Two-Sided Coordination: Conflict in Collective Action*

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November 25, 2014

Abstract

We study a coordination problem where two distinct groups of individuals are in competition with each other. One group (regime opponents) prefers a change in regime, and can participate in an attack which, if sufficiently large, causes regime change. The other group (regime adherents) prefers the status quo and can support the regime, making it more resistant to attack. The outcome is dictated by the comparative mobilization of opponents and adherents. We show how changes in the incentives of one group influence the coordination of the other. We isolate the coordinating effect of public information which results from the two-sidedness of the coordination problem, and show that coordination between regime adherents intensifies the already disproportionate effect of public information. Specifically, we show that public information affects the actions of individuals in each group identically, regardless of disparities in the quality of private information between group members, implying that commonly observed sources of information will coordinate both regime adherents and regime opponents in exactly the same way. We then extend the analysis to study the effects of an information source that is commonly observed by only one group, regime adherents, which we term club information.

JEL-Classification: D74, D82, C71
Keywords: Coordination, Conflict, Global Games, Political Revolution

*We thank Ethan Bueno de Mesquita, Jim Morrow, Mehdi Shadmehr, Ken Shotts, and seminar participants at the University of Michigan for useful comments and suggestions.
1 Introduction

Coordination problems arise in a variety of social contexts. A particularly important kind of coordination problem is one where two groups of individuals with competing interests collectively compete to bring about their preferred outcome. Examples include competition between teams, political party competition, or conflict between political lobbies. It is commonly held that in such two-sided coordination settings the outcome is dictated by the group who coordinates more effectively — meaning they better align their actions with each other, as well as aligning their group action to an underlying state of the world.

In this paper we examine the context of collective action in a setting that exhibits both within-group coordination and between-group conflict.

To understand two-sided coordination we build on the global-game approach and develop a model of regime change where one group wants to maintain the status quo and the other wants to bring about regime change. Those who desire regime change, regime opponents, can actively try to bring down the regime, and those who enjoy the status quo, regime adherents, can actively increase the ability of the regime to withstand attacks from opponents. Regime opponents and regime adherents engage in a dueling coordination problem. The structure of outcomes implies that the preferred outcome can only be achieved for a single group, hence the interests of the groups are diametrically opposed. What complicates the coordination efforts of individuals in one group is that the mobilization of the other group impairs their ability to determine the outcome.

We distinguish between within-group coordination, that is the alignment of actions within a specific group, and between-group coordination, that is the alignment of actions between groups. Within-group coordination arises from the complementarities between individuals who are members of the same group. For instance, an opponent’s incentive to rebel increases as other opponents rebel, and similarly, an adherent’s incentive to support the regime increases as other adherents support the regime. In addition to an individual’s desire to coordinate with other individuals within their group, there is also
an incentive to align with the underlying state of the world. Opponents want to rebel whenever the regime fails and adherents want to support the regime whenever the regime survives. However, this alignment with outcomes creates an incentive to coordinate between groups. Specifically, the act of rebellion for an opponent and the act of supporting the regime for a regime adherent are strategic substitutes. Hence, an opponent’s incentive to rebel decreases as more adherents support the regime, and an adherent’s incentive to support the regime increases as fewer opponents rebel.

We derive the necessary and sufficient conditions under which there is a unique equilibrium in our model, and we find that the conditions guaranteeing uniqueness in a two-sided coordination problem are more demanding than in existing one-sided games (Morris and Shin 2003, 2004, e.g.). In particular, we show that in addition to restrictions on the level of common information, when one group is large relative to the other, equilibrium uniqueness fails. When there is a large enough disparity between the size of groups, a single group can, by aligning their actions within the group, create a strong incentive for alignment within the other group. Specifically, when active support from regime adherents comprises a large enough component of the regime’s strength, structural characteristics of the regime are relatively unimportant, implying that coordination is the primary motivation of adherents and opponents, yielding multiple equilibria.

We detail the strategic environment that arises whenever two groups, who must act in a decentralized manner, come into direct conflict. Our model shows that commonly known factors that help one group coordinate necessarily help the other group coordinate as well. We examine this feature from several angles. First, we examine comparative statics with respect to the cost parameters of each group and show how the mobilization of each group is influenced by changes in one group’s incentives. In particular, an increase in the opportunity cost of participating in a rebellion for an opponent depresses the level of rebellion, but also increases the level of adherent support for the regime. Repression works — and it is more effective than conventionally thought. Repression of regime opponents directly decreases turnout in a rebellion, and in addition, indirectly increases
the level of adherent support for the regime. Regime adherents, expecting fewer opponents to rebel, have an increased incentive to support the regime. These effects reinforce each other and further compound from the incentive individuals have to coordinate, both within and between groups. Similarly, an increase in the support cost to an adherent depresses adherent support, and also increases the size of the rebellion.

As in other models that exhibit strategic uncertainty in a coordination environment, public information plays a disproportionate and important role. Interestingly, we find that public information affects each group identically, regardless of disparities in the quality of private information for members of each group. Although the informativeness of the public signal is greater for the less informed group, the strategic response of the more informed group is larger. The net effect is that each group is equally responsive to the public signal, regardless of the quality of their private information. Since the public signal is more informationally relevant to the poorly informed, public information is informative of their aggregate behavior. As a consequence, the well informed react strongly to public information even when the disparity between their private information and public information is extreme. Strategic uncertainty causes members of both groups to focus more on public information than they would in the case of a one-sided coordination problem. As a consequence, the comparative mobilization between groups intensifies the disproportionate influence of public information. Furthermore, an attractive feature of our model is that it allows us to isolate the compounding effect that results from between-group coordination.

Finally, to further analyze the difference between within-group coordination and between-group coordination, we extend our two-sided global game of regime change to include an information source which is commonly observed by one group but unavailable to the other. Such information, which we call club information, highlights a distinction that is novel to the study of coordination problems and collective action. An individual can coordinate well, in that she aligns her action with her group, and an individual can coordinate correctly, in that she aligns her action with the underlying state of the world.
Better private information helps individuals better coordinate with each other while also helping a group correctly align with the underlying state. In contrast, a club signal helps individuals to coordinate well, but does not necessarily help them align correctly with an underlying state.

The global game approach originally developed by Carlsson and Van Damme (1993) has been fruitfully applied to currency crises (Morris and Shin 1998, 2004), as well as political contexts. Atkeson (2001) suggests using the model of Morris and Shin (2001) as a fruitful approach to study riots.\(^1\) In this direction, Angeletos, Hellwig and Pavan (2007) examine dynamics in a regime change game where rebels must coordinate to overthrow a regime; over time failed revolts provide information as to the fixed level of regime strength. Other applications of global games of regime change include studies of endogenous information (Angeletos and Werning 2006; Angeletos, Hellwig and Pavan 2006; Angeletos and Pavan 2013), investment dynamics (Dasgupta 2007; Kovács and Steiner 2012), strategic voting (Myatt 2007), party leadership (Dewan and Myatt 2007), political propaganda (Edmond 2013), international conflict (Chassang and Padró i Miquel 2010), civil war (Chassang and Padró i Miquel 2009), and coordination cascades (Mathevet and Steiner 2013).\(^2\) More directly related, Shadmehr and Bernhardt (2011) look at a two-player regime change game and find that uncertainty regarding the payoff of successful revolt leads to novel interactions between repression and incidence of protest, which they term punishment dilemmas.

The model presented is an *interrelated coordination problem* where two distinct groups of actors face a coordination dilemma where there is a strategic connection between the actors in each group. The importance of interrelated coordination problems, in both economic and political contexts, is stressed by Schelling (1960), but such coordination dynamics has received little theoretical treatment. A novel feature of our model is the presence of between-group coordination which is absent from other models. The distinction between within-group coordination and between-group coordination is present in

\(^1\)See also Hellwig (2002).

\(^2\)See Bueno De Mesquita (2010) for an application of similar techniques to terrorist violence.
models where there are at least two coordination problems that share some link. Casper and Tyson (2014b) examine a model where a set of protesters and a set of coup plotters each face distinct coordination problems but the collective action of protesters provide information to coup plotters since the thresholds determining success for each group are correlated. Using a similar structure, Casper and Tyson (2014a) study how the inability of a set of civilians to perfectly align their actions in a rebellion create a coordination problem among government agents tasked with the implementation of policy.

The paper proceeds as follows. Section 2 presents the model and section 3 presents the equilibrium and discusses equilibrium multiplicity. Section 4 presents comparative statics while Section 5 examines the effects of public information, and Section 6 examines the model with club information. Section 7 applies the model to political instability and the final section concludes, with proofs contained in the appendix.

2 The Model

Consider an environment where there are two groups of actors. First, there is a unit mass of individuals who are unhappy with the current regime and prefer regime change, we call this group regime opponents. Second, there is a unit mass of individuals who benefit from the current regime relative to the alternative, and therefore prefer the status quo, we call these individuals regime adherents. Opponents can take an action that actively promotes regime change, while regime adherents can take an action that improves the regime’s ability to withstand pressure from opponents, thereby strengthening the status quo.

The coordination environment is also characterized by an underlying state of the world \( \theta \in \mathbb{R} \), which we call the regime fundamental; it influences the strength of the status quo regime through an exogenous structural relationship. One can interpret the regime fundamental as economic conditions, the regime’s ability to purchase mercenaries, or the actions of groups not facing collective action problems. The state of the world
θ captures all relevant factors of the environment, domestic and international, which contribute to the regime’s structural ability to withstand pressure from opponents. We suppose that these factors are not commonly known by anyone. For convenience, we assume θ is drawn according to an improper uniform prior, where every real number is equally likely (DeGroot 1970).3

Each regime opponent can choose either to rebel against the regime or abstain. The payoffs for opponents are as follows, where actions are represented by rows and outcomes are represented by columns:

<table>
<thead>
<tr>
<th>Opponent Payoff</th>
<th>Status Quo</th>
<th>Regime Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rebel</td>
<td>u₀</td>
<td>u₁</td>
</tr>
<tr>
<td>Abstain</td>
<td>v₀</td>
<td>v₁</td>
</tr>
</tbody>
</table>

with u₀ < v₀, u₁ > v₁, and u₁ > v₀. The first two inequalities capture the coordination incentive and the third captures the preference for regime change. Without loss of generality, let C ≡ \(\frac{v₀ - u₀}{u₁ - v₁ + u₀ - v₀}\) ∈ (0, 1), then we can write the payoffs of opponents as:

<table>
<thead>
<tr>
<th>Opponent Payoff</th>
<th>Status Quo</th>
<th>Regime Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rebel</td>
<td>−C</td>
<td>1 − C</td>
</tr>
<tr>
<td>Abstain</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

In addition to regime opponents, there is also a set of regime adherents. Each regime adherent has binary action set: support the regime or desert it. Without loss of generality, the payoffs for a regime adherent are summarized by:

<table>
<thead>
<tr>
<th>Adherent Payoff</th>
<th>Status Quo</th>
<th>Regime Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Support Regime</td>
<td>1 − k</td>
<td>−k</td>
</tr>
<tr>
<td>Desert</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The payoff to being a loyal member of a regime that survives is normalized to 1. We normalize the payoff from deserting a regime to zero. An adherent pays a cost k ∈ (0, 1)

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3We do this for expositional convenience. If, for instance, we assumed a normal proper prior then its mean and variance could be incorporated into the information structure, but this would involve carrying extra parameters through all calculations.
to supporting the regime, regardless of whether or not the regime survives.\textsuperscript{4} Payoffs, both for regime opponents and regime adherents, can be substantially generalized; specifically the payoff conditions of Morris and Shin (2003, pp 12-13) are sufficient. We keep payoffs simple for expositional convenience.

\section{The Technology of Conflict}

Suppose that regime adherents have physical capacity $\lambda \geq 0$ relative to regime opponents. This parameter represents how a unit mass of regime adherents translates into physical regime strength. Denote the proportion of opponents who rebel by $R$, and let $E$ represent the proportion of regime adherents who support the regime.

The underlying state of the world $\theta$ determines the structural strength of the regime through the differentiable, Lipshitz continuous function $w(\theta, \lambda)$, which is strictly increasing in $\theta$. This function captures aspects of the environment that influence regime strength, but are not directly influenced by the collective action of regime adherents.

Together the weighted sum of structural factors and adherent support determine the proportion of opponents who must actively rebel in order to cause a change in the regime. If this combination of adherent support and structural factors is greater than the proportion of regime opponents who rebel, then the regime survives; otherwise the regime collapses. Specifically, given $\theta$ and $\lambda$, the regime survives if and only if

\begin{equation}
    w(\theta, \lambda) + \lambda E > R.
\end{equation}

Revolutionary success then depends upon the proportions of actors on each side rather than the turnout of a single group.

We make the following assumption regarding the relationship between structural fundamentals and outcomes:

\footnote{If $k > 1$ or $k < 0$ then all adherents have dominant strategies and the game reduces to a one-sided global game of regime change.}
Assumption 1  For every $\lambda$, there exists a $\theta_\lambda \in \mathbb{R}$ such that, if $\theta \leq \theta_\lambda$ the regime falls regardless of the actions of individuals, and there exists a $\bar{\theta}_\lambda \in \mathbb{R}$, with $\theta_\lambda < \bar{\theta}_\lambda$, such that, if $\theta > \bar{\theta}_\lambda$ the regime survives regardless of the actions of individuals. Moreover, $\theta_\lambda$ and $\bar{\theta}_\lambda$ are common knowledge.

This assumption implies that if $\theta > \bar{\theta}_\lambda$, and this were commonly known, then all individuals would have a dominant strategy to support the regime. Moreover, if $\theta \leq \theta_\lambda$, and this were commonly known, then all individuals would have a dominant strategy to abandon the regime or rebel. These regions are typically referred to as the dominance regions, because they induce dominant strategies for those who receive extreme signals. Importantly, the states $\bar{\theta}_\lambda$ and $\theta_\lambda$ are indexed by $\lambda$. Assumption 1 implies that it is possible for some regimes to survive without internal support, e.g. by hiring mercenaries to quell rebellion. Similarly, it is also possible for a regime to fail for reasons unrelated to opposition revolt. Since $\theta_\lambda < \bar{\theta}_\lambda$ there is a region of the complete information game which exhibits multiple equilibria (Morris and Shin 2003).

As a consequence of Assumption 1, we have the following restriction on the function, $w(\cdot)$, which relates structural factors into regime strength.

Lemma 1  Given Assumption 1, the function $w(\cdot)$ takes the following values: $w(\bar{\theta}_\lambda, \lambda) = 1$, and $w(\theta_\lambda, \lambda) = -\lambda$.

Lemma 1, whose proof is in the appendix, ties Assumption 1 into the technology of conflict. As can be seen, adherent strength shrinks the lower dominance region, but leaves the upper dominance region unaffected. This isolates an important distinction between our model and existing global game models of regime change. Specifically, the set of regime adherents influence the size of the lower dominance region without effecting the upper dominance region so that in our model the size of the lower dominance region is endogenously determined by the collective support of the set of regime adherents. We have assumed separability between structural factors, adherent mobilization, and opponent mobilization; this is adopted for convenience, our arguments rely on monotonicity of the
technology of conflict.

