac{m^PW}{m^PW + (1 - m^P)(1 - w^A)w^B} > m^P.
\]
Hence, the probability of re-election after \( p^A = 0, p^B = 1 \) is \( \pi_B > 1/2 \), and the probability of re-election after \( p^A = p^B = 0 \) is \( \pi_0 < 1/2 \). Note that the minority type’s first period payoff is
\[
-(1 - w^A)\gamma - w^B(1 - \gamma),
\]
and the probability of re-election is
\[
w^AY(\varepsilon, W) + (1 - w^A)w^B\pi_B + (1 - w^A)(1 - w^B)\pi_0 < w^AY(\varepsilon, W) + w^B\pi_B + (1 - w^A - w^B)\pi_0.
\]
If she deviates to effort allocation \((0, w^A + w^B)\) her first period payoff is
\[
-(\gamma - (w^A + w^B)(1 - \gamma),
\]
and the probability of re-election is
\[
(w^A + w^B)\pi_B + (1 - w^A - w^B)\pi_0.
\]
So, in order for \((0, w^A + w^B)\) not to be a profitable deviation we must have
\[
w^A > w^A(\pi_B - Y(\varepsilon, W))(\phi + (1 - q)m^P(1 - \gamma)W) > w^A\left(\frac{1}{2} - Y(\varepsilon, W)\right)(\phi + (1 - q)m^P(1 - \gamma)W),
\]

which contradicts the assumption that $\phi > \phi^A_0$. As such, we cannot have an equilibrium in which $w^A > 0$.

We can then conclude that when $\phi > \phi^A_0$ the minority type chooses $w^A = 0$ and $w^B < W$ in the unique pure strategy equilibrium. □

**Proof of Proposition 4:** We begin by showing that $\phi^*_{NA}$ from equation (2) is larger than $\phi^*$. By equations (1) and (6) we have that

$$\phi^* = \max \left\{ \frac{1 + (W - 2)q}{2} (2\gamma m^P - 1), \frac{2(1 - (W - 1)\gamma)}{\delta(1 - 2Y(\varepsilon,W))} - m^P(1 - \gamma)(1 + (W - 2)q) \right\}.$$  

Note immediately that

$$\frac{2(1 - (W - 1)\gamma)}{\delta(1 - 2Y(\varepsilon,W))} - m^P(1 - \gamma)(1 + (W - 2)q) < \frac{2}{\delta(1 - 2Y(\varepsilon,W))} - m^P(1 - \gamma)(1 + (W - 2)q) = \phi^*_{NA},$$

and

$$\frac{1 + (W - 2)q}{2} (2\gamma m^P - 1) < \gamma m^P < \frac{2}{\delta(1 - 2Y(\varepsilon,W))} - m^P(1 - \gamma) < \phi^*_{NA}.$$  

As $\phi^*$ is the maximum of the left hand side of the above inequalities, $\phi^*_{NA} > \phi^*$ as claimed.

We show that we can support a pooling equilibrium in which the minority type chooses allocation $w^B = 1, w^A = W - 1$ if and only if $\phi \geq \phi^*_{NA}$. First note that, given that in the purported equilibrium the incumbent is re-elected with probability $1/2$ if $p^B = 1$ regardless of $p^A$. The harshest possible beliefs, to support such an equilibrium, induce re-election probability $Y(\varepsilon,W)$ when $p^B = 0$. So, in order to prevent the minority type from deviating to $(w^A = 1, w^B = W - 1)$, we must have

$$(2 - W) \leq \delta(2 - W) \left( \frac{1}{2} - Y(\varepsilon,W) \right) (\phi + m^P(1 - \gamma)(1 + q(W - 2))),$$

or, equivalently, $\phi \geq \phi^*_{NA}$. So the above strategies cannot constitute an equilibrium if $\phi < \phi^*_{NA}$.

We now verify that there is no other profitable deviation, given these beliefs, if $\phi \geq \phi^*_{NA}$. In equilibrium, as we have pooling behavior, the voter does not update based on $p^A$. Hence, the probability of re-election is $1/2$ if $p^B = 1$ and $Y(\varepsilon,W)$ if $p^B = 0$. Suppose the minority type deviates to $(w^A, w^B)$ where $w^B < 1$. Then the benefit in terms of first period payoff is

$$(w^A + 1 - W)\gamma + (1 - w^B)(1 - \gamma) \leq (1 - w^B).$$

The cost, in terms of forgone re-election probability is

$$\delta(1 - w^B) \left( \frac{1}{2} - Y(\varepsilon,W) \right) (\phi + m^P(1 - \gamma)(1 + q(W - 2))).$$
Hence, this deviation is not profitable if

$$\delta \left(\frac{1}{2} - Y(\varepsilon, W)\right) (\phi + m^P(1 - \gamma)(1 + q(W - 2)) \geq 1,$$

or, equivalently, $$\phi \geq \phi_{NA}^*.$$ Hence, we have an equilibrium with both types pooling on $$(W - 1, 1)$$ if and only if $$\phi \geq \phi_{NA}^*.$$

We now show that there exists $$\phi^{**} < \phi_{NA}^*$$ such that, for all $$\phi \in (\phi^{**}, \phi_{NA}^*),$$ there exists an equilibrium in which the majority type chooses allocation $$(W - 1, 1)$$ and the minority type chooses effort allocation $$(1, W - 1)$$ with probability $$r \in (0, 1],$$ and allocation $$(W - 1, 1)$$ otherwise. Note that, given the above strategies,

$$\mu(p^A = 1, p^B = 1) = m^P,$$

$$\mu(p^A, p^B = 0) = 0,$$

$$\mu(p^A = 0, p^B = 1; r) = \frac{m^P(1 - W)}{m^P(1 - W) + (1 - m^P)(1 - r)(1 - W)} = \frac{m^P}{m^P + (1 - m^P)(1 - r)}.$$

Notice that $$\mu(0, 1; r)$$ is increasing in $$r$$ and $$\mu(0, 1; 1) = 1.$$ We first show that the majority and minority type are optimizing conditional on choosing allocation $$(w^A, w^B) \in \{(1, W - 1), (W - 1, 1)\}.$$ For each $$r$$ we then have $$\pi_A = \pi_0 = Y(\varepsilon, W), \pi_{AB} = 1/2,$$ and $$\pi_B = \pi(r),$$ where $$\pi(r)$$ is increasing in $$r$$ with $$\pi(0) = 1/2$$ and $$\pi(1) = X(\varepsilon, W).$$ Now note that, in order to have the minority type willing to randomize we must have that her payoff from choosing $$(1, W - 1)$$ is the same as $$(W - 1, 1).$$ As the difference in first period utility is $$W - 1,$$ and the change in re-election probability is $$(W - 1)(\pi_B - \pi_A)$$ randomization is optimal if and only if

$$1 = (\pi(r) - Y(\varepsilon))(\phi + m^P(1 - \gamma)(1 + q(W - 2)).$$

Note that, since $$\phi < \phi_{NA}^*$$ we have

$$1 > \delta \left(\frac{1}{2} - Y(\varepsilon, W)\right) (\phi + m^P(1 + q(W - 2)) = \delta(\pi(0) - Y(\varepsilon, W))(\phi + m^P(1 - \gamma)(1 + q(W - 2)),$$

and since $$\pi(r)$$ is increasing, we have a solution with $$r \in (0, 1)$$ if and only if

$$1 < \delta(\pi(1) - Y(\varepsilon, W))(\phi + m^P(1 + q(W - 2)) = \delta(X(\varepsilon, W) - Y(\varepsilon, W))(\phi + m^P(1 - \gamma)(1 + q(W - 2)).$$