2.2 The Information Structure

We now describe the information structure of our game. As is common in global games, we incorporate a lack of common knowledge into our environment with idiosyncratic noisy signals of the underlying state of the world.

All actors see a common public signal \( Q = \theta + \tau^Q \), where \( \tau^Q \) is independent of \( \theta \), and normally distributed with mean zero and variance \( \frac{1}{\alpha} \). As is common convention, we refer to \( \alpha \) as the precision (DeGroot 1970). In addition to the public signal, opponent member \( j \) receives a private signal \( Y_j = \theta + \nu_j \) where \( \nu_j \) is independent of \( \theta, Q \), and across individuals, and is normally distributed with mean zero and precision \( \gamma \).

We model the information of regime adherents in a similar fashion to that of opponents. Each adherent, in addition to observing the public signal \( Q \), also receives a private signal \( Z_i = \theta + \varepsilon_i \), where \( \varepsilon_i \) is independent of \( \theta, Q \), and across all individuals, and is normally distributed with mean zero and precision \( \beta \).

Actors update their beliefs in a straightforward way upon seeing the private and public signals. These first-order beliefs, meaning they are beliefs about the regime fundamental \( \theta \), follow directly from Bayes’ rule. These signals shape not only an actor’s belief regarding the value of the regime fundamental, but more importantly, they shape what actors perceive about the actions of other actors. Importantly, this information structure captures the strategic uncertainty between individuals within each group, and also between each group.

Opponent \( j \), after observing his private signal \( Y_j \), and the public signal \( Q \) believes that \( \theta \) is normally distributed with mean \( y_j \) and precision \( \alpha + \gamma \), where \( y_j = (1 - \eta)Q + \eta Y_j \) and \( \eta = \frac{\gamma}{\alpha + \gamma} \) is the relative importance of a opponent’s private information in their posterior belief regarding the regime fundamental.\(^5\) Similarly, having seen her private signal \( Z_i \),

\(^5\)Linearity is a consequence of Bayesian updating of information generated from normal distributions, see DeGroot (1970). For notational clarity, let \( \Phi(x) = \int_x^{\infty} \phi(\theta) d\theta \) be the distribution function of a
and the public signal $Q$, an adherent’s posterior beliefs about $\theta$ are normally distributed with mean $z_i = \kappa Z_i + (1 - \kappa)Q$, and precision $\alpha + \beta$, where $\kappa = \frac{\beta}{\alpha + \beta}$ is the relative importance of an adherent’s private signal in her posterior expectation. For the remainder of the analysis we express results in terms of the posterior expectation of individuals.

We note here that in the two-sided game we study there are two differentially informed groups whose “active” action has different consequences. An opponent who supports the regime cannot make the regime stronger through his action, he can only upset the mobilization potential of other opponents. Likewise, a regime adherent who abandons the regime cannot contribute to precipitating the regime’s collapse, she only prevents the regime from putting down a marginally larger rebellion. If one switched the active actions of one group, then the game would reduce to a one-sided game of regime change in which there are two groups with different precisions of information. Our framework can be extended to capture these possibilities by allowing $\lambda$ to take negative values, however we do not examine these cases here because we focus on the coordination dynamic that arises when there is conflict between groups.

3 Dueling Coordination

A strategy in this model is a measurable function which maps from an individual posterior expectation of $\theta$ to the binary action set of an individual. We consider symmetric Bayesian Nash equilibria in strategies which are monotonic in posterior expectations, otherwise called monotone strategies; this is a straightforward class of strategies to focus on, and we show that it is without loss of generality in Theorem 2 below.\footnote{In another specification, a strategy would map from the public signal and the private signal into the action set. Then beliefs would be required to be consistent with Bayes rule, restricting the posterior expectation of an opponent to $y$ and the posterior expectation of an adherent to $z$.}

Focusing on monotone strategies, suppose that each opponent chooses to rebel if and only if his posterior expectation of the regime fundamental drops below some specific standard normal variate, where $\phi(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$ is the density function, and let $x = \Phi^{-1}(p)$ be its inverse function, where $x$ is the $p$th quantile.
value, i.e. rebel if and only if \( y \leq \bar{y}(Q) \). Similarly, suppose that each regime adherent chooses to abandon the regime if and only if her posterior expectation drops below some specific value, i.e. desert the regime if and only if \( z \leq \bar{z}(Q) \). Note that the reverse inequalities are strictly dominated. Denoting the extended real numbers as \( \mathbb{R} = \mathbb{R} \cup \{ \infty \} \cup \{-\infty\} \), a monotone strategy profile can be expressed as a pair \((\bar{z}(Q), \bar{y}(Q)) \in \mathbb{R}^2\). The monotone strategies \((\bar{z}, \bar{y})\) depend on the public signal \( Q \), but we do not explicitly track this dependence when there is no confusion.

All individuals in our model have normally distributed posterior expectations of \( \theta \) that differ according to their mean. Consequently, the cross-section of opponent posteriors is characterized by a normal distribution with mean \( \eta \theta + (1 - \eta)Q \) and precision \( \alpha + \gamma \). This distribution provides an aggregate measure of the dispersion of opponent beliefs by second-order stochastic dominance. For a given regime fundamental \( \theta \), and given a cutoff strategy for opponents \( \bar{y} \), the proportion of opponents who rebel is

\[
\mathcal{R}(\theta, \bar{y}) = \int_{-\infty}^{\bar{y}} \sqrt{\gamma} \phi\left(\frac{\sqrt{\gamma}}{\eta}(y - \eta \theta - (1 - \eta)Q)\right)dy = \Phi\left(\frac{\sqrt{\gamma}}{\eta}(\bar{y} - \eta \theta - (1 - \eta)Q)\right).
\]

Similarly, given \( \theta \) the cross-sectional posteriors of regime adherents are normally distributed with mean \( \kappa \theta + (1 - \kappa)Q \) and precision \( \alpha + \beta \), so the proportion of regime adherents who support the regime is

\[
\mathcal{E}(\theta, \bar{z}) = 1 - \Phi\left(\frac{\sqrt{\beta}}{\kappa}(\bar{z} - \kappa \theta - (1 - \kappa)Q)\right) = \Phi\left(\frac{\sqrt{\beta}}{\kappa}(\kappa \theta + (1 - \kappa)Q - \bar{z})\right).
\]

It is important to note that whenever adherents are more informed regarding regime fundamentals than opponents, i.e. \( \beta > \gamma \), then the aggregate beliefs of regime adherents second-order stochastically dominate the aggregate beliefs of opponents.

To begin, we first note that a given monotone strategy profile, characterized by \((\bar{z}, \bar{y})\), induces a critical state \( \bar{\theta}(\bar{z}, \bar{y}) \) that determines the strongest regime that fails. In other words, for a given monotone strategy pair \((\bar{z}, \bar{y})\) the revolution succeeds if and only if
\[ \theta \leq \overline{\theta}(z, y). \] Although \( \overline{\theta}(z, y) \) also depends on the public signal \( Q \), as above, we suppress this dependence unless doing so causes confusion. Formally,

**Lemma 2** For a given \( Q \), and a given pair of cutoff strategies \((z, y) \in \mathbb{R}^2\), one for regime adherents \( z \), and one for opponents \( y \), there is a unique critical state \( \overline{\theta}(z, y) \in [\theta_{\lambda}, \overline{\theta}_{\lambda}] \) such that the regime survives if and only if \( \theta > \overline{\theta}(z, y) \).

With a monotone strategy profile, increasing the state \( \theta \) decreases the proportion of opponents who rebel and increases the proportion of regime adherents who support the regime. An immediate implication is that whenever strategies are monotone, outcomes are dictated by a critical state, meaning that equilibrium outcomes are discontinuous in states.

A consequence of Lemma 2 is that a monotone Bayesian Nash equilibrium in our game is a monotone strategy profile characterized by the triple, \((z, y, \overline{\theta}(z, y))\), which consists of a cutoff strategy for regime adherents \( z \), a cutoff strategy for opponents \( y \), and the critical threshold, or tipping point, \( \overline{\theta}(z, y) \), induced by \( z \) and \( y \) in which the regime survives if and only if \( \theta > \overline{\theta}(z, y) \).

From the monotone strategy of regime adherents \( z \), we can compute the induced strength of the regime, which is the threshold regime opponents face, as a function of the monotone best-response of regime adherents and an adherent conjecture of the critical state.

**Lemma 3** Fix \( Q \). For a symmetric regime adherent critical state conjecture \( \theta^\dagger \), there exists a unique function \( W \), that is a function of the critical state conjecture \( \theta^\dagger \) and the state variable \( \theta \), that expresses the induced strength of the regime. The induced strength of the regime is strictly increasing in the regime fundamental \( \theta \), and strictly decreasing in the critical state conjecture \( \theta^\dagger \).

The proof (in the appendix) follows a simple argument. Holding the monotone strategy of opponents fixed at \( y \), and the conjectured regime critical state fixed at a specific value \( \theta^\dagger \), we find the best-response cutoff for adherents, \( \overline{z}(\theta^\dagger) \). After finding the best-response
cutoff for adherents, we substitute the adherent monotone best-response into the turnout of adherent support, denoted by $E(\theta | \theta^\ddagger)$. This value determines the level of adherent support for a given state $\theta$ and a conjectured critical state $\theta^\ddagger$. We then obtain the induced strength of the regime as a function of $\theta$ and $\theta^\ddagger$,

$$W(\theta | \theta^\ddagger) = w(\theta, \lambda) + \lambda E(\theta | \theta^\ddagger) = w(\theta, \lambda) + \lambda \Phi \left( \frac{\sqrt{\beta}}{\kappa} \left( \kappa \theta + (1 - \kappa)Q - \theta^\ddagger - \frac{1}{\sqrt{\alpha + \beta}} \Phi^{-1}(k) \right) \right).$$

The function $W$ expresses the threshold that rebels must exceed in order to achieve regime change. If the proportion of opponents is less than $W(\theta | \theta^\ddagger)$, then the status quo prevails. The function $W$ measures the induced strength of the regime, which depends on the critical state conjecture $\theta^\ddagger$ that adherents use to compute their expectations of the regime’s fate; entailing that the strength of the regime is self-fulfilling. Since the function $W$ is strictly decreasing in $\theta^\ddagger$, if adherents believe the regime will fail under a large set of states, i.e. $\theta^\ddagger$ is high, then the induced threshold $W$ is low and the regime is indeed weak. In contrast, when adherents expect the regime to be robust to threats, i.e. $\theta^\ddagger$ is low, then the induced threshold $W$ is high and the size of the opponent rebellion must be large in order to overturn the status quo. A group that is able to mobilize larger participation is harder to defeat in the sense that in all states of the world the other group requires (weakly) more participation to meet the critical state.

An important quantity of interest is the induced strength of the 	extit{critical regime} which gives the induced strength of the strongest regime that fails. At a symmetric regime adherent critical state conjecture $\theta^\ddagger$, the induced strength of the critical regime is

$$W(\theta^\ddagger) \equiv W(\theta^\ddagger | \theta^\ddagger) = w(\theta^\ddagger, \lambda) + \lambda \Phi \left( \frac{\sqrt{\beta}}{\kappa} \left( (1 - \kappa)(Q - \theta^\ddagger) - \frac{1}{\sqrt{\alpha + \beta}} \Phi^{-1}(k) \right) \right).$$
3.1 Equilibrium

Next we state the main result of this section, which gives the necessary and sufficient condition for the monotone Bayesian Nash equilibrium to be the unique iterated dominance solution of the two-sided global game.

**Theorem 1** An equilibrium is characterized by the triple \((z^*, y^*, \theta^*)\), which are the values that simultaneously solve the following equations,

\[
P(\theta \leq \theta^* \mid y^*) = C \quad \text{Opponent Indifference;}
\]

\[
P(\theta > \theta^* \mid z^*) = k \quad \text{Adherent Indifference;}
\]

\[
R(\theta^*, y^*) = w(\theta^*, \lambda) + \lambda E(\theta^*, z^*) \quad \text{Critical Threshold.}
\]

The equilibrium characterized by \((z^*, y^*, \theta^*)\) is unique for all \(Q\) if and only if

\[
\Omega(W \mid \theta^*) \equiv \eta \left(1 + \min_{\theta^* \in (\mathcal{Z}, \mathcal{F})} \frac{W(\theta^*)}{\sqrt{\gamma \phi(\Phi^{-1}(W(\theta^*)))}}\right) - 1 > 0. \tag{4}
\]

Heuristically, the adherent indifference equation ensures that given an expectation that the regime survives if and only if the state is above the conjectured critical state \(\theta^\dagger\), then the adherent’s best-response is to use the cut-off strategy \(z(\theta^\dagger)\), where an adherent supports the regime if and only if \(z > z(\theta^\dagger)\).

The opponent indifference condition plays a similar role by characterizing the posterior belief that makes an opponent indifferent between rebelling and not, as a consequence of the conjecture that the regime survives in states above \(\theta^\dagger\). The posterior belief satisfying this indifference constitutes a best-response monotone strategy \(y(\theta^\dagger)\).

In equilibrium all individuals, both opponents and adherents, must forecast the critical state correctly, and hence the critical state conjecture used to find the indifferent opponent posterior, and the critical state conjecture used to locate the indifferent regime adherent, must be the same. Furthermore, the common critical state conjecture that individuals
use to compute expectations must be the true state defining the strongest regime that fails. This logic underlies the third condition, the critical threshold condition, which characterizes the critical state $\theta^*$ at which the regime survives if and only if $\theta > \theta^*$. In equilibrium the adherent and opponent indifference equations ensure that $\theta^*$ induces $z^* = \bar{z}(\theta^*)$ and $y^* = \bar{y}(\theta^*)$. The critical threshold equation ensures that $z^*$ and $y^*$ indeed induce the critical threshold $\theta^* = \bar{\theta}(z^*, y^*)$, implying $\theta^1 = \theta^2 = \theta^*$. 

As a benchmark for comparison, consider as a corollary of Theorem 1, the standard one-sided global game of regime change where the threshold function is given by $w$ and the equilibrium is characterized by an opponent cutoff $\hat{y}$ and a critical threshold $\hat{\theta}_w$ which depends on the threshold function.

**Corollary 1** *(Hellwig 2002)* If $\lambda = 0$, then the one-sided regime change game has an equilibrium, characterized by a cutoff, below which opponents rebel and above which opponents abstain, and a critical state below which the regime falls and above which the regime survives, $(\hat{y}, \hat{\theta}_w)$, which solves opponent indifference,

$$P(\theta < \hat{\theta}_w | \hat{y}) = C$$

and a critical threshold condition

$$w(\hat{\theta}_w) = R(\hat{\theta}_w, \hat{y}).$$

Moreover, this equilibrium is the unique iterated dominance solution for all $Q$ if and only if

$$\Omega(w| \hat{\theta}_w) = \eta \left( 1 + \min_{\hat{\theta}_w \in (\hat{y})} \frac{w'(\hat{\theta}_w)}{\sqrt{\bar{\phi}^2(\Phi^{-1}(w((\hat{\theta}_w))))}} \right) - 1 > 0.$$ 

The proof is a special case of Theorem 1, where the structural threshold function is $w$ and $\lambda = 0$. The proof that the strategy profile is the unique iterated dominance solution can be found in Morris and Shin (2004).\textsuperscript{7}

\textsuperscript{7}The case focused on in Morris and Shin (2004) is where $\lambda = 0$ and $w(\theta) = \theta$. 