So if $$\phi \geq \frac{1}{\delta(X(\varepsilon, W) - Y(\varepsilon, W))} - m^P(1 + q(W - 2))$$ there exists a unique $$r \in (0, 1)$$ to make the minority type indifferent. If $$\phi < \frac{1}{\delta(X(\varepsilon, W) - Y(\varepsilon, W))} - m^P(1 - \gamma)(1 + q(W - 2))$$ then the minority type’s strategy must involve $$r = 1.$$
Now note that, if the minority type is randomizing, the majority type has a strict preference for allocation \((W - 1, 1)\) over \((1, W - 1)\). If \(\phi < \frac{1}{\delta(X(\varepsilon, W) - Y(\varepsilon, W))} - m^P(1 + q(W - 2))\) and so \(r = 1\), we must check that the majority type prefers allocation \((W - 1, 1)\) to \((1, W - 1)\). This requires that
\[
2\gamma - 1 \leq \delta(\pi(1) - Y(\varepsilon, W))(\phi + m^P(1 - \gamma)(1 + q(W - 2))).
\]
Notice that this is satisfied if and only if
\[
\phi > \frac{2\gamma - 1}{\delta(X(\varepsilon, W) - Y(\varepsilon, W))} - (1 - m^P)(1 - \gamma)(1 + q(W - 2)),
\]
which, when \(\phi > \phi^\ast\) is strictly lower than \(\frac{1}{\delta(X(\varepsilon, W) - Y(\varepsilon, W))} - m^P(1 - \gamma)(1 + q(W - 2))\) or \(\phi^\ast_{NA}\).

Defining
\[
\phi^{**} = \max\{\phi^*, \frac{2\gamma - 1}{\delta(\frac{1}{2} - Y(\varepsilon, W))} - (1 - m^P)(1 - \gamma)(1 + q(W - 2))\},
\]
we have established that neither the majority or minority type wants to deviate between \((W - 1, 1)\) and \((1, W - 1)\) when \(\phi > \phi^{**}\). It is immediate that \(\phi^{**} < \phi^*_{NA}\).

We now must show that neither the majority or minority type can benefit from deviating to \((w^A, w^B) \notin \{(1, W - 1), (W - 1, 1)\}\) when \(\phi \in (\phi^{**}, \phi^*_{NA})\). Recall that under the above strategies \(\pi_0 = \pi_A = Y(\varepsilon, W), \pi_{AB} = 1/2, \text{ and } \pi_B \in (1/2, X(\varepsilon, W))\). So the probability of re-election from choosing \(w^A, w^B\) is
\[
w^B(1 - w^A)\pi_B + w^A w^B \pi_{AB} + w^A(1 - w^B)\pi_A + (1 - w^A)(1 - w^B)\pi_0 =
\]
\[
w^B(1 - w^A)\pi_B + \frac{1}{2} w^A w^B + (1 - w^B)Y(\varepsilon, W).
\]

To see that the minority type has no incentive to choose \((w^A, w^B) \notin \{(1, W - 1), (W - 1, 1)\}\), note that if the minority type chooses allocation \((w^A, w^B)\) her first period payoff is
\[
-\gamma(1 - w^A) - (1 - \gamma)w^B.
\]
Her first period payoff from allocation \((1, W - 1)\) however is \(-(1 - \gamma)(W - 1)\) and her re-election probability is \((W - 1)\frac{1}{2} + (2 - W)Y(\varepsilon, W)\). Hence, for the minority type to prefer \((w^A, w^B)\) to \((1, W - 1)\) requires that
\[
\gamma(1 - w^A) + (1 - \gamma)(1 + w^B - W) <
\]
\[
\delta(w^B(1 - w^A)\pi_B + (1 + w^A w^B - W)\frac{1}{2} - (1 + w^B - W)Y(\varepsilon, W))(\phi + m^P(1 - \gamma)(1 + q(W - 2))).
\]
To verify that this inequality is violated for all $w^A$ it is sufficient to check for $w^A = \min\{1, W - w^B\}$. Note that when $w^A = W - w^B$ this inequality reduces to

$$(1+w^B-W) < (1+w^B-W)(w^B(\pi_B-Y(\varepsilon, W))+(1-w^B)(\frac{1}{2}-Y(\varepsilon, W)))(\phi+m^P(1-\gamma)(1+q(W-2))).$$

But because the minority type weakly prefers $(1, W-1)$ to $(W-1, 1)$, and $\phi < \phi_{NA}^*$, this inequality cannot be satisfied. Similarly, when $w^A = 1$, since $1 + w^B - W \leq 0$ this reduces to

$$(1-\gamma) > \left(\frac{1}{2} - Y(\varepsilon, W)(\phi + m^P(1-\gamma)(1+q(W-2)))\right),$$

which violates the assumption that $\phi > \phi^{**} \geq \phi^*$. So it is not optimal for the minority type to deviate to any $(w^A, w^B) \notin \{(1, W-1), (W-1, 1)\}$ when $\phi \in (\phi^{**}, \phi_{NA}^*)$.