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The dominance regions imply that there always exists some positive measure of individuals who are sufficiently confident that the regime will fall(survive), that they have a dominant strategy. Given this measure of extreme individuals whose actions are independent of the behavior of others, there exists some state in which the regime falls(survives) with enough probability so that some measure of individuals then have an iteratively dominant strategy. This logic generates two monotonic sequences which converge to the monotone equilibrium, whenever the monotone equilibrium is itself unique. When there are multiple monotone equilibria, the iterated dominance argument generates a sequence converging to the lowest equilibrium, and a sequence converging to the highest equilibrium. Formally,

Theorem 2 If $\Omega(W \mid \theta^*) > 0$ then $(z^*, y^*, \theta^*)$ characterizes the unique strategy profile which survives iterated deletion of strictly dominated strategies.

The proof is in the appendix. The condition which characterizes uniqueness of the monotone equilibrium in our model, namely $\Omega(W \mid \theta^*) > 0$, belies an important subtlety which arises as a consequence of between-group coordination. The iterated dominance argument requires that the induced strength of the regime is locally increasing around the equilibrium critical state. Importantly, this condition fails whenever the capacity of one group is sufficiently strong, or public information is sufficiently precise. Specifically, consider the precision of commonly observed information, $\alpha$, and the contribution of regime adherents to regime strength, $\lambda$. The set of possible pairs for these values are $\mathbb{R}^2_+$. We next parse out this space to show that the subset of $\mathbb{R}^2_+$ for which there is a unique equilibrium is generic. This is summarized by:

Theorem 3 For a fixed private information structure, $(\beta, \gamma)$, there exists a nonempty, bounded, open set $\Delta \subset \mathbb{R}^2_+$, such that the two-sided global game of regime change has a unique monotone equilibrium if and only if $(\alpha, \lambda) \in \Delta$.

---

8Observe that we have normalized the size of opponents to 1. Since $w(\cdot)$ depends on $\lambda$, this is without loss of generality.
In a one-sided model, uniqueness follows as a consequence of dominance regions, and sufficiently imprecise public information. By contrast, we show that when the importance of adherents in determining the induced regime strength is sufficiently high, meaning that \( \lambda \) is large, then there are multiple monotone equilibria in the two-sided game, regardless of the precision of public information and the presence of dominance regions.

The source of the multiplicity of equilibria arises because of the between-group coordination between regime opponents and regime adherents. An important component of \( \Omega(W | \theta^*) \) is the marginal effect of the induced threshold function \( W \) evaluated at \( \theta^* \). This measures the marginal increase in the strength of the regime at the critical threshold \( \theta^* \) as the critical threshold changes. More specifically,

\[
W'(\theta^*) = \frac{\partial w(\theta^*, \lambda)}{\partial \theta} - \lambda \frac{\alpha}{\sqrt{\beta}} \phi \left( \frac{\alpha}{\sqrt{\beta}} (Q - \theta^*) - \frac{\sqrt{\beta}}{\kappa \sqrt{\alpha + \beta}} \Phi^{-1}(k) \right).
\]

It is composed of two parts. The first, \( A \), comprises the standard marginal increase from structural factors. Since \( w \) is an increasing function this component is always positive, and implies that when the critical state increases, so does the strength of the critical regime.

Due to strategic influences on the induced threshold, the marginal effect of increasing the critical threshold has a secondary effect. The second part, \( B \), comprises the marginal effect from changes in the level of adherent support, which arises as a consequence of a change in the critical threshold. Increasing the critical state \( \theta^* \) decreases the set of regimes that survive rebellion and thus implies that a regime adherent requires a higher signal of regime strength in order to be willing to support the regime. As a consequence, the level of elite support around the critical state is strictly decreasing. Uniqueness requires that the diminishing level of adherent support does not overwhelm the marginal improvement in structural factors, so that whenever the critical state increases, then the induced strength of the critical regime also increases. It is worth emphasizing that in one-sided models \( B = 0 \), and hence the presence of this effect is a result of the feedback
between the coordination dilemma of regime adherents and the coordination dilemma of regime opponents.

4 Comparative Statics

A novel feature of our model is the interrelation of distinct coordination problems, i.e. the connection between the coordination problem of regime opponents and the coordination problem of regime adherents that arises because both coordination problems jointly determine outcomes. In particular, the actions of individuals in each group are statistically correlated through private signals, but are also strategically related by the technology of conflict. In this section we analyze the strategic connection by conducting a comparative static analysis on the payoff parameters for members in each group, as well as the parameter $\lambda$ that measures the contribution of regime adherents to regime strength.

Recall the opponent relative cost ratio $C$, and the adherent participation cost $k$. The critical threshold $\theta^*$, characterized in Theorem 1, allows for a simple comparison of status quo regimes in terms of their ability to survive an opposition revolt.

**Proposition 1** If $\Omega(W | \theta^*) > 0$, then the critical state $\theta^*$ is strictly decreasing in the cost ratio $C$, and strictly increasing in cost $k$. The cutoff for opponents, and the cutoff for adherents, are strictly decreasing in the opponent cost ratio and strictly increasing in the adherent participation cost.

This result details how the participation cost of one group influences the collective action of both groups. Increasing the cost of participating in a failed revolt, or decreasing the benefit of participating in a successful revolt, raises the opportunity cost of participating in rebellion, which implies that more regimes survive rebellion.

So repression works — the more a regime can punish rebels, or prevent them from enjoying the prize to successful collective action, the less willing is an individual opponent to rebel. But in addition, the more a regime can punish rebels the more willing are adherents to support the regime, and taken together, the more robust the regime is
to revolutions. Repression benefits the regime, and to a greater extent than might be expected.

An increase in the opportunity cost of rebelling for an opponent directly increases the incentive for an opponent to abstain. This direct effect implies that some set of opponents, who would have participated at a lower cost, now abstain because they face a greater cost of coordination failure. This direct reduction leads to two indirect effects: one from within-group coordination among opponents and another from between-group coordination with adherents. From the incentive to coordinate within the group of opponents, the direct decrease in opponent participation makes regimes more likely to survive and therefore opponents need lower signals of regime strength to be willing to rebel. Additionally there is a third effect, with fewer opponents expected to rebel at any given state, adherents have a greater incentive to support the regime. Knowing that the regime survives under a larger set of states, adherents are more confident in regime survival at any given level of $\theta$ which provides an increased incentive to support the regime. These three effects reinforce each other and entail a decrease in the critical state.

The decision of opponents to rebel, and the decision of adherents to support the regime, are strategic substitutes and therefore amplify any direct effect on the mobilization of either group. As participation in rebellion by opponents becomes harder to motivate, regime support by adherents becomes easier to motivate. Specifically, an increase in the opportunity cost of supporting the regime for an adherent, $k$, implies that adherents require higher signals of regime fundamentals in order to be willing to support. In response, regime opponents also are willing to rebel for higher signals. These affects show how aligning the actions within one group necessarily leads to an improvement in the alignment of actions within the other group.

Recall the parameter $\lambda$, which measures the contribution of adherent support to the level of opponent resistance the status quo can withstand. We have not made assumptions about the relationship between $\lambda$ and the structural ability of the regime to combat rebellion $w(\theta,\lambda)$ (other than the convenient assumption that this relationship is smooth).
Although the exact considerations motivating the structural relationship between the physical strength of adherents, $\lambda$, and the strength of the regime, given by $w(\cdot, \lambda)$, is exogenous in our model, we retain a large degree of flexibility in this relationship. First, there is the case where the structural strength of the regime is independent of the physical strength of adherents, i.e., $\frac{\partial w(\cdot)}{\partial \lambda} = 0$.

Second, adherent strength could be a net substitute with structural characteristics, i.e., $\frac{\partial w(\cdot)}{\partial \lambda} < 0$. In this case, an increase in the strength of adherent supporters implies a weakening of the structural strength of the regime. A microfounded account of this relationship could result from a prior stage, where the leadership must invest resources into structural characteristics or adherent strength but faces a budget constraint over the use of these resources. This would imply that more adherent strength leads to less structural strength, thus yielding the net substitution.

Finally, consider the case where adherent strength is a net complement with structural characteristics, i.e., $\frac{\partial w(\cdot)}{\partial \lambda} > 0$. This could result from a situation with significant technological complementarities to improving structural features because such improvements also increase the effectiveness of coordinated adherent support. This would reflect how certain technological improvements, such as improved surveillance, can improve the effectiveness of adherent support as well as the structural strength of the regime.

The equilibrium effect of $\lambda$ depends on the (local) relationship of the critical regime’s structural strength and the physical capacity of coordinated adherent support.

**Proposition 2** If $\Omega(W | \theta^*) > 0$, then there exists a bound $H < 0$ such that the critical state $\theta^*$ is (locally) decreasing with $\lambda$ if and only if $\frac{\partial w(\theta^*, \lambda)}{\partial \lambda} < H$ for $\lambda$ around the critical state $\theta^*$.

If the structural strength of the regime increases in the strength of adherent support, $\frac{\partial w(\theta^*, \lambda)}{\partial \lambda} > 0$, then an increase in the physical capacity of regime supporters increases the set of states for which the status quo survives. The more interesting case arises when there is a tradeoff between the contribution of $\lambda$ to the structural strength of the regime and the collective support of regime adherents. Provided that the structural strength of the regime
does not greatly decrease in the strength of adherents, i.e. \( \frac{\partial w(\theta^*, \lambda)}{\partial \lambda} \) is negative but small in magnitude, then increasing the strength of regime supporters increases the set of states for which the status quo survives. If there is a strong enough level of substitutability between the structural strength of the regime and the physical strength of adherents, then increasing the physical strength of the set of supporters actually decreases the set of regimes that can survive in equilibrium. If the net substitution effect is motivated by budgetary concerns, then this result suggests that regimes facing extreme financial pressures will disempower adherents. The relationship captured by \( w(\cdot, \lambda) \) is crucial to understanding the survival, and consequently, the incentives of regimes.

5 Public Information in a Two-Sided Global Game

As in most global game applications, the role of public information in our model is subtle but important. Public information motivates behavior beyond its ability to inform individuals regarding structural characteristics. Public signals serve to coordinate as well as inform. Specifically, the publicity of the public signal \( Q \) informs an opponent (adherent) about information possessed by other opponents (adherents) and hence, how they are likely to act — and importantly, the public signal provides information about how members of the other group will act. We restrict attention to the case when the equilibrium is unique so that conclusions remain sharp.

To make a comparison, we consider a benchmark where the set of regime opponents are the only strategic players, and the threshold level of participation required to incite regime change is the same as the induced threshold function from Lemma 3, \( W \). In other words, we take a function which depends only on \( \theta \), but whose values agree with the function \( W \). The corresponding equilibrium critical threshold \( \hat{\theta} \) is derived from Corollary 1.\(^9\) This is the critical threshold at which the value of the critical state matches that in our two-sided model with the exception that the threshold is induced through a

\(^9\)Note that the uniqueness restriction is the same, namely \( \Omega(W | \theta^*) > 0 \).
one-sided coordination problem, rather than a dueling coordination problem. This allows a comparison with a regime that has the same level of regime strength, but where the regime’s strength is not endogenously determined. We do this to account for the difference that is attributable to the feedback between coordination problems and not the difference that results from the added strength of regime adherents.

**Proposition 3** If \( \Omega(W \mid \theta^*) > 0 \), then the marginal effect of public information on the critical state is given by

\[
\frac{d\theta^*(Q)}{dQ} = \left( \frac{d\hat{\theta}_W(Q)}{dQ} \right) \cdot \left( 1 + \lambda \sqrt{\frac{\gamma}{\beta}} \left( \frac{\sqrt{\frac{\gamma}{\beta}} (Q - \theta^*) - \frac{1}{\sqrt{\kappa}} \Phi^{-1}(k)}{\phi(\Phi^{-1}(W(\theta^*)))} \right) \right),
\]

where

\[
\frac{d\hat{\theta}_W(Q)}{dQ} = -\frac{1 - \eta}{\Omega(W \mid \hat{\theta}_W)}.
\]

As regime adherents become arbitrarily well-informed relative to opponents, i.e. as \( \frac{\gamma}{\beta} \rightarrow 0 \), then \( \frac{d\theta^*(Q)}{dQ} \rightarrow \frac{d\hat{\theta}_W(Q)}{dQ} \).

Signals of regime strength which are commonly observed push up, in the sense of first-order stochastic dominance, all posteriors, both for opponents and adherents. In the unique equilibrium case, this decreases the set of states under which the regime fails, thereby improving the ability of the status quo to survive. Each individual is motivated by three distinct things. First, an increase in the public signal \( Q \) suggests the regime is stronger, and provides adherents with an incentive to support the regime because it is more likely to survive rebellion. Second, an individual adherent, knowing that other adherents will support, has a further incentive to support because of the improved robustness of the regime. A similar dynamic occurs within the set of opponents. These effects together, for adherents and opponents, detail the within-group coordination effect. Third, there is also a between-group coordination effect resulting from the relationship between the adherent cutoff have on the opponent cutoff. Increasing adherent support for the regime provides an added incentive for opponents to abstain because successful
rebellion is less likely.

To see the aggregate effect of public information on the survival prospects of the regime we examine (5). The first term of (5) measures the effect on the critical state resulting from the coordination within the set of regime opponents. It is the marginal effect of the public signal on the critical threshold one obtains from a canonical one-sided global game using $W$ as the threshold function. It measures how changes in opponent behavior induced by a change in the public signal translate into the critical state.

From the model we see that the conventional effect of public information obtained from a one-sided model, with a suitably adjusted threshold function, does not fully account for the effects of public information in a context which involves between-group coordination as well as within-group coordination. The second term of the product in (5) acts as a multiplier (since it always exceeds 1) that intensifies the impact of public information on individual behavior that results from anticipating the behavior of the other group. It depends on three things. First, it depends on the physical capacity of the set of regime adherents $\lambda$; larger $\lambda$ implies a larger contribution of regime adherents to regime strength. The coordination dilemma of regime adherents has a larger impact on outcomes as the importance of adherent support to regime strength grows. Second, the information disparity between groups, $\frac{\gamma}{\beta}$, intensifies the between-group coordination effect. The last term measures the proportion of the change in adherent support resulting from a change in the critical state.

We now focus on the level at which individuals in each group respond to public information in their action choices. A well known result in global game applications is that individuals “over-react” to public signals, since commonly observed signals provide reasonably good information about how others will behave. This result is stressed in Morris and Shin (2003), and applied to explain media effects in the relationship between protests and coups in Casper and Tyson (2014b). The setup of our model, and the discussion above, leads to a natural question: how does each group exploit public information?

Recall that the posterior expectation of $\theta$ for an opponent, given private signal $Y$, and
the posterior expectation of $\theta$ for an adherent, given private signal $Z$, are respectively given by

$$E[\theta|Y] = (1 - \eta)Q + \eta Y$$
$$E[\theta|Z] = (1 - \kappa)Q + \kappa Z.$$

Suppose that $\gamma < \beta$, implying that regime adherents’ private signals are more tightly clustered around the true value of the regime fundamental than the private signals of opponents. When $\gamma < \beta$, then $1 - \eta > 1 - \kappa$, implying that the public signal affects the posterior expectation of $\theta$ for opponents more than for adherents. Put simply, public information is relatively more informative to opponents regarding regime fundamentals than it is to regime adherents.

The total effect of a public signal on the behavior of a regime opponent, and a regime adherent, results from the total change in the signal an individual must receive in order to remain indifferent between his or her action. We first present the following lemma, which details how changes in the public signal influence the indifferent opponent and the indifferent adherent.

**Lemma 4** The equilibrium posterior cutoff for opponents and for adherents react identically to public signals, i.e.

$$\frac{\partial y^*(Q)}{\partial Q} = \frac{\partial \theta^*(Q)}{\partial Q} = \frac{\partial z^*(Q)}{\partial Q}.$$

The indifference criterion for each group, adherents and opponents, depends on cost considerations for the group and the critical state $\theta^*(Q)$. Since regime opponents (adherents) strategically respond to the influence that regime adherents (opponents) have on the critical state they must react to the total influence the public signal has on aggregate behavior. Importantly, this implies that individuals internalize the effect that each group’s collective action has on the critical state. Since individuals in our model are risk neutral, the monotone best-response for each group responds identically to changes in
the public signal.

To isolate the strategic response to public information from the informational response to public signals we follow Morris and Shin (2003, pg. 82) and examine what they call the “publicity multiplier.” Consider a marginal change in the public signal $Q$. The publicity multiplier is the ratio of the change in the private signal an individual must possess in order to remain indifferent between their actions, and the change in the private signal an individual must receive in order to retain the same posterior belief of $\theta$. In our model the publicity multiplier is different for each group. We denote the publicity multiplier for opponents by $\xi_y$, and the publicity multiplier for adherents by $\xi_z$.

**Proposition 4** If regime adherents are better informed than opponents, i.e. $\beta > \gamma$, then the strategic response of regime adherents to the public signal is larger than that for opponents, i.e. $\xi_z > \xi_y > 1$. In contrast, if opponents are more informed than adherents $\gamma > \beta$, then the strategic response of opponents to the public signal is larger than that for adherents, i.e. $\xi_y > \xi_z > 1$.

Commonly observed information often plays a role disproportionate to its informational content in global games, and in a two-sided global game with conflict this role is further exacerbated by the fact that each group is trying to anticipate the overreaction of the other group. Proposition 4 suggests that in a game with conflict between two groups, each of which faces both within-group and between-group strategic uncertainty, the effect of public information is greatly amplified relative to its impact in a one-sided global game.

When $\gamma < \beta$, meaning opponents are less well informed about the regime characteristics than adherents, the public signal has a stronger influence on opponent posterior beliefs of $\theta$ than on adherent posteriors of $\theta$. However, public signals induce a stronger strategic reaction by a regime adherent than by an opponent — because adherents are more informed about the underlying state. Although the public signal is relatively uninformative about the state of the world $\theta$ for adherents, it is still informative about the average expectations of opponents. As such, the public signal provides adherents with a strong signal of average opponent posteriors, causing adherents to respond to public
information more strongly than is warranted purely by informational content, or even the one-sided coordination of adherents. As opponents react to public information, regime adherents react to the anticipated reaction of opponents. Opponents also anticipate the reaction of adherents to their own reaction to the public signal $Q$. This process continues *ad infinitum*, and elucidates why regimes are so sensitive to media outlets. A weak public signal can cause regime adherents to ignore even privately held beliefs about regime strength due to fears of opponent confidence in regime change and the aggressive behavior that results. The regime outcome then has a relatively weak relationship with underlying regime fundamentals because both opponents and regime adherents react strongly to public sources of information.

6 Club Information

To better contrast the behavior of the two groups in our model, in this section we examine an information source that is commonly observed by one group but not the other. We call this style of information *club information* since it is available to a strict subset of the player set, i.e. a club. There are three different cases in which club information can be introduced into our baseline model: one could endow adherents with a club signal, one could endow opponents with a club signal, and one could give a distinct club signal to each group. For simplicity, we focus on the setting where adherents receive a club signal of the regime fundamental in addition to the public signal and conditionally independent private signals, and opponents only receive conditionally independent private signals and the public signal.

Formally, there is a signal $S$, which is obtained from $S = \theta + \tau_S$, where $\tau_S$ is normally distributed with mean 0 and precision $\delta$. This signal is commonly observed by adherents, but opponents do not observe it. Adherents observe the club signal and know that every other adherent observes the club signal, and so on. Simultaneously, every opponent

\[^{10}\text{If both groups observed the signal then its realization would incorporate into the public signal according to standard techniques.}\]
knows that adherents see a club signal, and every opponent knows that other opponents
know that adherents see the club signal, and so on.

Opponents (who do not observe the club signal) must form an expectation of the
behavior of adherents, which requires them to form an expectation of the value of the
club signal. Since opponents have heterogeneous beliefs about the underlying regime
fundamental $\theta$, they also have heterogeneous beliefs about the value of the club signal $S$,
i.e.

$$E[S \mid Y, Q] = E[\theta + \tau S \mid Y, Q] = \frac{\alpha Q + \gamma Y}{\alpha + \gamma} = y.$$ 

Since the club signal provides an added source of information, it also entails a second-
order stochastic dominant improvement in the alignment of actions within the group of
individuals that observe the club signal. However, this need not always result in a better
alignment of adherent actions with the true state of the world. For instance, if the regime
fundamental is sufficiently low, but the club signal is high, then the size of adherent sup-
port is high even though the regime ultimately fails. In this example, a large proportion
of adherents miscoordinate with the state of the world (although not necessarily with
each other).\textsuperscript{11} In contrast, providing adherents with better private signals simultane-
ously improves the alignment of adherent actions with each other, and the alignment of
adherent actions with the regime fundamental. A club signal aligns the actions of adher-
ents by helping them act in concert, but it does not always align adherent actions with
the state since the club signal can deviate from the true state of the world. Club signals
allow a group to coordinate better with each other without necessarily improving their
coordination with the underlying regime fundamental.

Although the analysis involves a number of technical issues (that we examine in the
appendix), our approach shares many of the features explored in the baseline two-sided
model presented above. The main difference is that there is an adherent indifference
condition and a critical threshold condition for each possible realization of the club sig-

\textsuperscript{11} We say that an individual miscoordinates whenever she ex post prefers to take a different ac-
tion (Chassang 2010).
nal, but there is a single opponent indifference condition that is insensitive to the club signal. To begin we detail the posterior beliefs of opponents, followed by a presentation of the induced strength of the regime. We then characterize and analyze the monotone equilibria.

### 6.1 Posterior Beliefs

The key difference from the baseline model of Section 2 follows from the fact that the symmetric monotone strategy of regime adherents, and consequently, the regime’s critical state, are sensitive to the value of the club signal, while the symmetric monotone strategy for opponents is not. A symmetric monotone strategy for an individual is a mapping from an individual posterior expectation of the regime fundamental to a cutoff. It is important to note that the monotone strategies, as well as the critical state, depend on the value of the public signal \( Q \); although, as above, we explicitly detail this dependence only when needed to avoid confusion. We will decompose the arguments for a regime adherent strategy into two parts: (1) the posterior depending on the private signal and public signal, and (2) the club signal. We do this separately so as to isolate the part of an adherent’s strategy that depends explicitly on the club signal. Taking as given an arbitrary symmetric monotone strategy for adherents, denoted by \( \pi(S) \), and an arbitrary symmetric monotone strategy for opponents, denoted \( \bar{y} \), a similar argument as Lemma 2 implicitly defines a function \( \bar{\theta}(\pi(S), \bar{y}) \) by

\[
w(\bar{\theta}(\pi(S), \bar{y}), \lambda) + \lambda E(\bar{\theta}(\pi(S), \bar{y}), \pi(S)) = R(\bar{\theta}(\pi(S), \bar{y}), \bar{y}).
\]  

(6)

This gives the unique critical regime strength that determines the strongest regime that fails, which follows by application of the Intermediate Value Theorem and its full proof is omitted. As a result, an outcome function is dictated by a critical state \( \bar{\theta}(S) : \mathbb{R}^2 \rightarrow [\theta_\lambda, \bar{\theta}_\lambda] \). Observe from (6), and the Implicit Function Theorem, that the outcome function \( \bar{\theta}(S) \) is a continuously differentiable function of the club signal \( S \).
We characterize a monotone Bayesian Nash equilibrium. A monotone equilibrium in our model with a club signal is characterized by a triple \((\tilde{z}^*(S), \tilde{y}^*, \tilde{\theta}^*(S))\), which gives the equilibrium monotone strategies for regime adherents and opponents, and the critical state at which the regime fails.

An adherent receives a private signal, a public signal, and a club signal. From these, she forms a posterior expectation of the regime fundamental \(\theta\) and decides to support the regime whenever she believes the regime fundamental is sufficiently high. The club signal affects adherent behavior through two channels. First, the club signal provides information about the regime fundamental, i.e. higher club signals suggest stronger regimes. Second, the club signal provides information about the actions of other adherents, i.e. higher club signals suggest that more aggregate adherent support for the regime will materialize. The sensitivity of adherent strategies to the value of the club signal implies that the critical state also depends on the club signal. Importantly, the club signal only improves the within-group coordination of the set of adherents. A club signal is the only kind of information source that can induce a within-group coordination effect without simultaneously leading to a between-group coordination effect.

Opponents in our model face a complex strategic problem. In addition to the strategic uncertainty induced from the conditionally independent private signals, there is also a level of between-group strategic uncertainty which affects the best-responses of opponents, and hence the critical state. However, what compounds the strategic uncertainty faced by opponents is that opponents with different private signals have different posterior expectations of the club signal itself. The club signal acts as a device which pulls all adherent posteriors in the same direction, but does not affect opponent posteriors.

To keep track of posteriors in the extended model it is useful to introduce some notation. For both opponents and adherents, posterior expectations of the state are linear in signals. Denote \(\kappa_1 = \frac{\beta}{\alpha+\beta+\delta}, \kappa_2 = \frac{\delta}{\alpha+\beta+\delta}\) and \(\kappa_3 = 1 - \kappa_1 - \kappa_2\), which give the importance in an adherent’s posterior of her private signal, the club signal, and the public signal, \footnote{Observe that the existence theorem of Milgrom and Weber (1985) guarantees the existence of a Bayesian Nash equilibrium.}
respectively. For opponents, denote \( \eta_1 = \frac{\gamma}{\alpha+\gamma+\delta} \), \( \eta_2 = \frac{\delta}{\alpha+\gamma+\delta} \), and \( \eta_3 = 1 - \eta_1 - \eta_2 \); which give similar quantities and are needed to express the conditional densities for opponents.

We first look at the joint distribution between the regime fundamental and the club signal from the perspective of opponents. Opponents are concerned with the random pair \((\theta, S)\), comprised of the regime fundamental and the club signal. For an opponent whose private posterior of \( \theta \) is given by \( y \), the covariance between the fundamental and the club signal is given by

\[
E[(\theta - y)(S - y)] = E[\theta(\theta + \tau_S)] = \frac{1}{\alpha + \gamma}.
\]

From this, the private opponent posterior of \( \theta \), characterized by \( y \), has a posterior distribution of the pair \((\theta, S)\) that is joint normal with mean \((y, y)\), and variance matrix\(^{13}\)

\[
\Sigma = \begin{pmatrix}
\frac{1}{\alpha+\gamma} & \frac{1}{\alpha+\gamma} \\
\frac{1}{\alpha+\gamma} & \frac{1}{\alpha+\gamma+\delta} & \frac{\alpha+\gamma+\delta}{\delta(\alpha+\gamma)}
\end{pmatrix}.
\]

It is important to note that the critical threshold depends on \( S \) as the club signal both informs and coordinates the actions of regime adherents.

### 6.2 Equilibrium

We follow the strategy of the argument from the baseline model. Let \( \tilde{\theta}^1(S) \) be the symmetric regime adherent conjecture of the critical state, which is function of \( S \) and provides the regime adherents’ conjectured mapping from a value of the club signal to a critical regime. We can then compute the induced strength of the regime.

**Lemma 5** Given a symmetric regime adherent conjecture of a critical regime \( \tilde{\theta}^1(S) \), the induced strength of the regime is

\[
\tilde{W}(\theta, S \mid \tilde{\theta}^1(S)) = w(\theta, \lambda) + \lambda \Phi \left( \frac{\sqrt{2}}{\kappa_1} \left( \kappa_1 \theta + \kappa_2 S + \kappa_3 Q - \tilde{\theta}^1(S) - \frac{1}{\sqrt{\alpha+\delta+\beta}} \Phi^{-1}(k) \right) \right),
\]

\(^{13}\)These expressions and the results on distributions are standard (Casella and Berger 2002, pg. 199)
which is strictly increasing in the regime fundamental θ, and strictly decreasing in the critical state conjecture $\tilde{\theta}^\dagger(S)$. Furthermore, the induced strength of the critical regime is

$$\tilde{W}(\tilde{\theta}^\dagger(S), S) = w(\tilde{\theta}^\dagger(S), \lambda) + \lambda \Phi \left( \frac{\sqrt{\gamma}}{\kappa_1} \left( (\kappa_1 - 1)\tilde{\theta}^\dagger(S) + \kappa_2 S + \kappa_3 Q - \frac{1}{\sqrt{\alpha + \beta + \delta}} \Phi^{-1}(k) \right) \right).$$

Lemma 5 expresses the induced strength of the regime which gives the level of opponent rebellion required to bring about regime change. From the perspective of an opponent, this threshold is a random variable of the realized value of the club signal and the regime fundamental. Further, the strength of the critical regime is uncertain to opponents because of changes in adherent support that result from noise in the club signal. In contrast to the baseline model, there is residual uncertainty over the coordination threshold. In particular, an opponent with private posterior $y$ has a posterior expectation of induced regime strength

$$E[\tilde{W}(\theta | \tilde{\theta}^\dagger(S)) | y] = \int_{-\infty}^{\infty} w(\tilde{\theta}^\dagger(S), \lambda) + \lambda E(\theta | \tilde{\theta}^\dagger(S)) \frac{\delta + \gamma + \delta}{\delta(\alpha + \gamma)} \phi \left( \sqrt{\delta(\alpha + \gamma)} (S - \tilde{y}^*) \right) dS.$$

Recall that in the baseline model all regime opponents held a common conjecture of the value of the critical regime $\theta^*$, but had heterogeneous beliefs about the likelihood of regime failure, i.e. the probability of the event \{\$\theta \leq \theta^*$\}. In contrast, in the model with club information opponents face a lack of agreement regarding the induced strength of the critical regime $\tilde{\theta}^*(S)$, as well as heterogeneous beliefs about the likelihood of the event \{\$\theta \leq \tilde{\theta}^*(S)$\}. In this sense, the presence of club information intensifies the coordination dilemma of regime opponents because it reduces the availability of common things to use in coordination.