We conclude by considering the majority type. The majority type’s first period payoff from allocation $(w^A, w^B)$ is

$$-\gamma(1-w^A) - (1-\gamma)w^B,$$

while the first period payoff from $(W-1, 1)$ is $-\gamma(2-W)$ with associated re-election probability $(W-1)^{\frac{1}{2}} + (2-W)\pi_B$. Note that, as the majority type’s first period payoff, and re-election probability, are both increasing in $w^B$ we can restrict attention to cases in which $w^B = \min\{1, W - w^A\}$ without loss of generality. The majority type would only benefit from deviating if $\gamma(W-1-w^A) + (1-\gamma)(1-w^B)$ is strictly less than

$$\delta[(w^B(1-w^A) + W - 2)\pi_B + (w^A w^B + 1 - W)\frac{1}{2} + (1 - w^B)Y(\varepsilon, W)]$$

$$(\phi + (1-m^P)(1-\gamma)(1+q(W-2))).$$

Note first that when $w^B = 1$ this reduces to

$$\gamma(W-1-w^A) < \delta(\pi_B - \frac{1}{2})(W-1-w^A)(\phi + (1-m^P)(1-\gamma)(1+q(W-2)))$$

which, because $\gamma > 1/2$, can’t be satisfied when $\phi < \phi_{NA}^*$. And, if $w^B = W - w^A$, using the fact that $\pi_B > 1/2$, it implies that

$$(2\gamma - 1)(1 + w^A - W) > (1 + w^A - W)(\frac{1}{2} - Y(\varepsilon, W))(\phi + (1-m^P)(1-\gamma)(1+q(W-2))),$$

which can’t be satisfied when $\phi > \phi^{**}$. So the majority type doesn’t have an incentive to deviate to any $(w^A, w^B) \notin \{(1, W-1), (W-1, 1)\}$. 


We can then conclude that it is an equilibrium for both types to choose \((W - 1, 1)\) if and only if 
\[ \phi \geq \phi_{NA}^* > \phi^* , \]  
and when \( \phi \in (\phi^{**}, \phi_{NA}^*) \) it is an equilibrium for the minority type to randomize 
between \((W - 1, 1)\) and \((1, W - 1)\) while the majority type chooses \((W - 1, 1)\). \( \square \)
Proof of Lemma 3: Suppose there exists an allocation \((w^A_*, w^B_*)\) chosen with positive probability on the equilibrium path by the majority type, and suppose \(w^A_* + w^B_* < W\). Let \(\pi^* \in [Y(\varepsilon, W), X(\varepsilon, W)]\) be the probability with which the politician is re-elected after choosing \((w^A_*, w^B_*)\). Now consider a different allocation \((w', w'')\) with \((w' \geq w^A_*, w'' \geq w^B_*)\), and at least one of the inequalities strict. Now define \(u^x(w^A, w^B, \pi)\) to be the utilities to the politicians of each type, \(x \in \{0, 1\}\), from implementing a given policy \((w^A, w^B)\) if the probability of re-election after choosing policy \((w^A, w^B)\) is \(\pi\). Now define

\[
\pi_1 = \inf \{ \pi' : u^1(w', w'', \pi') > u^1(w^A_*, w^B_*, \pi^*) \}
\]

and

\[
\pi_2 = \min \{ \pi' : u^0(w', w'', \pi') \geq u^0(w^A_*, w^B_*, \pi^*) \}
\]

Then \(\pi_1\) defines the probability of re-election for which the majority type would have a strict incentive to choose \((w', w'')\) if \(\pi > \pi_1\). Similarly \(\pi_2\) defines the minimum probability of re-election for which the minority type would have a weak incentive to choose \((w', w'')\).