Below, we show how this more involved problem can be reduced to a problem of a similar structure as the baseline model. Since opponents do agree on the mapping from realizations of the club signal to critical states, we can collapse the lack of agreement

\footnote{Specifically, individuals had heterogeneous $p$-beliefs (Monderer and Samet 1989), see also Hellwig (2002).}
between opponents onto private posteriors that depend on private signals, the public signal, and the correlation structure between the club signal and private posteriors.

We first present the characterization of symmetric monotone equilibria.

**Theorem 4** A symmetric monotone equilibrium exists and is characterized by a triple \((\tilde{z}^*(S), \tilde{y}^*, \tilde{\theta}^*(S))\) which simultaneously satisfies

\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\tilde{\theta}^*(S)} \sqrt{\frac{2}{\pi}} \phi \left( \frac{\theta - \frac{\alpha + \gamma}{\alpha + \gamma + \delta} \tilde{y}^* - \frac{\delta}{\alpha + \gamma + \delta} S}{\sqrt{\frac{8(\alpha + \gamma)}{\alpha + \gamma + \delta}}} \right) \phi \left( \frac{\delta(\alpha + \gamma)}{\alpha + \gamma + \delta} (S - \tilde{y}^*) \right) d\theta dS = C, \tag{7}
\]

\[
P(\theta > \tilde{\theta}^*(S) \mid \tilde{z}^*(S)) = k, \tag{8}
\]

\[
w(\tilde{\theta}^*(S), \lambda) + \lambda \mathcal{E}(\tilde{\theta}^*(S), \tilde{z}^*(S)) = \mathcal{R}(\tilde{\theta}^*(S), \tilde{y}^*). \tag{9}
\]

The first condition, (7), characterizes the opponent posterior which makes an opponent indifferent between rebelling and not. The opponent indifference condition contains two components. As can be seen from the inner integral, the value of the club signal provides information about the critical state, hence there is information about the critical state in the club signal.\(^{15}\) Opponents must incorporate this into their expectations and then integrate over \(S\) with respect to their privately held belief over the club signal. An indifferent opponent is one for whom the cost \(C\) equates with his posterior belief of the likelihood of regime failure.

The second condition, (8), characterizes the posterior expectation of the regime fundamental which makes an adherent indifferent between supporting the regime and abandoning the regime. It is similar to the condition determining adherent indifference in the baseline model. An adherent uses the public signal \(Q\), the club signal \(S\), and her private signal \(Z\) to judge the likelihood of regime failure and thus her optimal action. The last condition, (9), characterizes the critical state which determines the exact state at which the regime fails. The critical state \(\tilde{\theta}^*(S)\) is sensitive to the club signal because adherents are sensitive to the club signal. The club signal affects the proportion of adherents who support the regime and thus determine the level of opponent rebellion needed to overturn

\(^{15}\)See DeGroot (1970, p. 55) for derivations of conditional normal densities.
the regime through the induced strength of the regime.

It is important to note that opponents use the cutoff $\tilde{y}^*$ which is sensitive to the distribution of the club signal, but not to the content of the club signal. The introduction of a club signal puts restrictions on the set of parameters for which the monotone equilibrium of Theorem 4 is unique, as detailed in the appendix.

**Theorem 5** *For a fixed private information structure, $(\beta, \gamma)$, there are multiple monotone equilibria if and only if public information, $\alpha$, or club information, $\delta$, are sufficiently precise or if regime adherents are sufficiently powerful, i.e. $\lambda$ is sufficiently large.*

Theorem 5 shows that multiplicity of monotone equilibria results from three distinct things: (1) too precise public information, (2) too precise club information, or (3) a large enough disparity in the physical contribution of groups. Although the argument in the club signal case is more complicated (since it requires keeping track of a functional derivative), it is conceptually similar to the baseline case. Importantly, club signals can also, if sufficiently precise, induce multiple monotone equilibria. This model provides an example that shows that common knowledge between a strict subset of actors is sufficient to create a multiplicity of equilibria in a coordination game.

### 6.3 Club Information vs Public Information

Club signals are qualitatively similar to public signals, although the magnitude of their equilibrium effect is different. To complete our analysis of the club signal extension, we present a comparative static result detailing the impact of the club signal on the critical state.

**Proposition 5** *Consider the case where the monotone equilibrium is unique. The critical state is strictly decreasing in the club signal. Furthermore, if public information and club information are equally precise, i.e. $\alpha = \delta$, then the public signal has the larger equilibrium effect on the critical state. The club signal has a larger influence on the critical state than*
the public signal, i.e.
\[
\frac{d\tilde{\theta}^*(S, Q)}{dS} > \frac{d\tilde{\theta}^*(S, Q)}{dQ},
\]

if and only if \( \delta \) is sufficiently larger than \( \alpha \).

This result establishes that the critical state at which the regime fails decreases in the value of the club signal. However, this effect results solely from a change in the behavior of regime adherents. An increase in the club signal mobilizes the support from adherents by providing information and facilitating coordination. The aggregate effect is that it increases the set of regimes that survive.

Club information has a similar effect to public information with regard to the within-group coordination of adherents. However, the club signal does not produce a between-group coordination effect because it provides no information to opponents, or information about opponent behavior both to opponents and adherents. In order for a club signal to have the same effect on the equilibrium critical state as the public signal it must be sufficiently more informative to adherents. Although adherents use the club signal to coordinate, they do not use it as strongly as they use public information. A public signal is informative of both the behavior of adherents, and the behavior of opponents, whereas a club signal informs adherents only of the behavior of other adherents.

Our model details how different sources of information affect coordinated behavior. At the same level of precision, a public signal has a larger effect on equilibrium behavior than a club signal because commonly observed, and commonly interpreted, information is the most informative of strategic behavior. These results highlight the importance of the observability, and common interpretability, of information in coordination settings.

7 Political Instability

We now apply our model and its results to the context of political instability. Recently, the Arab Spring has refocused attention on the causes and dynamics of political rebellion and much attention has been given to the role of social media in coordinating rebels.
across a country and within major urban areas.\textsuperscript{16} Although social media sites like Facebook, Twitter, and YouTube, as well as media outlets such as Al Jazeera, Al Jazeera English, and BBC News contributed significantly, scholars and journalists alike have yet to fully understand the role of social media in the unfolding of these events.\textsuperscript{17} As events in Egypt, as well as Tunisia, Algeria, Russia, and Eastern Europe (Garton Ash 1990) have emphasized, it is the collective abandonment of regime elites that immediately precedes the fall of a regime. This observation suggests that the mobilization of citizens does not fully encapsulate the dynamics of rebellion, and by incorporating the comparative mobilization of both citizens and the set of elites who comprise the government into a model of coordination to better understand the factors underlying these events.

Existing accounts of political rebellion highlight the coordinated efforts of citizens who mobilize to overthrow the regime (Tilly 1978).\textsuperscript{18} But, as suggested by Karklins and Peterson (1993), political instability is typically a two-sided problem, and considering the internal dynamics between regime elites is an important step forward toward understanding how political regimes fail.

The set of regime elites in our model are meant to represent the different factions that comprise the government. These factions can be housed across branches of government, bureaucracies, or military branches. Regimes that have a well coordinated coalition of support among elites are generally more difficult to topple through popular action. Our results have several implications important for understanding situations of mass political action and political instability.

The results of the model suggest that repression by the regime simultaneously decreases a citizen’s willingness to challenge the regime and increases the willingness of a regime elite to support. Our results suggest that more attention should be given to

\textsuperscript{16}Regime change associated with the Arab Spring have occurred in Tunisia (January 2011), Egypt (February 2011), Libya (October 2011), and at the time of writing this the fighting in Syria has not yet resolved. For a survey of these events see Lynch (2012).

\textsuperscript{17}See Morozov (2011) for a discussion of the broad implications of internet and social media in authoritarian politics.

\textsuperscript{18}For a critique of the one-sidedness of existing accounts see Tullock (1987).
examining the internal effects of repressive strategies.\textsuperscript{19} Our results also suggest that the strategic response on the part of adherents to public information suggests why political leaders are so sensitive to public information. A commonly observed signal of regime weakness not only encourages rebellion, it also encourages regime adherents to abandon the regime, which further weakens the regime, thereby further emboldening opponents. Public signals of regime weakness create a self-fulfilling pessimism in the future viability of the regime among both the citizens on the street (opponents) and the elites that comprise the government (adherents).

Finally, we note that analysts and journalists alike typically base their assessments of political instability on information which is publicly available (since they presumably do not receive private information). At first glance, the analysts’ difficulty in making accurate predictions could be resolved by better quality public information. Unfortunately, this is not necessarily so. If public information is relatively accurate, then adherents and opponents are also able to condition their behavior on commonly observed signals. This belies an important implication for the study of revolutions which is not generally highlighted. When the precision of public information is too large, everyone has a good assessment of the true regime fundamental and, as a consequence of the complementarities between actors, there are multiple equilibria and the analyst’s ability to predict revolutionary outcomes requires the analyst to know also the equilibrium selection. But there is a tradeoff in predicting revolutionary outcomes from public information, the analyst faces a discontinuity at some particular level of public information. If the quality of public information is poor then predictions are consequently weak. Improving the quality of public information improves the analyst’s predictions — but only up to a point. Once public information is sufficiently precise there are multiple equilibria, and the theoretical prediction is indeterminant.\textsuperscript{20}

\textsuperscript{19}For a more involved analysis of repressive strategies see Casper and Tyson (2014a) and Shadmehr (2014).

\textsuperscript{20}It is important to distinguish our fictional analyst from a fictional econometrician. A fictional econometrician would observe a cross-section of political regimes and then make inferences drawing on this sample. As a consequence, a fictional econometrician can make estimates of the equilibrium selection (Angeletos and Pavan 2013; Grieco 2014). The analyst we discuss is akin to the type discussed
8 Conclusion

Building on the global-game approach, we develop a model of a two-sided, dueling coordination problem. The model we present contributes to the literature on global games. We extend the canonical global game of regime change (commonly employed in economics and political science) to capture conflict between two groups who must act in a decentralized manner. In our model there are two groups who are distinguished by their preference toward a status quo. One group (adherents) prefer the status quo while the other group (opponents) prefer the alternative. Our model provides a formal analysis of the effects introduced by imperfect coordination dynamics into a conflict between groups. Our model is a model of interrelated coordination where each group takes into account the collective action of another group, who in turn accounts for the behavior of the original group and explore implications of this feedback. Finally, we apply this model to the study of political instability, providing a model that incorporates a model of citizen rebellion with internal regime dynamics.

We have shown that in addition to precise public information, the relative strength of one group with respect to the other can introduce multiplicity. We examine how changes in the incentive parameters of one group influence the coordination problems of both groups. For instance, repression, by raising the cost of opponent participation in rebellion, has both a direct and two indirect effects. First, repression increases the opportunity cost of participating in mass political action and thus reduces opponent involvement. Second, the coordination between regime opponents and regime adherents results in a strategic feedback, which further reduces opponent rebellion and increases adherent support for the regime. Adherents expect less opponent involvement and thus some adherents prefer to support the regime who would not have done so if repression were lower.

We have also examined the effects of public information between two differentially informed groups in a dueling coordination problem. We show that the equilibrium re-

in Simon (1984), corresponding to a case study analyst or journalist on the ground.
sponse to public information is identical in both groups. Hence, regime adherents (who are better informed of regime fundamentals) strategically react more strongly to public information precisely because it is less informative to them about the state, but more informative about the actions of opponents.

Finally, we examine an extension of the model with information that is partially public, which we call club information. Club information is information that is available to one side, thus providing a commonly observed signal that one group uses to condition their behavior. The other side, having heterogeneous beliefs about the value of the club signal are at a disadvantage. We contrast the equilibrium impact of club information with public information. We find that club information, which is only informative of the behavior of one group, and thus does not exhibit between-group coordination, has a smaller effect on equilibrium outcomes than public information when the precision of these information sources are similar. Club information shuts down an important source of strategic feedback — between-group coordination — showing that club information cannot influence outcomes to the same extent as public information.

Appendix

Proof of Lemma 1: By definition, a regime which survives regardless of the actions of individuals must survive for all \( R \) and all \( E \). By monotonicity, if a regime survives when \( R = 1 \) and \( E = 0 \), then the regime survives for all values of \( R \) and \( E \). Since the regime survives if and only if \( w(\theta, \lambda) + \lambda E > R \), then it must be that \( w(\theta, \lambda) > 1 \) for all \( \theta > \bar{\theta}_\lambda \) and \( w(\theta, \lambda) \leq 1 \) for all \( \theta \leq \bar{\theta}_\lambda \). By continuity of \( w(\cdot) \) we have that \( w(\bar{\theta}_\lambda, \lambda) = 1 \).

By definition, a regime which fails regardless of the actions of individuals must survive for all \( R \) and all \( E \). By monotonicity, if a regime fails when \( R = 0 \) and \( E = 1 \), then the regime fails for all values of \( R \) and \( E \). Since the regime fails if and only if \( w(\theta, \lambda) + \lambda E \leq R \), then it must be that \( w(\theta, \lambda) + \lambda < 0 \) for all \( \theta \leq \bar{\theta}_\lambda \) and \( w(\theta, \lambda) + \lambda > 0 \) for all \( \theta > \bar{\theta}_\lambda \). By continuity of \( w(\cdot) \) we have that \( w(\bar{\theta}_\lambda, \lambda) = -\lambda \).
**Proof of Lemma 2:** Fix any monotone strategy profile, which is characterized by a pair \((z, y)\). From (3), it is clear that the left-hand side of (1) is strictly increasing in \(\theta\) and from (2) it is clear that the right-hand side is strictly decreasing in \(\theta\). By Assumption 1, notice the left-hand side of (1) exceeds 1 for some values of \(\theta\), and also is negative for some values of \(\theta\). Since both sides of (1) are continuous monotone functions of \(\theta\), by the Intermediate Value Theorem, there exists a unique \(\bar{\theta}(z, y)\) such that the regime fails if \(\theta \leq \bar{\theta}(z, y)\), and survives if \(\theta > \bar{\theta}(z, y)\). ■

**Proof of Lemma 3:** Holding \(y\) fixed, take \(\theta^1(y)\) implicitly defined by

\[w(\theta^1(y), \lambda) + \lambda\mathcal{E}(\theta^1(y), z) = \mathcal{R}(\theta^1(y), y).\]

Existence of \(\theta^1(y)\) follows from Lemma 2 and the Implicit Function Theorem. Given \(\theta^1(y)\), a regime adherent prefers to support the regime if her posterior expectation of \(\theta\) is such that

\[P(\theta > \theta^1(y) \mid z) > k,\]

and a regime adherent prefers to abandon the regime if her posterior expectation of \(\theta\) is such that

\[P(\theta > \theta^1(y) \mid z) < k.\]

Since beliefs are ordered according to posterior means by first-order stochastic dominance, and are continuous functions of \(z\), by the Intermediate Value Theorem there exists a unique \(z(\theta^1)\), which characterizes the regime adherent who is indifferent between supporting the leadership and not, specifically

\[P(\theta > \theta^1 \mid z(\theta^1)) = k.\]

Using the Gaussian information structure, this can be rewritten as

\[\Phi(\sqrt{\alpha + \beta(\theta^1 - z(\theta^1)}) = k,\]

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which rearranges to
\[ z(\theta^\dagger) = \theta^\dagger + \frac{1}{\sqrt{\alpha + \beta}} \Phi^{-1}(k). \] (10)

Recall that the turnout of adherents who support the regime, as a function of a monotone adherent strategy \( z \), can be written as
\[ E(\theta \mid z) = \Phi \left( \frac{\sqrt{\alpha}}{\kappa} (\kappa \theta + (1 - \kappa)Q - z) \right) \]
which by substitution from (10) is written in terms of the conjecture \( \theta^\dagger \),
\[ E(\theta \mid \theta^\dagger) = \Phi \left( \frac{\sqrt{\alpha}}{\kappa} \left( \kappa \theta + (1 - \kappa)Q - \theta^\dagger - \frac{1}{\sqrt{\alpha + \beta}} \Phi^{-1}(k) \right) \right), \]
which is the level of adherent support, as a function of the conjecture \( \theta^\dagger \). Define the function
\[ \mathcal{W}(\theta \mid \theta^\dagger) \equiv w(\theta, \lambda) + \lambda \Phi \left( \frac{\sqrt{\alpha}}{\kappa} \left( \kappa \theta + (1 - \kappa)Q - \theta^\dagger - \frac{1}{\sqrt{\alpha + \beta}} \Phi^{-1}(k) \right) \right), \] (11)
which is the induced threshold opponents face as a function of \( \theta^\dagger \). Uniqueness follows by construction. That \( \mathcal{W} \) is strictly increasing in \( \theta \) and strictly decreasing in \( \theta^\dagger \) follows by inspection. \( \blacksquare \)

**Proof of Theorem 1:** The regime opponent who is indifferent between rebelling and not is one whose posterior satisfies
\[ P(\theta \leq \theta^* \mid y^*) = C, \]
which using the Gaussian information structure implies
\[ \Phi(\sqrt{\alpha + \gamma}(\theta^* - y^*)) = C. \]
Rearranging, this yields the opponent indifference condition

\[ y_{OI}^* = \theta^* - \frac{1}{\sqrt{\alpha + \gamma}} \Phi^{-1}(C). \]

Now applying Lemma 3, which incorporates the best-response of regime adherents, the critical proportion of opponents needed for regime change is characterized by \( W(\theta \mid \theta^*). \) The opponent posterior expectation for which the size of the rebel force equals the induced threshold, call this posterior \( y_{CT}^* \), is given implicitly by

\[ W(\theta^*) = R(\theta^*, y_{CT}^*). \]

By substitution for \( R \) we have

\[ W(\theta^*) = \Phi \left( \frac{\sqrt{\gamma}}{\eta} (y_{CT}^* - \eta \theta^* - (1 - \eta)Q) \right), \]

which by substitution for \( W \) is

\[ w(\theta^*, \lambda) + \lambda \Phi \left( \frac{\alpha}{\sqrt{\beta}} (Q - \theta^*) - \frac{1}{\sqrt{\kappa}} \Phi^{-1}(k) \right) = \Phi \left( \frac{\sqrt{\gamma}}{\eta} (y_{CT}^* - \eta \theta^* - (1 - \eta)Q) \right). \]

Rearranging this equation to solve for \( y_{CT}^* \), yields the opponent critical threshold

\[ y_{CT}^* = (1 - \eta)Q + \eta \theta^* + \frac{\eta}{\sqrt{\beta}} \Phi^{-1} \left( w(\theta^*, \lambda) + \lambda \Phi \left( \frac{\alpha}{\sqrt{\beta}} (Q - \theta^*) - \frac{1}{\sqrt{\kappa}} \Phi^{-1}(k) \right) \right). \]

First, observe that

\[ W'(\theta^*) = \frac{\partial w(\theta^*, \lambda)}{\partial \lambda} - \lambda \frac{\alpha}{\sqrt{\beta}} \Phi \left( \frac{\alpha}{\sqrt{\beta}} (Q - \theta^*) - \frac{\sqrt{\beta}}{\kappa \sqrt{\alpha + \beta}} \Phi^{-1}(k) \right), \]

and so

\[ \frac{\partial y_{CT}^*(\theta^*, Q)}{\partial \theta^*} = \eta \left( 1 + \frac{W'(\theta^*)}{\sqrt{\gamma} \Phi \left( \Phi^{-1}(W(\theta^*)) \right)} \right). \quad (12) \]
By Lemma 1, the opponent critical threshold satisfies the limit condition,

\[
\lim_{\theta \to \overline{\theta}} y_{CT}^*(\theta, Q) = (1 - \eta)Q + \eta \Phi^{-1} \left( w(\theta, \lambda) + \lambda \Phi \left( \frac{\alpha}{\sqrt{\beta}} (Q - \theta) - \frac{1}{\sqrt{\alpha}} \Phi^{-1}(k) \right) \right) = -\infty,
\]

which holds since \( w(\lambda) = -\lambda \), establishing the limit. Similarly, by Lemma 1, \( w(\lambda) = 1 \), so

\[
\lim_{\theta \to \overline{\theta}} y_{CT}^*(\theta, Q) = (1 - \eta)Q + \eta \Phi^{-1} \left( w(\lambda) + \lambda \Phi \left( \frac{\alpha}{\sqrt{\beta}} (\theta - \lambda) - \frac{1}{\sqrt{\alpha}} \Phi^{-1}(k) \right) \right) = +\infty.
\]

These limit conditions, along with continuity, imply there exists a \( \theta^* \) such that \( y_{OI}^*(\theta^*) = y_{CT}^*(\theta^*, Q) \), establishing that a symmetric cutoff equilibrium exists.

Combining these limit conditions with the corresponding limits for \( y_{OI}^* \), which are finite at both ends, we note that uniqueness holds for all \( Q \) under the condition that \( \frac{\partial y_{OI}^*(\theta^*)}{\partial \theta} > \frac{\partial y_{CT}^*(\theta^*)}{\partial \theta^*} \). By substitution in (12), and since \( \frac{\partial y_{OI}^*(\theta^*)}{\partial \theta^*} = 1 \), the necessary and sufficient condition for uniqueness is,

\[
\min_{\theta^* \in (\overline{\theta}, \lambda)} \frac{\partial w(\theta^*, \lambda)}{\partial \theta} \Phi^{-1} \left( \phi^{-1} \left( w(\theta^*, \lambda) + \lambda \Phi \left( \frac{\alpha}{\sqrt{\beta}} (Q - \theta^*) - \frac{1}{\sqrt{\alpha}} \Phi^{-1}(k) \right) \right) \right) > \frac{\alpha}{\sqrt{\gamma}}, \tag{13}
\]

establishing the theorem.

Note, the unique equilibrium is characterized by \( y_{CT}^*(\theta^*, Q) = y_{OI}^*(\theta^*, Q) \), which is

\[
\sqrt{\frac{\alpha}{\beta}} \Phi^{-1} \left( w(\theta^*, \lambda) + \lambda \Phi \left( \frac{\alpha}{\sqrt{\beta}} (Q - \theta^*) - \frac{1}{\sqrt{\alpha}} \Phi^{-1}(k) \right) \right) = (\theta^* - Q) - \sqrt{\frac{\alpha + \gamma}{\alpha}} \Phi^{-1}(C). \tag{14}
\]
Proof of Theorem 2: Assume that regime adherents are following a strategy of always support, and regime opponents are following a strategy of never rebel. Call these cutoff strategies,

\[ z_0 = -\infty, \quad y_0 = -\infty, \]

which implies that the regime fails if and only if \( \theta \leq \theta_0 = \theta_\lambda \). An adherent has a dominant strategy to desert the regime if her posterior expectation of the regime fundamental \( z \), is such that

\[ P(\theta \leq \theta_0 \mid z) < k. \]

Define

\[ z_1 = \sup_z P(\theta < \theta_\lambda \mid z) < k, \]

and compute the proportion of regime adherents who do not have a dominant strategy to abandon the regime

\[ \mathcal{E}(\theta, z_1) = P(z \geq z_1 \mid \theta). \]

From this, find the induced threshold faced by regime opponents,

\[ \mathcal{W}(\theta, z_1) = w(\theta, \lambda) + \lambda \mathcal{E}(\theta, z_1). \]

Since \( \eta \left(1 + \min_{\theta^* \in (0, \theta_\lambda)} \frac{\mathcal{W}'(\theta^*)}{\sqrt{\sigma(\mathcal{W}(\theta^*))}}\right) - 1 = \Omega(\mathcal{W} \mid \theta^*) > 0 \), then the numerator \( \mathcal{W}'(\theta^*) > 0 \), thus \( \mathcal{W}(\theta, z_1) \) is strictly increasing in \( \theta \), and hence since by definition \( \mathcal{W}(\theta_\lambda, z_0) = 0 \), it must be that

\[ \hat{\theta}_1 = \sup_{\theta} w(\theta, \lambda) + \lambda \mathcal{E}(\theta, z_1) < 0. \]

Now moving on to regime opponents. An opponent who has received a signal \( y \) has a dominant strategy to rebel if

\[ P(\theta \leq \hat{\theta}_1 \mid y) > C. \]
Compute the proportion of opponents who rebel

\[ R(\theta, y_1) = P(y \leq y_1 \mid \theta) > 0, \]

then define

\[ y_1 = \sup_y P(\theta \leq \hat{\theta}_1 \mid y) > C. \]

And then define

\[ \theta_1 = \sup_{\theta} W(\theta, z_1) < R(\theta, y_1). \]

Since \( R \) is strictly decreasing in \( \theta \), and \( W \) is strictly increasing in \( \theta \) (a consequence of \( \Omega > 0 \)), we have that \( \theta_0 < \hat{\theta}_1 < \theta_1 \).

Likewise, we can define cutoff strategies in which no adherent supports, and every opponent rebels,

\[ z^0 = +\infty, \quad y^0 = +\infty, \]

which implies the regime fails if and only if \( \theta \leq \theta^0 = \bar{\theta}_0 \). A regime adherent has a dominant strategy to support the regime if and only if

\[ P(\theta \leq \theta^0 \mid z) < 1 - k. \]

Define

\[ z^1 = \inf_z P(\theta \leq \theta^0 \mid z) < 1 - k, \]

and compute the proportion of adherents who do not have a dominant strategy to abandon the regime,

\[ \mathcal{E}(\theta, z^1) = P(z \geq z^1 \mid \theta). \]

From this, find the induced threshold faced by opponents,

\[ W(\theta, z^1) = w(\theta, \lambda) + \lambda \mathcal{E}(\theta, z^1). \]
Since $\Omega(W \mid \theta^*) > 0$, then $W(\theta, z^1)$ is increasing in $\theta$, and hence since by definition $W(\bar{\theta}_\lambda, z^0) = 1$, it must be that

$$\hat{\theta}^1 = \inf_{\theta} w(\theta, \lambda) + \lambda E(\theta, z^1) > 1.$$ 

Moving on to regime opponents, an opponent who has received a signal $y$ has a dominant strategy to support if

$$P(\theta \leq \hat{\theta}^1 \mid y) > 1 - C,$$

so define

$$y^1 = \inf_y P(\theta \leq \hat{\theta}^1 \mid y) > 1 - C.$$ 

And then define

$$\theta^1 = \inf_{\theta} W(\theta, z^1) < R(\theta, y^1).$$

Now since $R$ is strictly decreasing in $\theta$, and $W$ is strictly increasing in $\theta$, we have that $\bar{\theta}_\lambda > \hat{\theta}^1 > \theta^1$.

Proceeding in this way, we recursively define the following sequences

\begin{align*}
z_n &= \sup_z P(\theta < \theta_{n-1} \mid z) < k \\
\hat{\theta}_n &= \sup_{\theta} w(\theta, \lambda) + \lambda E(\theta, z_n) < R(\theta, y_{n-1}) \\
y_n &= \sup_y P(\theta < \hat{\theta}_n \mid y) > C \\
\theta_n &= \sup_{\theta} W(\theta, z_n) < R(\theta, y_n)
\end{align*}
and

\[ z^n = \inf_z P(\theta < \theta^{n-1} \mid z) < 1 - k \]

\[ \hat{\theta}^n = \inf_{\theta} w(\theta, \lambda) + \lambda \mathcal{E}(\theta, z^n) > \mathcal{R}(\theta, y^{n-1}) \]

\[ y^n = \inf_y P(\theta < \hat{\theta}^{n-1} \mid z) > 1 - C \]

\[ \theta^n = \inf_{\theta} \mathcal{W}(\theta, z^n) < \mathcal{R}(\theta, y^n) \]

Denote these two sequences, \( \varrho_n = (z_n, y_n, \theta_n) \) and \( \varrho^n = (z^n, y^n, \theta^n) \), and the point \( \varrho^* = (z^*, y^*, \theta^*) \). The proof proceeds by two lemmas. The first lemma establishes that the two sequences, \( \{\varrho_n\}_{n=1}^{\infty} \) and \( \{\varrho^n\}_{n=1}^{\infty} \), are monotonic in the componentwise order on \( \mathbb{R}^3 \). The second lemma demonstrates that whenever there is a unique monotone equilibrium then \( \{\varrho_n\}_{n=1}^{\infty} \) and \( \{\varrho^n\}_{n=1}^{\infty} \) have a common limit.

**Lemma 6** If \( \Omega(\mathcal{W} \mid \theta^*) > 0 \), then the sequence \( \{\varrho_n\}_{n=1}^{\infty} \) is a strictly increasing convergent sequence, and the sequence \( \{\varrho^n\}_{n=1}^{\infty} \) is a strictly decreasing convergent sequence.

**Proof:** We proceed by induction. The base case is established in the construction of \( \varrho_n = (z_n, y_n, \theta_n) \) and \( \varrho^n = (z^n, y^n, \theta^n) \). Now, suppose \( \varrho_0 < \varrho_1 < \cdots < \varrho_{n-1} \) and \( \varrho^0 > \varrho^1 > \cdots \varrho^{n-1} \), so then by the componentwise ordering on \( \mathbb{R}^3 \),

\[ y_0 < \cdots < y_{n-1} \quad , \quad y^0 > \cdots > y^{n-1} \]

\[ z_0 < \cdots < z_{n-1} \quad , \quad z^0 > \cdots > z^{n-1} \]

\[ \theta_0 < \cdots < \theta_{n-1} \quad , \quad \theta^0 > \cdots > \theta^{n-1} \]

for all \( \varrho_i \), with \( 1 \leq i \leq n - 1 \).