We now show that \(\pi_1 < \pi_2\). First, note that \(u^1(w', w'', \pi') > u^1(w^A_*, w^B_*, \pi^*)\) if and only if

\[
(1 - \gamma)(w' - w^A_*) + \gamma(w'' - w^B_*) > \delta(\pi^* - \pi')[\phi + (1 - q)(1 - m^P)(1 - \gamma)W],
\]

when \(W \leq 1\), and if and only if

\[
(1 - \gamma)(w' - w^A_*) + \gamma(w'' - w^B_*) > \delta(\pi^* - \pi')[\phi + (1 - m^P)(1 - \gamma)(1 + q(W - 2))],
\]

when \(W > 1\). Conversely, \(u^0(w', w'', \pi') > u^0(w^A_*, w^B_*, \pi^*)\) if and only if

\[
(1 - \gamma)(w' - w^A_*) + \gamma(w'' - w^B_*) > \delta(\pi^* - \pi')[\phi + (1 - q)m^P(1 - \gamma)W],
\]

when \(W \leq 1\), and if and only if

\[
(1 - \gamma)(w' - w^A_*) + \gamma(w^B_ - w^B_*) > \delta(\pi^* - \pi')[\phi + m^P(1 - \gamma)(1 + q(W - 2))],
\]

when \(W > 1\). Now since \(w' \geq w^A_*, w'' \geq w^B_*\), with at least one inequality strict, we can see immediately that \((1 - \gamma)(w' - w^A_*) + \gamma(w'' - w^B_*) > 0\), and so \(\pi_1 < \pi^* \leq X(\varepsilon, W)\). Similarly, because

\[
\phi + (1 - q)m^P(1 - \gamma)W > \phi + (1 - q)(1 - m^P)(1 - \gamma)W,
\]

\[
\phi + (1 + q(W - 2))m^P(1 - \gamma)W > \phi + (1 + q(W - 2))(1 - m^P)(1 - \gamma)W,
\]
and

\[(1 - \gamma)(w' - w^A) + \gamma(w'' - w^B) \geq (1 - \gamma)(w' - w^A) + \gamma(w^B - w') ,\]

we must have \(\pi_1 < \pi_2\). So we can conclude that \(\pi_1 < \pi_2\).

Finally, given that \(\pi_1 < \pi_2\), note we cannot have an equilibrium satisfying criterion D1, in which the majority type ever chooses \((w^A, w^B)\). To see this, note that \((w', w'')\) cannot be on path: As \(\pi_1 < \min\{\pi_2, X(\varepsilon, W)\}\), if the majority type ever chooses \((w^A, w^B)\) over \((w', w'')\) the minority type must strictly prefer \((w^A, w^B)\) over \((w', w'')\) and so the minority type can never choose \((w', w'')\). As the voters would then assign beliefs that the politician is the majority type with certainty, she would be re-elected with probability \(X(\varepsilon, W)\), and, as \(\pi_1 < X(\varepsilon, W)\), the politician would have a strict incentive to choose \((w', w'')\) over \((w^A, w^B)\). Further, \((w', w'')\) cannot be off the equilibrium path – if it were, by criterion D1 the voters must believe the politician is the majority type with probability 1. As the probability of re-election would then be \(X(\varepsilon, W)\), the majority type would have a strict incentive to deviate to \((w', w'')\).

So we can conclude that in any PBE with criterion D1 the majority type must choose \(w^A + w^B = W\) in period 1. □

**Proof of Lemma 4:** Consider an allocation \(w^B > 0\) and \(w^A = W - w^B\) and another allocation \(w', w''\) where \(w'' < w^B\), and let \(\pi \in [Y(\varepsilon, W), X(\varepsilon, W)]\) be the probability of being re-elected by implementing \(w^B > 0, w^A = W - w^B\). We must show that, the set of beliefs the voters could hold after observing \(w', w''\) for which the minority would prefer \((w', w'')\) to \(w^B > 0, w^A = W - w^B\) is either a proper superset of the beliefs for which the majority type would weakly prefer \((w', w'')\), or alternatively that, for both types, \((w', w'')\) is preferred for either all beliefs, or for no beliefs, that the voters could hold.