By definition

\[ z_{n-1} = \sup_z P(\theta < \theta_{n-2} \mid z) < k, \]

and

\[ z_n = \sup_z P(\theta < \theta_{n-1} \mid z) < k. \]
By the inductive assumption $\theta_{n-1} > \theta_{n-2}$, which by monotonicity implies

$$P(\theta < \theta_{n-2} \mid z) < P(\theta < \theta_{n-1} \mid z)$$

for all $z$. Since $P(\theta < \theta_{n-2} \mid z_{n-1}) = P(\theta < \theta_{n-1} \mid z_n) = k$, it must be that $z_{n-1} < z_n$.

Note that by definition,

$$\hat{\theta}_{n-1} = \sup_\theta w(\theta, \lambda) + \lambda E(\theta, z_{n-1}) < \mathcal{R}(\theta, y_{n-2}),$$

and

$$\hat{\theta}_n = \sup_\theta \mathcal{W}(\theta, z_n) < \mathcal{R}(\theta, y_{n-1}).$$

Notice since $\Omega(\mathcal{W} \mid \theta^*) > 0$, the left-hand side is strictly increasing in $\theta$ and strictly decreasing in $z$, while the right-hand side is strictly decreasing in $\theta$ and strictly increasing in $y$, thus $\hat{\theta}_n > \hat{\theta}_{n-1}$; and furthermore, $\hat{\theta}_n > \theta_{n-1}$.

Similarly, by definition

$$y_{n-1} = \sup_y P(\theta < \hat{\theta}_{n-1} \mid y) > C,$$

and

$$y_n = \sup_y P(\theta < \hat{\theta}_n \mid y) > C.$$

Since $\hat{\theta}_n > \hat{\theta}_{n-1}$, by strict monotonicity,

$$P(\theta < \hat{\theta}_n \mid y) > P(\theta < \hat{\theta}_{n-1} \mid y)$$

for all $y$, hence $y_n > y_{n-1}$.

Now consider

$$\theta_n = \sup_\theta \mathcal{W}(\theta, z_n) < \mathcal{R}(\theta, y_n).$$
Since $R(\theta, y_n) = W(\theta, z_n)$ for all $n$, we have

$$R(\theta_{n-1}, y_n) > R(\theta_{n-1}, y_{n-1}) = W(\theta_{n-1}, z_{n-1}) > W(\theta_{n-1}, z_n),$$

so

$$R(\theta_{n-1}, y_n) > W(\theta_{n-1}, z_n).$$

Now since $R$ is strictly decreasing in $\theta$, and $W$ is strictly increasing in $\theta$ (a consequence of $\Omega > 0$), we have that $\theta_{n-1} < \theta_n < \theta_n$. Combining these establishes that $\{\varrho_n\}_{n=1}^\infty$ is a strictly increasing sequence. A symmetric argument shows that $\{\varrho^n\}_{n=1}^\infty$ is a strictly decreasing sequence.

For convergence, note that by construction $\varrho_n$ is bounded above by $\varrho^*$ for all $n$. Similarly, $\varrho^n$ is bounded below by $\varrho^*$ for all $n$. Since $\{\varrho_n\}_{n=1}^\infty$ is a strictly increasing sequence which is bounded above, and $\{\varrho^n\}_{n=1}^\infty$ is a strictly decreasing sequence which is bounded below, both sequences converge.

Lemma 7 If $\Omega(W | \theta^*) > 0$, then the sequences $\{\varrho_n\}_{n=1}^\infty$ and $\{\varrho^n\}_{n=1}^\infty$ have a common limit point $\varrho^*$.

Proof: We proceed by contradiction, and show that when $\Omega(W | \theta^*) > 0$, if the sequences $\varrho^n$ and $\varrho_n$ do not converge to a common limit point, then one can eliminate more strategy profiles. Suppose that $\varrho^*$ is not a limit point of $\varrho_n$. Then since $\varrho_n$ converges, there must exist another limit point $\varrho_\infty < \varrho^*$. By monotonicity, $\varrho_n < \varrho_\infty < \varrho^*$ for all $n$, which means that

$$z_\infty < z^*, \quad y_\infty < y^*, \quad \theta_\infty < \theta^*.$$  

Taking $z_\infty$ and $y_\infty$, consider the following point:

$$\theta' = \sup_\theta W(\theta, z_\infty) < R(\theta, y_\infty), \quad (15)$$

whose existence is ensured by Lemma 2. Note that $R(\theta', y_\infty) = W(\theta', z_\infty)$, and incorpo-
rating $\theta'$ into the following:

$$z' = \sup_z P(\theta < \theta' \mid z) < k$$

$$y' = \sup_y P(\theta < \theta' \mid y) > C.$$  

There are two cases. First, if either $z' \neq z_\infty$ or $y' \neq y_\infty$, then this shows there is another point in the sequence which exceeds the supremum of the sequence, contradicting that $z_\infty$ or $y_\infty$ was indeed a limit point. Second, if $z' = z_\infty$ and $y' = y_\infty$, then by (15), $\theta' = \theta^*$, which contradicts that $\theta^* \neq \theta_\infty$. A symmetric argument establishes that $\theta^*$ is the limit point of $\{\theta^n\}_{n=1}^\infty$ which proves the lemma. ■

Combining our construction above, with Lemma 6 and Lemma 7, establishes that whenever $\Omega(W \mid \theta^*) > 0$, meaning condition (13) is satisfied, then iterated elimination of strictly dominated strategies eliminates all strategy profiles except for the strategy profile given by the monotone equilibrium identified in Theorem 1. ■

**Proof of Theorem 3:** We show that the set of parameters for which uniqueness of the monotone equilibrium holds is an open set contained in a compact subset of $\mathbb{R}_+^2$. From Theorem 1 the condition

$$\min_{\theta^* \in (\Phi\theta)} \frac{\partial w(\theta^*, \lambda)}{\partial \theta} - \lambda \frac{\alpha}{\sqrt{\beta}} \phi(\frac{\alpha}{\sqrt{\beta}}(Q - \theta^*) - \frac{1}{\sqrt{\kappa}}\Phi^{-1}(k)) \leq \frac{\alpha}{\sqrt{\gamma}}$$

is necessary and sufficient for equilibrium multiplicity. Taking limits as $\alpha \to \infty$, then

$$\min_{\theta^* \in (\Phi\theta)} \frac{\partial w(\theta^*, \lambda)}{\partial \theta} - \lambda \frac{\alpha}{\sqrt{\beta}} \phi(\frac{\alpha}{\sqrt{\beta}}(Q - \theta^*) - \frac{1}{\sqrt{\kappa}}\Phi^{-1}(k)) \to -\infty$$

$$\lim_{\alpha \to \infty} \frac{\alpha}{\sqrt{\gamma}} = \infty,$$

thus satisfying the necessary and sufficient condition for equilibrium multiplicity. Note the value $\bar{\alpha}$ such that multiplicity maintains if $\alpha > \bar{\alpha}$ is given by $\bar{\alpha} = \sqrt{\beta} \left( \frac{\partial w(\theta^*, \lambda)}{\partial \theta} \right)_{\lambda \phi(\frac{\alpha}{\sqrt{\beta}}(Q - \theta^*) - \frac{1}{\sqrt{\kappa}}\Phi^{-1}(k))}$.  

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Taking limits as $\lambda \to \infty$, then

$$\lim_{\lambda \to \infty} \min_{\theta^* \in (\theta, \theta)} \frac{\partial w(\theta^*, \lambda)}{\partial \theta} - \lambda \frac{\alpha}{\sqrt{\alpha}} \frac{\partial}{\partial \theta} \left( \frac{\alpha}{\sqrt{\beta}} (Q - \theta^*) - \frac{1}{\sqrt{\kappa}} \Phi^{-1}(k) \right) < 0 < \frac{\alpha}{\sqrt{\gamma}},$$

thus satisfying the necessary and sufficient condition for equilibrium multiplicity. Since $w$ is strictly increasing and Lipschitz continuous, $\frac{\partial w(\theta^*, \lambda)}{\partial \theta}$ is bounded by a positive number.

Note that for $\lambda$ large enough, the numerator in (16) is negative. This implies that there exists a $\Lambda$ such that if $\lambda > \Lambda$ then (16) is satisfied.

Consider the set

$$\{(\alpha, \lambda) \in \mathbb{R}_+^2 | \alpha \leq \alpha_0 \text{ and } \lambda \leq \Lambda\}$$

which is a compact set in $\mathbb{R}_+^2$. It is nonempty since it contains the origin. Notice that the set of parameters $(\alpha, \lambda)$ for which uniqueness maintains is the interior of the set defined in (17).

**Proof of Proposition 1:** Total differentiation with equation (14) shows

$$\frac{\partial \theta^*}{\partial C} = -\frac{1}{\sqrt{\alpha + \gamma} \phi(\Phi^{-1}(C))} \Omega(W | \theta^*) < 0.$$

Now total differentiation with equation (14) shows

$$\frac{\partial \theta^*}{\partial k} = \frac{\lambda}{\sqrt{\pi}} \frac{\Phi \left( \frac{\alpha}{\sqrt{\beta}} (Q - \theta^*) - \frac{1}{\sqrt{\kappa}} \Phi^{-1}(k) \right)}{\phi(\Phi^{-1}(W(\theta))) \phi(\Phi^{-1}(k))} \Omega(W | \theta^*) > 0.$$

From above, observe that

$$y^* = \theta^* - \frac{1}{\sqrt{\alpha + \gamma}} \Phi^{-1}(C),$$

and

$$z^* = \theta^* + \frac{1}{\sqrt{\alpha + \beta}} \Phi^{-1}(k), \quad (18)$$
so the comparative statics of the cutoffs follow by simple calculus.

**Proof of Proposition 2:** Total differentiation with equation (14) shows

\[
\frac{\partial \theta^*}{\partial \lambda} = \frac{\sqrt{\gamma}}{\alpha \Omega \Phi(W(\theta^*)))} \cdot \left[ \frac{\partial w(\theta^*, \lambda)}{\partial \lambda} + \Phi \left( \frac{\alpha}{\sqrt{\beta}} (Q - \theta^*) - \frac{1}{\sqrt{\kappa}} \Phi^{-1}(k) \right) \right],
\]

which is positive as long as \( w(\theta, \lambda) \) is increasing in \( \lambda \), or, if negative, it is sufficiently flat in \( \lambda \) so as not to dominate. In contrast, (19) is negative if \( \frac{\partial w(\theta^*, \lambda)}{\partial \lambda} \) is negative and of sufficiently large magnitude.

**Proof of Proposition 3:** Total differentiation with equation (14) shows

\[
\frac{\partial \theta^*}{\partial Q} = \frac{(1 - \eta) + \lambda \eta}{\frac{\phi \left( \frac{\alpha}{\sqrt{\beta}} (Q - \theta^*) - \frac{1}{\sqrt{\kappa}} \Phi^{-1}(k) \right)}{\phi(\Phi^{-1}(W(\theta^*)))}} \Omega(W | \theta^*) < 0.
\]

From Corollary 1 one obtains

\[
\frac{\partial \hat{\theta}_W^*}{\partial Q} = -\frac{(1 - \eta)}{\Omega(W | \theta^*)}.
\]

The result follows by substitution in (5), and rearranging. The limit result follows by taking limits in (5).

**Proof of Lemma 4:** Notice that the indifferent opponent posterior can be written as

\[
y^*(Q) = \theta^*(Q) - \frac{1}{\sqrt{\alpha + \gamma}} \Phi^{-1}(C).
\]

Combining this with

\[
z^*(Q) = \theta^*(Q) + \frac{1}{\sqrt{\alpha + \beta}} \Phi^{-1}(k),
\]

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and using simple calculus gives the result. ■

**Proof of Proposition 4:** We begin by finding the change in the private signal which compensates for a change in $Q$. Let $L$ be a constant. Holding the posterior expectation of $\theta$ for an opponent fixed at $L$, total differentiation in

$$(1 - \eta)Q + \eta Y = L,$$

gives that for an opponent to remain informationally indifferent, his private signal must change by $-\frac{1 - \eta}{\eta}$. Similarly, from

$$(1 - \kappa)Q + \kappa Z = L$$

for an adherent, her private signal must change by $-\frac{1 - \kappa}{\kappa}$ for her to remain informationally indifferent.

The indifferent posterior for an opponent is given by

$$y^*(Q) = \theta^*(Q) - \frac{1}{\sqrt{\alpha + \gamma}} \Phi^{-1}(C),$$

which implies that there exists a unique private signal $Y^*$ for which this indifference condition is true. By substitution, and rearranging, the private signal which leaves an opponent indifferent at a given $Q$ is

$$Y^* = \frac{1}{\eta} \theta^*(Q) - \frac{1 - \eta}{\eta} Q - \frac{1}{\eta \sqrt{\alpha + \gamma}} \Phi^{-1}(C).$$

which by simple calculus, the marginal effect of the public signal on the indifferent opponent private signal is

$$\frac{\partial Y^*}{\partial Q} = \frac{1}{\eta} \cdot \frac{\partial \theta^*(Q)}{\partial Q} - \frac{1 - \eta}{\eta}.$$
Similarly, for the adherent indifferent posterior $z^*(Q)$, there exists a unique private signal $Z^*$ for a given $Q$. The marginal effect of the public signal on the indifferent adherent private signal is

$$\frac{\partial Z^*}{\partial Q} = \frac{1}{\kappa} \cdot \frac{\partial \theta^*(Q)}{\partial Q} - \frac{1 - \kappa}{\kappa}.$$

The publicity multiplier is the ratio of the change in the private signal which makes an individual indifferent between their actions, taken from above, and the change in the private signal which makes the posterior expectation of $\theta$ unchanged. Specifically,

$$\xi_y = -\frac{\partial Y^*}{\partial Q}/\frac{1 - \eta}{\eta}$$

$$\xi_z = -\frac{\partial Z^*}{\partial Q}/\frac{1 - \kappa}{\kappa}$$

The result follows by showing that the magnitude of the publicity multiplier for adherents is strictly larger than the magnitude of the publicity multiplier for opponents, i.e. $|\xi_z| > |\xi_y|$. This is true if and only if

$$1 - \left(\frac{1}{1 - \kappa}\right)\left(\frac{\partial \theta^*}{\partial Q}\right) > 1 - \left(\frac{1}{1 - \kappa}\right)\left(\frac{\partial \theta^*}{\partial Q}\right).$$

Canceling terms gives the condition

$$\left(\frac{1}{1 - \kappa}\right)\left(-\frac{\partial \theta^*}{\partial Q}\right) > \left(\frac{1}{1 - \eta}\right)\left(-\frac{\partial \theta^*}{\partial Q}\right).$$

Since $\frac{\partial \theta^*}{\partial Q} < 0$ from Proposition 3, this condition reduces to

$$1 - \eta > 1 - \kappa$$

which is true if and only if $\gamma < \beta$, and reverses if and only if $\gamma > \beta$. ■
Proof of Lemma 5: First, for a given critical state $\tilde{\theta}^\dagger(S)$, consider the indifferent adherent whose posterior is such that it satisfies

$$P(\theta > \tilde{\theta}^\dagger(S) \mid \tilde{z}(S)) = k,$$

which by the Gaussian information structure can be written

$$\Phi(\sqrt{\alpha + \beta + \delta(\tilde{z}(S) - \tilde{\theta}^\dagger(S))}) = k.$$ 

Rearranging this, we have

$$\tilde{z}(S) = \tilde{\theta}^\dagger(S) + \frac{1}{\sqrt{\alpha + \beta + \delta}}\Phi^{-1}(k), \tag{22}$$

which defines, as a function of a given critical state, the posterior value for which a regime adherent is indifferent between supporting the regime and abandoning the regime.

Using that $\tilde{z}(S)$ is a monotone strategy, and substituting it into $\mathcal{E}(\theta, \tilde{z}(S))$, for a given opponent monotone strategy $\tilde{y}$, there is a critical state $\tilde{\theta}^*(S, \tilde{y})$, implicitly defined by

$$w(\tilde{\theta}^*(S, \tilde{y}), \lambda) + \lambda \mathcal{E}(\tilde{\theta}^*(S, \tilde{y}), \tilde{\theta}^*(S, \tilde{y}) + \frac{1}{\sqrt{\alpha + \beta + \delta}}\Phi^{-1}(k)) = \mathcal{R}(\tilde{\theta}^*(S, \tilde{y}), \tilde{y}). \tag{23}$$

Using the Implicit Function Theorem, $\tilde{\theta}^*(S, \tilde{y})$ is continuously differentiable in $S$ and $\tilde{y}$. Observe that as $\tilde{y} \to \infty$, $\mathcal{R} \to 1$ and so $\tilde{\theta}^*(S) \leq \tilde{\theta}_\lambda$ for all $S$, and as $\tilde{y} \to -\infty$, $\mathcal{R} \to 0$ and so $\tilde{\theta}^*(S) \geq \theta_\lambda$ for all $S$. This implies that the set of critical state functions is a subset of the set of bounded continuously differentiable functions.

Finally by substitution from (22) we have that the aggregate turnout of adherents is given by

$$\mathcal{E}(\theta, S \mid \tilde{\theta}^\dagger(S)) = \Phi\left(\frac{\sqrt{\beta}}{\kappa_1} \left(\kappa_1 \theta + \kappa_2 S + \kappa_3 Q - \tilde{\theta}^\dagger(S) - \frac{1}{\sqrt{\alpha + \beta + \delta}}\Phi^{-1}(k)\right)\right),$$
and then by substitution, we compute the induced strength of the regime:

\[
\tilde{W}(\theta, S \mid \tilde{\theta}(S)) = w(\theta, \lambda) + \lambda \Phi \left( \frac{\sqrt{\kappa}}{\kappa_1} \left( \kappa_1 \theta + \kappa_2 S + \kappa_3 Q - \frac{1}{\sqrt{\alpha+\delta+\delta}} \Phi^{-1}(k) \right) \right).
\]

That it is strictly increasing in \( \theta \) and strictly decreasing in \( \tilde{\theta}(S) \) follows by inspection.

Considering \( \theta = \tilde{\theta}(S) \), the induced strength of the critical regime is

\[
\tilde{W}(\tilde{\theta}(S), S) = w(\tilde{\theta}(S), \lambda) + \lambda \Phi \left( \frac{\sqrt{\kappa}}{\kappa_1} \left( (\kappa_1 - 1) \tilde{\theta}(S) + \kappa_2 S + \kappa_3 Q - \frac{1}{\sqrt{\alpha+\delta+\delta}} \Phi^{-1}(k) \right) \right).
\]

\[\blacksquare\]

**Proof of Theorem 4:** To characterize a monotone equilibrium, we must determine the monotone best-reply for opponents. From DeGroot (1970, pg. 55), the density of \( \theta \), conditional on \( S \) and opponent private posterior \( y \), can expressed as

\[
(2\pi)^{-\frac{1}{2}} \left( \frac{|\Sigma_{22}|}{|\Sigma|} \right)^{-\frac{1}{2}} \exp \left( \frac{-X_1}{2} \right), \tag{24}
\]

where

\[
X_1 = \left( \theta - y - \frac{1}{\alpha + \gamma} \left( \frac{\delta}{\alpha + \gamma + \delta} \right) (S - y) \right)^2 (\alpha + \gamma + \delta).
\]

By substitution, the density (24) can be expressed as

\[
\sqrt{\frac{\alpha}{\eta_1}} \phi \left( \sqrt{\frac{\alpha}{\eta_1}} \left( \theta - (\eta_1 + \eta_2) y - \eta_3 S \right) \right),
\]

and the density for an opponent with posterior \( \tilde{y}^* \) for the club signal is given by

\[
\sqrt{\frac{\delta(\alpha+\gamma)}{\alpha+\gamma+\delta}} \phi \left( \sqrt{\frac{\delta(\alpha+\gamma)}{\alpha+\gamma+\delta}} (S - \tilde{y}^*) \right).
\]
Putting these together with the implicit function $\tilde{\theta}^*(S, \tilde{y}^*)$ defined by (23), the posterior which satisfies opponent indifference is characterized by

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sqrt{\frac{\gamma}{\eta_1}} \phi \left( \sqrt{\frac{\gamma}{\eta_1}} (\theta - \frac{\alpha + \gamma}{\alpha + \gamma + \delta} \tilde{y}^* - \frac{\delta}{\alpha + \gamma + \delta} S) \right) d\theta \sqrt{\frac{\delta(\alpha + \gamma)}{\alpha + \gamma + \delta}} \phi \left( \sqrt{\frac{\delta(\alpha + \gamma)}{\alpha + \gamma + \delta}} (S - \tilde{y}^*) \right) dS = C. \quad (25)$$

As $\tilde{y}^* \to \infty$, $\tilde{\theta}^*(S, \tilde{y}^*) \to \bar{\theta}_\lambda$, and the inner integral of the left-hand side converges to 0, making the left-hand side 0. As $\tilde{y}^* \to -\infty$, $\tilde{\theta}^*(S, \tilde{y}^*) \to \underline{\theta}_\lambda$, and the inner integral of the left-hand side converges to 1, thus the left-hand side is 1. Since the left-hand side is continuous in $\tilde{y}^*$, by the Intermediate Value Theorem, there is a $\tilde{y}^*$ satisfying (25), completing the proof of existence. □

**Proof of Theorem 5:** Proceeding in a manner similar to the baseline model, consider the critical mass condition,

$$w(\tilde{\theta}^*(S), \lambda) + \lambda \Phi \left( \frac{\sqrt{\gamma}}{\kappa_1} \left( (\kappa_1 - 1)\theta^* + \kappa_2 S + \kappa_3 Q - \frac{1}{\sqrt{\alpha + \beta + \delta}} \Phi^{-1}(k) \right) \right) = \Phi \left( \frac{\sqrt{\gamma}}{\eta} \left( \tilde{y}^* - \eta \tilde{\theta}^*(S) - (1 - \eta)Q \right) \right)$$

which rearranging for $\tilde{y}$ we obtain

$$\tilde{y}_{CT}(S) = \eta \tilde{\theta}^*(S) + (1 - \eta)Q + \frac{\eta}{\sqrt{\gamma}} \Phi^{-1} \left( w(\tilde{\theta}^*(S), \lambda) + \lambda \Phi \left( \frac{\sqrt{\gamma}}{\kappa_1} \left( (\kappa_1 - 1)\theta^* + \kappa_2 S + \kappa_3 Q - \frac{1}{\sqrt{\alpha + \beta + \delta}} \Phi^{-1}(k) \right) \right) \right).$$

The posterior which makes an opponent indifferent is the $\tilde{y}_{OI}$ that solves

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sqrt{\frac{\gamma}{\eta_1}} \phi \left( \sqrt{\frac{\gamma}{\eta_1}} (\theta - \frac{\alpha + \gamma}{\alpha + \gamma + \delta} \tilde{y}_{OI}^* - \frac{\delta}{\alpha + \gamma + \delta} S) \right) d\theta \sqrt{\frac{\delta(\alpha + \gamma)}{\alpha + \gamma + \delta}} \phi \left( \sqrt{\frac{\delta(\alpha + \gamma)}{\alpha + \gamma + \delta}} (S - \tilde{y}_{OI}^*) \right) dS = C.$$

By the Implicit Function Theorem, $\tilde{\theta}^*(S)$ is continuously differentiable and hence Fréchet differentiable. By (25) and the Implicit Function Theorem on Banach Spaces (Nijenhuis 1974), $\tilde{y}^*$ is continuously Fréchet differentiable on $C^\infty([\underline{\theta}_\lambda, \bar{\theta}_\lambda])$. Since it is a continuous linear functional it is bounded. By the Hahn-Banach theorem (e.g. Royden 1988, pg. 223) there exists a linear extension $\frac{d\psi}{d\theta}$ to $L_1([\underline{\theta}_\lambda, \bar{\theta}_\lambda])$. Then, by the Riesz Representation
Theorem (e.g. Royden 1988, pg. 284) there exists a unique function \( h \) such that

\[
\frac{d\psi}{d\theta^*} = \int \hat{\theta}^*(S)h(S)dS
\]

for every \( \hat{\theta}^*(S) \). Moreover, the Riesz Representation Theorem also gives that \( \|\frac{d\psi}{d\theta^*}\|_{L_1} = \|h\|_{\infty} \).

The monotone equilibrium is unique if and only if \( \eta \) exists for sufficiently large \( \delta \). Hence, multiplicity obtains for sufficiently large \( \lambda, \alpha \), or \( \delta \).

**Proof of Proposition 5:** Consider the critical mass condition,

\[
w(\hat{\theta}^*(S), \lambda) + \lambda \Phi \left( \frac{\sqrt{\gamma}}{\kappa_1} \left( (\kappa_1 - 1)\hat{\theta}^*(S) + \kappa_2S + \kappa_3Q - \frac{1}{\sqrt{\alpha + \beta + \delta}} \Phi^{-1}(k) \right) \right) = \Phi \left( \frac{\sqrt{\gamma}}{\eta} \left( \hat{y} - \eta \hat{\theta}^*(S) - (1 - \eta)Q \right) \right)
\]

By total differentiation in (27) and slightly abusing notation by calling \( \frac{\partial w(\cdot)}{\partial \theta} = w_{\theta}(\cdot) \),

\[
\frac{d\hat{\theta}^*(S)}{dS} = -\frac{\lambda \Phi \left( \frac{\sqrt{\gamma}}{\kappa_1} \left( (\kappa_1 - 1)\hat{\theta}^*(S) + \kappa_2S + \kappa_3Q - \frac{1}{\sqrt{\alpha + \beta + \delta}} \Phi^{-1}(k) \right) \right) \sqrt{\pi} \delta_{\theta}^{\hat{\theta}^*_S(S)} - w_{\theta}(\theta^*(S), \lambda) + \lambda \Phi \left( \frac{\sqrt{\gamma}}{\kappa_1} \left( (\kappa_1 - 1)\hat{\theta}^*(S) + \kappa_2S + \kappa_3Q - \frac{1}{\sqrt{\alpha + \beta + \delta}} \Phi^{-1}(k) \right) \right) \sqrt{\pi} \delta_{\theta}^{\hat{\theta}^*_S(S)} + \Phi \left( \frac{\sqrt{\gamma}}{\eta} \left( \hat{y} - \eta \hat{\theta}^*(S) - (1 - \eta)Q \right) \right) - \frac{\partial w(\cdot)}{\partial S} dS
\]

Since the numerator is positive, and the denominator is positive when the monotone equilibrium is unique, we have that \( \frac{d\hat{\theta}^*(S)}{dS} < 0 \), meaning that an increase in the club
signal increases the set of regimes that survive.

By total differentiation in (27), we have

\[
\frac{d\hat{\theta}^*(S)}{dQ} - \frac{d\hat{\theta}^*(S)}{dS} = D^{-1}\left(\lambda \sqrt{\beta} \left(\frac{\kappa_1 - \kappa_2}{\kappa_1} \phi \left(\frac{\sqrt{\beta}}{\kappa_1} \left((\kappa_1 - 1)\hat{\theta}^*(S) + \kappa_2 S + \kappa_3 Q - \frac{1}{\sqrt{\alpha + \beta + \gamma}} \Phi^{-1}(k)\right)\right)\right) + \sqrt{\gamma} \phi \left(\frac{\sqrt{\gamma}}{\eta} \left(\hat{y}^* - \eta\hat{\theta}^*(S) - (1 - \eta)Q\right)\right)\right) - \frac{d^2\hat{\theta}^*(S)}{dQ^2}.
\]

Let the denominator be represented by

\[
D = w_0(\hat{\theta}^*(S), \lambda) + \lambda \phi \left(\frac{\sqrt{\beta}}{\kappa_1} \left((\kappa_1 - 1)\hat{\theta}^*(S) + \kappa_2 S + \kappa_3 Q - \frac{1}{\sqrt{\alpha + \beta + \gamma}} \Phi^{-1}(k)\right)\right)
+ \sqrt{\gamma} \phi \left(\frac{\sqrt{\gamma}}{\eta} \left(\hat{y}^* - \eta\hat{\theta}^*(S) - (1 - \eta)Q\right)\right) - \frac{\sqrt{\gamma}}{\eta} \frac{d\hat{y}^*}{d\hat{\theta}^*(S)}.
\]

Taking the difference

\[
\frac{d\hat{\theta}^*(S)}{dQ} - \frac{d\hat{\theta}^*(S)}{dS} = D^{-1}\left(\lambda \sqrt{\beta} \left(\frac{\kappa_1 - \kappa_2}{\kappa_1} \phi \left(\frac{\sqrt{\beta}}{\kappa_1} \left((\kappa_1 - 1)\hat{\theta}^*(S) + \kappa_2 S + \kappa_3 Q - \frac{1}{\sqrt{\alpha + \beta + \gamma}} \Phi^{-1}(k)\right)\right)\right) + \sqrt{\gamma} \phi \left(\frac{\sqrt{\gamma}}{\eta} \left(\hat{y}^* - \eta\hat{\theta}^*(S) - (1 - \eta)Q\right)\right)\right)
and simplifying

\[
\frac{d\hat{\theta}^*(S)}{dQ} - \frac{d\hat{\theta}^*(S)}{dS} = D^{-1}\left(\lambda \sqrt{\beta} \left(\frac{\kappa_1 - \kappa_2}{\kappa_1} \phi \left(\frac{\sqrt{\beta}}{\kappa_1} \left((\kappa_1 - 1)\hat{\theta}^*(S) + \kappa_2 S + \kappa_3 Q - \frac{1}{\sqrt{\alpha + \beta + \gamma}} \Phi^{-1}(k)\right)\right)\right) + \sqrt{\gamma} \phi \left(\frac{\sqrt{\gamma}}{\eta} \left(\hat{y}^* - \eta\hat{\theta}^*(S) - (1 - \eta)Q\right)\right)\right).
\]

The club signal has a larger marginal effect on the critical state if and only if (28) is negative. Observe from (28) that if \(\alpha = \delta\) then \(\frac{d\hat{\theta}^*(S)}{dQ} - \frac{d\hat{\theta}^*(S)}{dS} > 0\), and that if \(\delta\) is sufficiently large then (28) is negative. ■

References