We prove this separately for the case in which \(W \leq 1\) and when \(W > 1\). Consider first the case in which \(W \leq 1\). Then the minority type would have a strict incentive to implement \((w', w'')\) if and only if the re-election probability \(\pi'\) is such that

\[(w' + w^B - W)\gamma + (w^B - w'')(1 - \gamma) > \delta(\pi - \pi')[\phi + (1 - \gamma)(1 - q)m^PW] ,\]

or equivalently

\[\pi' - \pi > \pi_0 \equiv \frac{-w' + w^B - W)\gamma - (w^B - w'')(1 - \gamma)}{\delta[\phi + (1 - \gamma)(1 - q)m^PW]} .\]
Now consider the majority type. She will have a weak incentive to prefer \((w', w'')\) if and only if the re-election probability \(\pi'\) is such that
\[
(w' + w^B - W)\gamma - (w^B - w'') (1 - \gamma) \geq \delta(\pi - \pi')[\phi + (1 - \gamma)(1 - q)(1 - mP)W],
\]
or equivalently
\[
\pi' - \pi \geq \pi_1 \equiv \frac{-(w' + w^B - W)\gamma + (w^B - w'') (1 - \gamma)}{\delta[\phi + (1 - \gamma)(1 - q)(1 - mP)W]}.
\]

We now show that \(\pi_0 < \pi_1\). To see this, note that we can write
\[
\pi_0 = \frac{(W - w'' - w') - (w^B - w'') \gamma - (w^B - w'') (1 - \gamma)}{\delta[\phi + (1 - \gamma)(1 - q)(1 - mP)W]}
\]
\[
= \frac{W - w'' - w'}{\delta[\phi + (1 - \gamma)(1 - q)(1 - mP)W]} - \frac{w^B - w''}{\delta[\phi + (1 - \gamma)(1 - q)(1 - mP)W]},
\]
and
\[
\pi_1 = \frac{(W - w'' - w') - (w^B - w'') \gamma + (w^B - w'') (1 - \gamma)}{\delta[\phi + (1 - \gamma)(1 - q)(1 - mP)W]}
\]
\[
= \frac{W - w'' - w'}{\delta[\phi + (1 - \gamma)(1 - q)(1 - mP)W]} - \frac{(w^B - w'' (2 \gamma - 1))}{\delta[\phi + (1 - \gamma)(1 - q)(1 - mP)W]}.
\]

So, since
\[
\frac{W - w'' - w'}{\delta[\phi + (1 - \gamma)(1 - q)(1 - mP)W]} \leq \frac{W - w'' - w'}{\delta[\phi + (1 - \gamma)(1 - q)(1 - mP)W]}
\]
and \(w^B > w''\) it is sufficient to show that
\[
\frac{1}{\phi + (1 - \gamma)(1 - q)mP W} > \frac{(2 \gamma - 1)}{\phi + (1 - \gamma)(1 - q)(1 - mP)W}.
\]

Cross multiplying, this holds whenever
\[
\phi > \hat{\phi}(W) = \frac{1 - q}{2}(2 \gamma mP - 1)W.
\]

As we have now established that, when \(\phi > \hat{\phi}, \pi_0 < \pi_1\) and we can conclude that either the set of beliefs which give the minority type a strict preference for \((w', w'')\) are a proper subset of those which give the minority type a weak incentive – or that, for both types, \((w', w'')\) is preferred for either all beliefs, or for no beliefs, that the voters could hold.

Now consider the case in which \(W > 1\). Then the minority type would have a strict incentive to preference for \((w', w'')\) if and only if the re-election probability \(\pi' \in [Y(\varepsilon, W), X(\varepsilon, W)]\) is such
that
\[(w' + w^B - W)\gamma + (w^B - w'')(1 - \gamma) > \delta(\pi - \pi')[(\phi + m^P(1 - \gamma)(1 + (W - 2)q)),\]

or equivalently
\[\pi' - \pi > \pi_0 \equiv \frac{-(w' + w^B - W)\gamma - (w^B - w'')(1 - \gamma)}{\delta[\phi + m^P(1 - \gamma)(1 + (W - 2)q)]}.

Now consider the majority type. She will have a weak incentive to prefer \((w', w'')\) if and only if the re-election probability \(\pi'\) is such that

\[(w' + w^B - W)\gamma - (w^B - w'')(1 - \gamma) \geq \delta(\pi - \pi')[\phi + (1 - m^P)(1 - \gamma)(1 + (W - 2)q)],\]

or equivalently
\[\pi' - \pi \geq \pi_1 \equiv \frac{-(w' + w^B - W)\gamma - (w^B - w'')(1 - \gamma)}{\delta[\phi + m^P(1 - \gamma)(1 + (W - 2)q)]}.

We now show that \(\pi_0 < \pi_1\), as we did for the case \(W \leq 1\). To see that this holds, note that

\[\pi_0 = \frac{W - w'' - w'}{\delta[\phi + m^P(1 - \gamma)(1 + (W - 2)q)]} - \frac{w^B - w''}{\delta[\phi + m^P(1 - \gamma)(1 + (W - 2)q)]},\]

and

\[\pi_1 = \frac{W - w'' - w'}{\delta[\phi + (1 - m^P)(1 - \gamma)(1 + (W - 2)q)]} - \frac{(w^B - w'')(2\gamma - 1)}{\delta[\phi + (1 - m^P)(1 - \gamma)(1 + (W - 2)q)]}.

As
\[\frac{W - w'' - w'}{\delta[\phi + m^P(1 - \gamma)(1 + (W - 2)q)]} \leq \frac{W - w'' - w'}{\delta[\phi + m^P(1 - \gamma)(1 + (W - 2)q)]},\]

and \(w^B > w''\) it is sufficient to show

\[\frac{2\gamma - 1}{\phi + (1 - m^P)(1 - \gamma)(1 + (W - 2)q)} < \frac{1}{\phi + m^P(1 - \gamma)(1 + (W - 2)q)}.\]

Simplifying, this holds when

\[\phi > \hat{\phi}(W) = \frac{1 + (W - 2)q}{2}(2\gamma m^P - 1).

As we have \(\pi_0 < \pi_1\) when \(\phi > \hat{\phi}\), we can conclude that either the set of beliefs which give the minority type a strict preference for \((w', w'')\) are a proper subset of those which give the minority type a weak incentive – or that, for both types, \((w', w'')\) is preferred for either all beliefs, or for no beliefs, that the voters could hold. □

Proof of Lemma 5: We show, by contradiction, that there cannot exist a Perfect Bayesian Equilibrium satisfying criterion D1 in which the majority type ever chooses an allocation \(w^B <
min\{W,1\} and is re-elected with probability less than \(X(\varepsilon,W)\) after taking that action. We prove this result by considering the case \(W \leq 1\) and \(W > 1\).

Suppose the majority type chooses allocation \(w^B < \min\{W,1\}\), \(w^A = W - w^B\) with positive probability, and suppose the probability of re-election after choosing that action is \(\pi \in [0,X(\varepsilon,W))\). Now consider the allocation \((W - w',w')\), with \(w' > w^B\). Note that, by Lemma 4, if \((W - w',w')\) is on the equilibrium path, the voters must believe the politician is the majority type with certainty, and so the probability of re-election is \(X(\varepsilon,W)\).

Consider the case in which \(W \leq 1\). The majority type would have a strict incentive to deviate to \((W - w',w')\) if and only if the probability of re-election, \(\pi'\) is such that

\[(w' - w^B)(2\gamma - 1) < \delta(\pi' - \pi)[\phi + (1 - q)(1 - \gamma)(1 - m^P)W],\]

or if

\[\pi' - \pi > \pi_1 \equiv \frac{(w' - w^B)(2\gamma - 1)}{\delta[\phi + (1 - q)(1 - \gamma)(1 - m^P)W]}\]

Similarly, the minority type has a weak incentive to deviate if and only if

\[w' - w^B < \delta(\pi' - \pi)[\phi + (1 - q)(1 - \gamma)m^PW],\]

which is equivalent to

\[\pi' - \pi \geq \pi_0 \equiv \frac{w' - w^B}{\delta[\phi + (1 - q)(1 - \gamma)m^PW]}\]

Now note that \(\pi_1 < \pi_0\). This follows because

\[\frac{(w' - w^B)(2\gamma - 1)}{\delta[\phi + (1 - q)(1 - \gamma)(1 - m^P)W]} < \frac{w' - w^B}{\delta[\phi + (1 - q)(1 - \gamma)m^PW]}\]

if and only if

\[\phi > \frac{1 - q}{2}(2\gamma m^P - 1),\]

which is always satisfied if \(\phi > \hat{\phi}(W)\). Finally, note that, as \(w' \to w^B\), \(\pi_1 \to 0\). Hence there exist \(w' > w^B\) such that the majority type would have a strict incentive to choose \((W - w',w')\) even if they would be re-elected with probability less than \(X(\varepsilon,W)\). Note that, by Lemma 4, since the minority type can never be choosing \(w'\), if the majority type chooses \(w'\), the voters must assign probability 1 to them being the majority type, giving the majority politician a strict incentive not to choose \(w^B\), breaking the purported equilibrium. Further, the set of beliefs under which the majority type would have a strict incentive to deviate are a proper superset of those for which the minority type would have a weak incentive to deviate.
Hence, by criterion D1, if \((W - w', w')\) the voters must assign probability 1 to any politician who chose \((W - w', w')\) being the majority type, and such a politician would be re-elected with probability \(X(\varepsilon, W)\). We have verified, however, the the majority type would then have a strict incentive to deviate. We can therefore conclude that there cannot exist a Perfect Bayesian Equilibrium in which the majority type chooses an allocation with \(w^B < \min\{W, 1\}\) and gets re-elected with probability less than \(X(\varepsilon, W)\) when \(W \leq 1\).

We now turn to the case where \(W > 1\). The majority type would have a strict incentive to deviate to \((W - w', w')\) if and only if the probability of re-election, \(\pi'\) is such that
\[
(w' - w^B)(2\gamma - 1) < \delta(\pi' - \pi)[\phi + (1 - \gamma)(1 - m^P)(1 + (W - 2)q)],
\]
or if
\[
\pi' - \pi > \pi_1 \equiv \frac{(w' - w^B)(2\gamma - 1)}{\delta[\phi + (1 - \gamma)(1 - m^P)(1 + (W - 2)q)]}.
\]
Similarly, the minority type has a weak incentive to deviate if and only if
\[
w' - w^B \leq \delta(\pi' - \pi)[\phi + m^P(1 - \gamma)(1 + (W - 2)q)],
\]
which is equivalent to
\[
\pi' - \pi \geq \pi_0 \equiv \frac{w' - w^B}{\delta[\phi + m^P(1 - \gamma)(1 + (W - 2)q)]}.
\]
Now note that \(\pi_1 < \pi_0\). This follows because
\[
\frac{(w' - w^B)(2\gamma - 1)}{\delta[\phi + (1 - \gamma)(1 - m^P)(1 + (W - 2)q)]} < \frac{w' - w^B}{\delta[\phi + (1 - \gamma)m^P(1 + (W - 2)q)]}
\]
if and only if
\[
\frac{2\gamma - 1}{\phi + (1 - \gamma)(1 - m^P)(1 + (W - 2)q)} < \frac{1}{\phi + (1 - \gamma)m^P(1 + (W - 2)q)}
\]
if and only if
\[
\phi > \frac{1 + (W - 2)q}{2\gamma m^P - 1},
\]
which is always satisfied if \(\phi > \hat{\phi}(W)\). Finally, note that, as \(w' \rightarrow w^B\), \(\pi_1 \rightarrow 0\). Hence there exist \(w' > w^B\) such that the majority type would have a strict incentive to choose \((W - w', w')\) even if they would be re-elected with probability less than \(X(\varepsilon, W)\). Note that, by Lemma 3, since the minority type can never be choosing \(w'\), if the majority type chooses it with positive probability in equilibrium, the voters must assign probability 1 to them being the majority type, giving the majority politician a strict incentive not to choose \(w^B\), breaking the purported equilibrium.
Further, the set of beliefs under which the majority type would have a strict incentive to deviate are a proper superset of those for which the minority type would have a weak incentive to deviate. Hence, by criterion D1, if \((W - w', w')\) the voters must assign probability 1 to any politician who chose \((W - w', w')\) being the majority type, and such a politician would be re-elected with probability \(X(\varepsilon, W)\). We have verified, however, that the majority type would then have a strict incentive to deviate. We can therefore conclude that there cannot exist a Perfect Bayesian Equilibrium in which the majority type chooses an allocation with \(w^B < \min\{W, 1\}\) and gets re-elected with probability less than \(X(\varepsilon, W)\) when \(W > 1\). \(\Box\)